Project 2 — Amortization Tables

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1 Introduction

In this activity you will create what is known as an *Amortization Table*. The idea is simple. You buy a car or a house. Based on the amount of the loan and the interest rate, you then compute your monthly payment. You then create a table which details the progress of the loan. Columns in the table track several important categories:

1. the payment number,
2. the amount of the payment credited to lowering the principal balance,
3. the amount of the payment goes for interest,
4. the cumulative payment amount toward the principal balance,
5. the cumulative interest, and
6. the principal balance.

The rows of the table list these details for each payment made. For example, suppose that

1. the amount of the loan is $100,000,
2. the yearly rate is 8%,
3. the payments are monthly, and
4. the loan is over a period of 30 years (360 monthly payments).

A simple mathematical formula determines that the monthly payments are $733.76. Here are a few rows of the amortization table for this loan.

Payment	**Principal**	**Interest**	**Cum Prin**	**Cum Int**	**Prin Bal**
1	67.09	666.67	67.09	666.67	99932.91
2	67.54	666.22	134.63	1332.89	99865.37
3	67.99	665.77	202.62	1998.66	99797.38

Table 1: Amortization Table.

Your job in this activity will be to produce all 360 rows of the amortization table. Of course, if you perform the task correctly, the principal balance at the end should total $0.00.

2 Mathematical Background

Of course, you need to know a little mathematics before your begin your project. We start with a little lesson on compound interest.

2.1 Compound Interest

Suppose that you invest \$1,000 at 8% yearly interest, compounded quarterly. This means that the interest on the investment is computed four times per year. Of course, you do not get the full 8% each compounding period. Rather, you are awarded 8%/4, or 2% each compounding period. This means that at the end of each compounding period, your investment is increased by 102%.

Let $A(k)$ represent the amount of the investment after k compounding periods. Because the initial investment is \$1,000, the balance after zero compounding periods is \$1,000. In symbols,

$$A(0) = 1,000.$$

At the end of one compounding period, the balance increases by 102%. That is, the balance at the end of one compounding period is 102% of the balance at the end of zero compounding periods. In symbols,

$$A(1) = \left(1 + \frac{0.08}{4}\right) A(0) = \left(1 + \frac{0.08}{4}\right) 1000.$$

At the end of two compounding periods, the balance will be 102% ofthe balance at the end of one compounding period. In symbols,

$$A(2) = \left(1 + \frac{0.08}{4}\right) A(1) = \left(1 + \frac{0.08}{4}\right)\left(1 + \frac{0.08}{4}\right) 1,000 = \left(1 + \frac{0.08}{4}\right)^2 1,000.$$

By induction, the balance after k compounding periods is given by

$$A(k) = \left(1 + \frac{0.08}{4}\right)^k 1,000.$$

There are four compounding periods each year. If the money is allow to grow for t years, then that will be exactly $k = 4t$ compounding periods. This allows us to right the balance as a function of time instead of a function of the number of compounding periods.

$$A(t) = 1,000\left(1 + \frac{0.08}{4}\right)^{4t}$$

It is traditional to place the principal at the front of the expression as shown above.

Consider the following table of variable assignments.

A	Balance after t years
P	Principal investment
r	Yearly interest rate
n	Number of compounding periods per year
t	Length of the investment in years

With these assignments, it's easy to generalize the results of our arguments above. Indeed, the balance after t years is given by the equation

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

2.2 Geometric Series

Consider the following series of numbers.

$$S = 1 + 2 + 4 + 8 + 16 + 32 \tag{1}$$

Note that each successive term is a scalar multiple of the term that proceeds it. Indeed, in the case of the series above, we move from term to term by multiplying by 2. In this case, we say that the *common ratio* is 2.

As a young child, Gauss multiplied both sides of equation (1) by the common ratio.

$$2S = 2 + 4 + 8 + 16 + 32 + 64 \tag{2}$$

Then he subtracted equation (2) from equation (1) to get

$$-S = 1 - 64.$$

Note how most of the terms cancel one another in this subtraction process, leading to the sum $S = 63$. The careful reader will check that this is the correct sum.

In general, a finite geometric series has the form

$$S = a + ar + ar^2 + \cdots + ar^{n-1}. \tag{3}$$

The variables in this sum and their meanings are summarized in the following table.

