(1 pt) $y' = y(y-5), \quad y(0) = 1$

Separate variables:

$$\frac{dy}{y(y-5)} = dt$$

Partial fraction decomposition:

$$\frac{1}{y(y-5)} = \frac{A}{y} + \frac{B}{y-5}$$

$$1 = A(y-5) + By$$

$\Rightarrow$ $y=0 \Rightarrow 1 = -5A$, $A = -\frac{1}{5}$

$y=5 \Rightarrow 1 = 5B$, $B = \frac{1}{5}$

Thus,

$$\left[\frac{-1/5}{y} + \frac{1/5}{y-5}\right] dy = dt$$

$$\left[\frac{1}{y} - \frac{1}{y-5}\right] dy = -5\,dt \quad \leftarrow \text{integrate}$$

$$\ln y - \ln(y-5) = -5t + C$$

$$\ln \frac{y}{y-5} = -5t + C$$

Exponentiate:

$$\frac{y}{y-5} = e^{-5t+C}$$

$$\frac{y}{y-5} = Ce^{-5t} \quad \leftarrow \quad e^C = C$$

Solve for C:

$$y(0) = 1$$

$$\frac{1}{1-5} = Ce^{-5(0)}$$

$$C = -\frac{1}{4}$$

Solve for y:

$$\frac{y}{y-5} = -\frac{1}{4}e^{-5t}$$

$$4y = (5-y)e^{-5t}$$

$$4y = 5e^{-5t} - ye^{-5t}$$

$$(4+e^{-5t})y = 5e^{-5t}$$

$$y = \frac{5e^{-5t}}{4+e^{-5t}}$$ ← Multiply top & bottom by e^{5t}

$$y = \frac{5}{1+4e^{5t}}$$

(2)

10pts $y' - y = \cos t$, $y(0) = 1$

Integrating factor: $\mu = e^{\int -1\,dt} = e^{-t}$

$$e^{-t}y' - e^{-t}y = e^{-t}\cos t$$

$$(e^{-t}y)' = e^{-t}\cos t$$

$$e^{-t}y = \int e^{-t}\cos t\,dt$$

Integrate by parts:

	D	I
+	e^{-t}	$\cos t$
−	$-e^{-t}$	$\sin t$
+	e^{-t}	$-\cos t$

$$\int e^{-t}\cos t\,dt$$

$$= e^{-t}\sin t - e^{-t}\cos t - \int e^{-t}\cos t\,dt$$

Hence, $2\int e^{-t}\cos t\,dt = e^{-t}\sin t - e^{-t}\cos t + C$

$$\int e^{-t}\cos t\,dt = \frac{1}{2}e^{-t}(\sin t - \cos t) + C$$

Thus,

$$e^{-t}y = \frac{1}{2}e^{-t}(\sin t - \cos t) + c$$

$$y = \frac{1}{2}(\sin t - \cos t) + ce^{t}$$

But $y(0) = 1$, so:

