

Project 1 — The Quadratic Formula

David Arnold

January 30, 2003

Background

In algebra, the graph of the quadratic function defined by

$$y = ax^2 + bx + c \tag{1}$$

is a parabola. To find the x -intercepts of the parabola, one sets $y = 0$.

$$0 = ax^2 + bx + c \tag{2}$$

You can now proceed to solve this last equation with a technique called *completing the square*. First, divide both sides of the equation by a , then move the constant term to the other side of the equation.

$$\begin{aligned} 0 &= x^2 + \frac{b}{a}x + \frac{c}{a} \\ x^2 + \frac{b}{a}x &= -\frac{c}{a} \end{aligned}$$

Take half of the coefficient of x and square, adding the result to both sides of the equation.

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a} \tag{3}$$

On the left, use the factoring pattern for squaring a binomial. On the right, find a common denominator and add the resulting fractions.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \tag{4}$$

This last equation has two square roots, one positive and one negative.

$$\begin{aligned} x + \frac{b}{2a} &= \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned} \tag{5}$$

This last result is usually called the *quadratic formula* and is used to find the solutions of equation (2). The term under the radical in equation (5) is called the *discriminant*, which we will denote by the variable D . That is, the discriminant is given by the formula

$$D = b^2 - 4ac. \tag{6}$$

Thinking geometrically, there are three cases to consider.

1. The parabola cuts the x -axis twice, leaving two x intercepts.
2. The parabola is tangent to the x -axis, in which case there is exactly one x -intercept.
3. The parabola lies entirely above or below the x -axis, in which case there are no x -intercepts.

Each of these cases is revealed in the quadratic formula (5). Each case is completely determined by the value of the discriminant, $D = b^2 - 4ac$.

1. If $b^2 - 4ac > 0$, then the equation yields two answers. That is, if the discriminant $D > 0$, there are two solutions. This corresponds to the item above where the parabola has two x -intercepts.
2. If $b^2 - 4ac = 0$, then the equation yields a single answer. That is, if the discriminant $D = 0$, there is exactly one solution. This corresponds to the item above where the parabola is tangent to the x -axis and has exactly one x -intercept.
3. If $b^2 - 4ac < 0$, then the equation has no real answers. That is, if the discriminant $D < 0$, then there are no real solutions. This corresponds to the item above where the parabola lies completely above or below the x -axis and has no x -intercepts.

The Program

You are to write a program that will report the solutions to an arbitrary quadratic equation having the form of equation (2). Make your program interactive, allowing the user to enter the coefficients a , b , and c of equation (2) at the terminal. Your program should then report back the solutions of equation (2) using the quadratic formula given in equation (5).

Note that your program should use the value of the discriminant to decide which of the three cases is present before reporting the solutions. In the case where there are no real solutions, your program should report this fact to the user; i.e., your program should report that “there are no real solutions.”

The Grading Rubric

The following rules will apply for this program, after which we will discuss and adjust the rubric during class.

1. (30 points) Will be awarded for adequate comments. Comments should include:
 - (a) A description of the program’s purpose.
 - (b) Name, date, version or revision number.
 - (c) A complete dictionary of all variables and parameters used in the program.
 - (d) Interprogram comments should proceed any code snippets explained by the comments. These should be adequately sprinkled throughout your code.
2. (50 points) Will be awarded if the program works and does what it was asked to do.
3. (10 points) Will be awarded for good program style. This includes good indentation practices, etc.
4. (10 points) Will be awarded for creativity and extra effort. Did you just do the bare minimum? Or did you stretch and reach a little higher? Did you put something cute or clever into your program that nobody else seemed to think of?

Penalties

Each program that is assigned during the term will have a due date. On that date, the program must be on the instructor’s desk before the start of class. Penalties will be assessed as follows.

1. (10 points) There will be a 10 point deduction for any program that is handed in after the class has begun.
2. (20 points) There will be a 20 point deduction per class period. That is, if you hand the program in one class period late, there is an automatic 20 point deduction. Two class periods warrants a 40 point deduction, etc. To be clear, if the program is in the instructor’s hands before the beginning of the next class, that is a 20 point deduction. If the program is in the instructor’s hands before the start of the second class period past the due date, that is a 40 point deduction, etc.

Managing Files and Folders

Each of you has been given personal space on the sci-math server to store your work, `/home/loginname`.¹ In our last class, you created a `fortran` subdirectory in your home directory. For this project, create a directory called `program1` in the `fortran` directory, then save your program as `quadratic.f90`.² When you receive your next project, create a new folder called `Program2` to hold that project, etc.

If you work at home, I still want you to place copies of your work in the space reserved for you on our system. Simply copy your home files onto a floppy disk and bring them with you to school. Use the Windows Explorer to copy the files on your disk into the proper folder; that is, `/home/loginname/fortran/program1/`.³

If everyone follows these simple rules, I can easily access your work from my office machine for purposes of assigning a grade.

Caveat

On this project, if you stop by my office with hardcopy of your program before the due date of this assignment, I will give a quick glance and critique of your source code. Somewhat like receiving a grade on a draft before submitting your final draft for assessment.

¹In windows, this space is mapped to the drive letter H. If you open the Windows Explorer (the file manager, not the internet browser), you can see that the drive letter has been mapped to your login name.

²Note that you must **never** use spaces in filenames. In the Windows operating system, filenames are not case-sensitive, which is exactly opposite to what happens in Unix and Linux, where filenames are case-sensitive.

³If you work in windows, then `H:\FortranPrograms\Program1`.