S	The sum of the series
a	The first term of the series
r	The common ratio
n	The number of terms in the series

Like Gauss, we multiply equation (3) by the common ratio r.

$$rS = ar + ar^2 + ar^3 + \cdots + ar^n \tag{4}$$

We then subtract equation (4) from equation (3) to get

$$\begin{aligned} S - rS &= a - ar^n \\ (1-r)S &= a(1-r^n) \\ S &= \frac{a(1-r^n)}{1-r}. \end{aligned} \tag{5}$$

This is the formula we will use for summing a finite geometric series. For example, if we use formula (5) on series (1), we arrive at the following result.

$$\begin{aligned} S &= \frac{a(1-r^n)}{1-r}. \\ &= \frac{1(1-2^6)}{1-2} \\ &= 63 \end{aligned}$$

Note that this agrees nicely with our previous calculation.

2.3 Calculating the Payment

Next, we must familiarize ourselves with the concepts of *present* and *future value* of our money. For example, if you make a payment of \$100 on the first of January, the future value of that payment will be greater than \$100 on the first of February. Indeed, if the yearly interest rate is 8% and interest is compounded monthly, then the value of the payment on first of February is $\$100(1 + 0.08/12)$. The value of the payment on the first of March is $\$100(1 + 0.08/12)^2$, and so on.

In general, if you make a payment of P dollars every month, on an amount borrowed at a yearly rate r compounded monthly, then the future value of the payment is given by $P(1 + r/12)^k$, where k is the number of months that the payment is pushed forward in time. To simplify notation, let the variable $i = r/12$. Then the future value of the payment is $P(1 + i)^n$.

Let's suppose that you borrow $A = \$1,000$ at 8% compounded monthly and you make monthly payments of P dollars. Letting $i = 0.08/12$, then the future value of each payment is listed in the following 12-month payment schedule.

1	2	3	$\cdots$	12
P	$P(1+i)$	$P(1+i)^2$	$\cdots$	$P(1+i)^{11}$
	P	$P(1+i)$	$\cdots$	$P(1+i)^{10}$
				$\vdots$
				P

This table warrants some explanation. A payment P is made at the end of the first month of the loan. At the end of the twelfth month of the loan, the future value of the payment is $P(1+i)^{11}$, as it is pushed forward 11 months in time. Similarly, the payment made at the end of the second month has future value $P(1+i)^{10}$ at the end of the twelfth month, and so on. Finally, one final payment is made at the end of the twelfth month, for a total of 12 payments on the year.

Thus, if you sum up the values in the last column, then you have the total future value of all 12 payments.

$$S = P + P(1+i) + P(1+i)^2 + \cdots + P(1+i)^{10} + P(1+i)^{11} \tag{6}$$

Now, the original amount of the loan was $A = \$1,000$. One month later, you make your first payment. Thus, at the end of the twelfth month, the loan amount is pushed forward 12 months and has future value $\$1,000(1+i)^{12}$. This money must be equal to the total in equation (6) because we are comparing the future values of both monies at the same date (at the end of the tewlfth month). Therefore, we can write

$$1000(1+i)^{12} = P + P(1+i) + P(1+i)^2 + \cdots + P(1+i)^{10} + P(1+i)^{11}. \tag{7}$$

This equation can be solved for the monthly payment P. In general, we can replace the loan amount with the variable A and the number of months by n and write

$$A(1+i)^n = P + P(1+i) + P(1+i)^2 + \cdots + P(1+i)^{n-2} + P(1+i)^{n-1}. \tag{8}$$

Next, note that the series on the right of equation (8) is geometric, so we can use equation (5) to find its sum.

$$\begin{aligned} A(1+i)^n &= \frac{P(1-(1+i)^n)}{1-(1+i)} \\ A(1+i)^n &= \frac{P((1+i)^n-1)}{i} \\ P &= \frac{Ai(1+i)^n}{(1+i)^n-1} \end{aligned}$$

If we divide top and both of the fraction on the right by $(1+i)^n$, we arrive at our final result for calculating the monthly loan payment.

$$P = \frac{Ai}{1-(1+i)^{-n}} \tag{9}$$

The calculations in Table 1 are based on a monthly payment of $P = \$733.76$. The amount of the loan was $\$100,000$, the yearly rate 8% compounded monthly, and the time of the loan is 360 months. Let's use formula (9) to verify the value of the loan payment. First, note that

$$i = \frac{r}{12} = \frac{0.08}{12} \approx 0.006667. \tag{10}$$

Next,

$$\begin{aligned} P &= \frac{Ai}{1-(1+i)^{-n}} \\ P &= \frac{100,000(0.006667)}{1-(1+0.006667)^{-360}} \\ P &\approx \$733.76. \end{aligned}$$

That's pretty good agreement!