$$1 = \frac{1}{2}(\sin(0) - \cos(0)) + ce^{0}$$

$$1 = -\frac{1}{2} + c$$

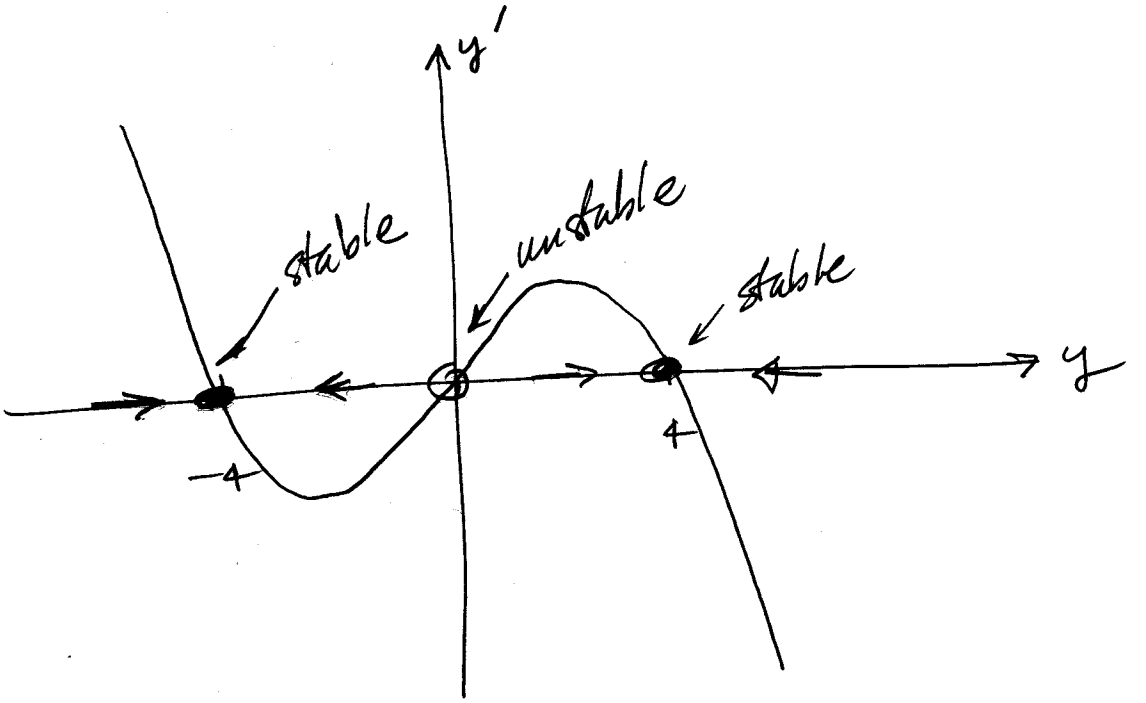
$$c = 3/2$$

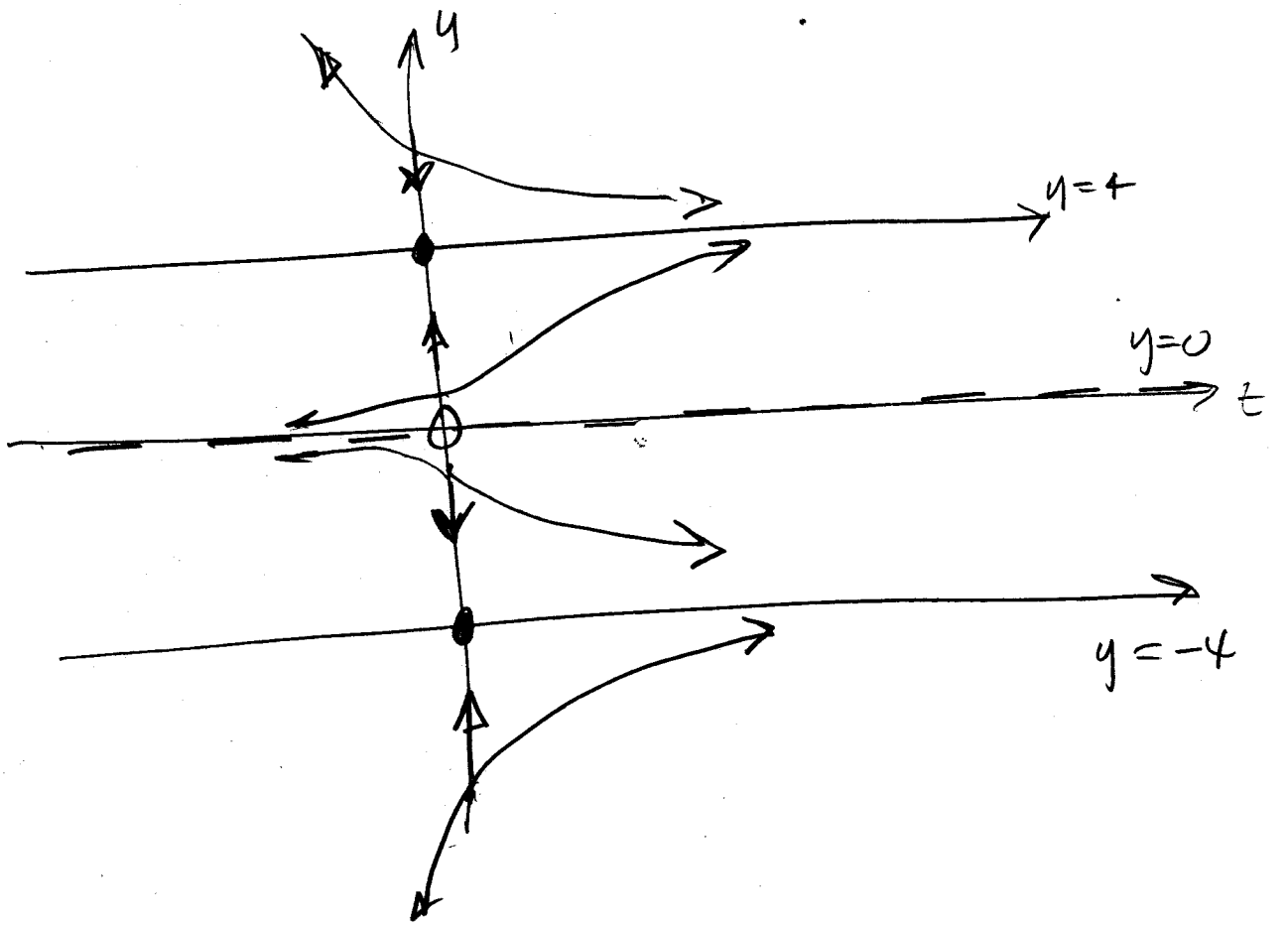
Hence:

