Math 25 — Activity #1

Instructions: Do **three** of the four problems.

1. **Number of Daylight Hours**

   March 21 is the vernal equinox, the date on which night and day are approximately of equal length. From March 21 to June 21 the number of hours of daylight in Eureka increases to a maximum of about 15. From June 21 until December 21 the number of hours of daylight decreases, reaching 12 again around September 21 (the autumnal equinox) and a minimum of about 9 on December 21. Then the number of hours of daylight starts to increase again.

   (a) Use the information above to find the sinusoidal function that best describes the number of daylight hours for Eureka. Have June 21 = 92 days after March 21, December 21= 275 days after March 21, and recall that there are 365 days in a non-leap year.

   (b) Predict the number of daylight hours for July 30th. (131st day after March 21)

   (c) Which is the first day after June 21 that has 10 hours of daylight?

   (d) Write the equation using January 1st as the starting point.

   (e) Graph the function and label it appropriately.

2. **Tsunami Problem**

   A *tsunami* (commonly called a *tidal wave* because of its effect is like a rapid change in the tide) is a fast moving ocean wave caused by an earthquake. The water first goes down from its normal level and then rises to an equal distance above its normal level, and finally returns to its normal level. In 1964 a deadly tsunami struck Crescent City. The period of the wave was about 30 min. Suppose that the tsunami, which had an amplitude of 6.3 m, approached the boat dock in Crescent City where the normal depth of the water is about 5 m.

   (a) Assuming that the depth of the water varies sinusoidally with time as the tsunami passes the dock, sketch the graph of the water depth as a function of time.

   (b) Write an equation of the graph and predict the depth of the water at the following times after the tsunami first reaches the dock:

      i. 2 min
      ii. 8 min
      iii. 24 min

   (c) According to your model, what will the *minimum* depth of the water be? How do you interpret this answer in terms of what will happen in the real world?

   (d) Between what two times is there *no* water at the dock?

   (e) The *wavelength* of a wave is the distance a crest of the wave travels in one period. It is also equal to the distance between two adjacent crests. If the tsunami travels at 600 km/hr, what is its wavelength?

3. **Bouncing Spring Problem**

   A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stop watch. When the stop watch reads 0.3 sec, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor occurs at 1.8 sec.

   (a) Sketch the graph of this sinusoidal function. (Assume no damping in the spring.)
   (b) Write an equation expressing distance from the floor in terms of the number of seconds the stopwatch reads.
   (c) Predict the distance from the floor when the stopwatch reads 17.2 sec.
   (d) What was the distance from the floor when you started the stopwatch?
   (e) Predict the first positive value of time at which the weight is 59 cm above the floor.

4. **Fox Population Problem**

   Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time $t = 0$ years. A minimum number, 200 foxes, occurred when $t = 2.9$ years. The next maximum, 800 foxes, occurred at $t = 5.1$ years.

   (a) Sketch the graph of the sinusoidal curve.
   (b) Find the amplitude, phase shift, period changes, and translations.
   (c) Write an equation of the graph expressing the number of foxes as a function of time.
   (d) Predict the population at the following times
      i. $t = 7$ years
      ii. $t = 0$ years
      iii. $t = 9.5$ years
   (e) What is the first year in which there are 600 foxes?