3 The Program

You are to write a program that will complete the amortization table started for you in Table 1. Assume a loan amount $A = \$100,000$ and a yearly interest rate $r = 8\%$ compounded monthly. Also, assume that the duration of the loan runs for 360 months.

Your program should:

1. calculate and report the monthly loan payment, and
2. complete the amortization table shown in Table 1, including records for each of the 360 payments.

The amortization table should be formatted nicely with carefully planned use of FORMAT statements in your program. Be sure to include headers for each column of your table.

In addition, your program should output your results to a file, then you are to print that file using Emacs. You are to turn in the following:

1. a hardcopy printing of your program,
2. an electronic form of your program,
3. a hardcopy printing of your amortization table, and
4. an electronic copy of the file holding the amortization table.

The electronic versions are to be stored on your H drive at school, as described below. I will access these files from my office and access a grade based on their contents.

4 The Grading Rubric

Consider this a first draft of the rubric that will be used to grade your programs throughout the term. The following rules will apply for this first program, after which we will discuss and adjust the rubric during class.

1. (30 points) Will be awarded for adequate comments. Comments should include:
   (a) A description of the program's purpose.
   (b) Name, date, version or revision number.
   (c) A complete dictionary of all variables and parameters used in the program.
   (d) Interprogram comments should proceed any code snippets explained by the comments. These should be adequately sprinkled throughout your code.
2. (50 points) Will be awarded if the program works and does what it was asked to do. This grade will include an assessment of the electronic version of your amortization table save on a file in your personal workspace at school.
3. (10 points) Will be awarded for good program style. This includes good indentation practices, etc.
4. (10 points) Will be awarded for creativity and extra effort. Did you just do the bare minimum? Or did you stretch and reach a little higher? Did you put something cute or clever into your program that nobody else seemed to think of?

5 Penalties

Each program that is assigned during the term will have a due date. On that date, the program must be on the instructor's desk before the start of class. Penalties will be assessed as follows.

1. (10 points) There will be a 10 point deduction for any program that is handed in after the class has begun.
2. (20 points) There will be a 20 point deduction per class period. That is, if you hand the program in one class period late, there is an automatic 20 point deduction. Two class periods warrants a 40 point deduction, etc. To be clear, if the program is in the instructor's hands before the beginning of the next class, that is a 20 point deduction. If the program is in the instructor's hands before the start of the second class period past the due date, that is a 40 point deduction, etc.

6 Managing Files and Folders

Each of you has been given personal space on the sci-math server to store your work. Typically, this space is mapped to the drive letter H. If you open the Windows Explorer (the file manager, not the internet browser), you can see that the drive letter has been mapped to your login name.

In this folder, create a new folder call FortranPrograms. Note that you must **never** use spaces in filenames[1]. In the Windows operating system, filenames are not case-sensitive, which is exactly opposite what happens in Unix and Linux, where filenames are case-sensitive.

In your FortranPrograms folder, create another folder called Program2. It is in this folder that you are to place the source code and executables for this current project. When you receive your next project, create a new folder called Program3 to hold that project, etc.

If you work at home, I still want you to place copies of your work in the space reserved for you on our system. Simply copy your home files onto a floppy disk and bring them with you to school. Use the Windows Explorer to copy the files on your disk into the proper folder (H:/FortranPrograms/Program2).

If everyone follows these simple rules, I can easily access your work from my office machine for purposes of assigning a grade.

7 Caveat

On this project, if you stop by my office with hardcopy of your program before the due date of this assignment, I will give a quick glance and critique of your source code. Somewhat like receiving a grade on a draft before submitting your final draft for assessment.

[1] MacIntosh users are particularly susceptible to this disease