College of the Redwoods
Mathematics Department

Math 55 — Differential Equations
Quiz #3

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Quiz Questions

Read Carefully! *You have until Monday of next week (4/7/08) to complete the quiz. The quiz is due, in my hands, at the beginning of class.*

This quiz is open notes, open book. This includes any supplementary texts or online documents. You must answer all of the exercises on your own. You are not allowed to work in groups or pairs on the quiz. You are not allowed to enlist the aid of a tutor or friend to help with the quiz. You are not allowed to read the exercises in the quiz, then seek help on similar questions. Once you open the quiz and read the questions, you may not seek any outside help of any kind.

I am not interested in reading pages and pages of calculations without accompanying narrative. It is essential that you include sound mathematical writing that both explains and justifies your solution or proof. Grammar and punctuation are important, as is the organization of your solution on the written page.

When working in the lab, please do not work at a terminal next to any other student who is also working on the quiz. For the sake of propriety, please separate yourselves when working on the quiz in the lab.

Place the solution to each exercise on a separate sheet of paper. On a good sheet of paper, write out (longhand) and sign the following honor pledge.

> I promise that all work found herein is my own. I have received no help from tutors, colleagues, or other teachers. I have honored all of the quiz constraints listed in the directions.

Arrange your solutions in order, place these quiz page(s) on top of your solutions, then place the honor pledge on top of the quiz as a cover sheet. Staple. Good luck!

Exercise 1. The augmented matrix for a system of linear equations, in reduced row echelon form, is given by

$$\begin{bmatrix} 1 & 0 & 2 & 4 & 1 \\ 0 & 1 & -2 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Without doing any work, state a *particular* solution of the system $A\mathbf{x} = \mathbf{b}$.

(b) Without doing any work, state the special solutions, i.e. state a basis, for the system $A\mathbf{x} = \mathbf{0}$.

(c) Without doing any work, state the general solution of the system $A\mathbf{x} = \mathbf{b}$ in parametric form.

Exercise 2. Consider the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{v}_3 = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}.$$

Determine whether the vectors are independent or dependent. If the vectors are dependent, state a specific linear combination of the vectors that equals the zero vector. Use Matlab to perform any calculations.

Exercise 3. Given the matrix

$$A = \begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix},$$

solve the equation

$$\det(A - \lambda I) = 0$$

for λ. Hand calculations only. Show all of your work!

Exercise 4. The system

$$x' = x(4 - 2x) - xy$$
$$y' = y(4 - 2y) - xy$$

models two populations x and y that are competing for resources. This type of model is known as a *competititon model*.

(a) By hand, compute the x- and y-nullclines. On a sheet of graph paper, set up an xy-coordinate system. Sketch the x-nullclines in red, the y-nullclines in blue.

(b) By hand, compute the equilibrium points for the system. Plot and label these on the xy-coordinate system on your graph paper.

(c) Use **pplane** to sketch the system, the nullclines, and the equilbrium points. Use the text capabilities of **pplane** to label the equilibrium points with their coordinates.

(d) Use **pplane** to sketch a few solution trajectories. Include a printout with your examination papers.

(e) Write a short description of what happens to each population regardless of their initial population size.

Exercise 5. Consider the linear system

$$\mathbf{x}' = \begin{bmatrix} 7 & 10 \\ -5 & -8 \end{bmatrix} \mathbf{x}.$$

(a) Use matrix calculations (hand calculations only) to show that

$$\mathbf{x}_1(t) = e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \text{and} \qquad \mathbf{x}_2(t) = e^{2t} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

are solutions of the system.

(b) Find a solution of the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 7 & 10 \\ -5 & -8 \end{bmatrix} \mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$