$$y = \frac{1}{2}(\sin t - \cos t) + \frac{3}{2}e^{t}.$$

(3) 10pts

$$y' = 16y - y^3 = y(4+y)(4-y)$$





④ 10pts

$c' + 0.52c = 0.02$

$c' = 0.02 - 0.52c$

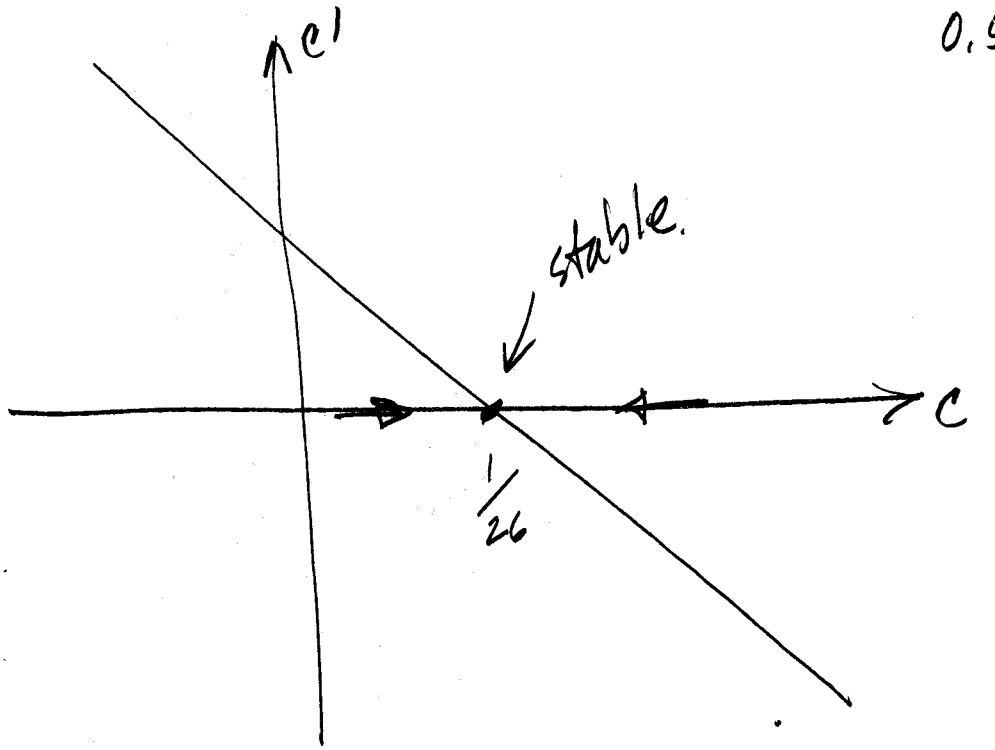
$0.02 - 0.52c = 0$

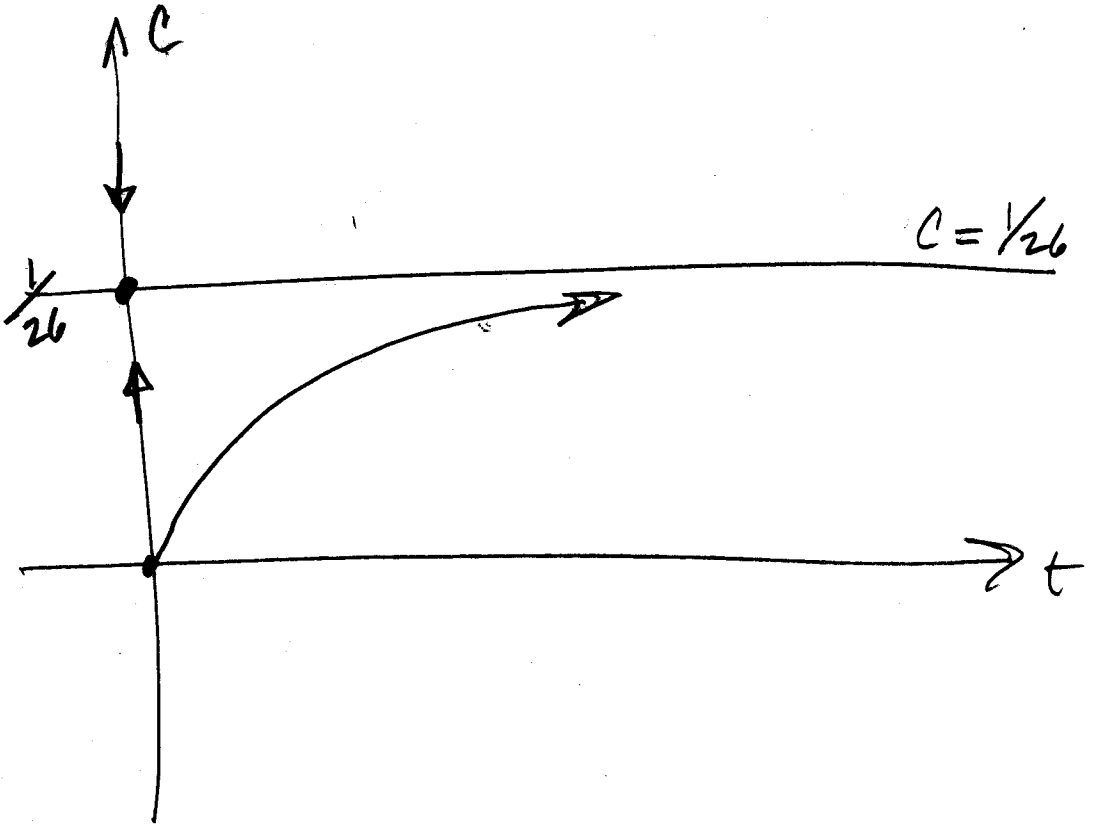
$0.52c = 0.02$

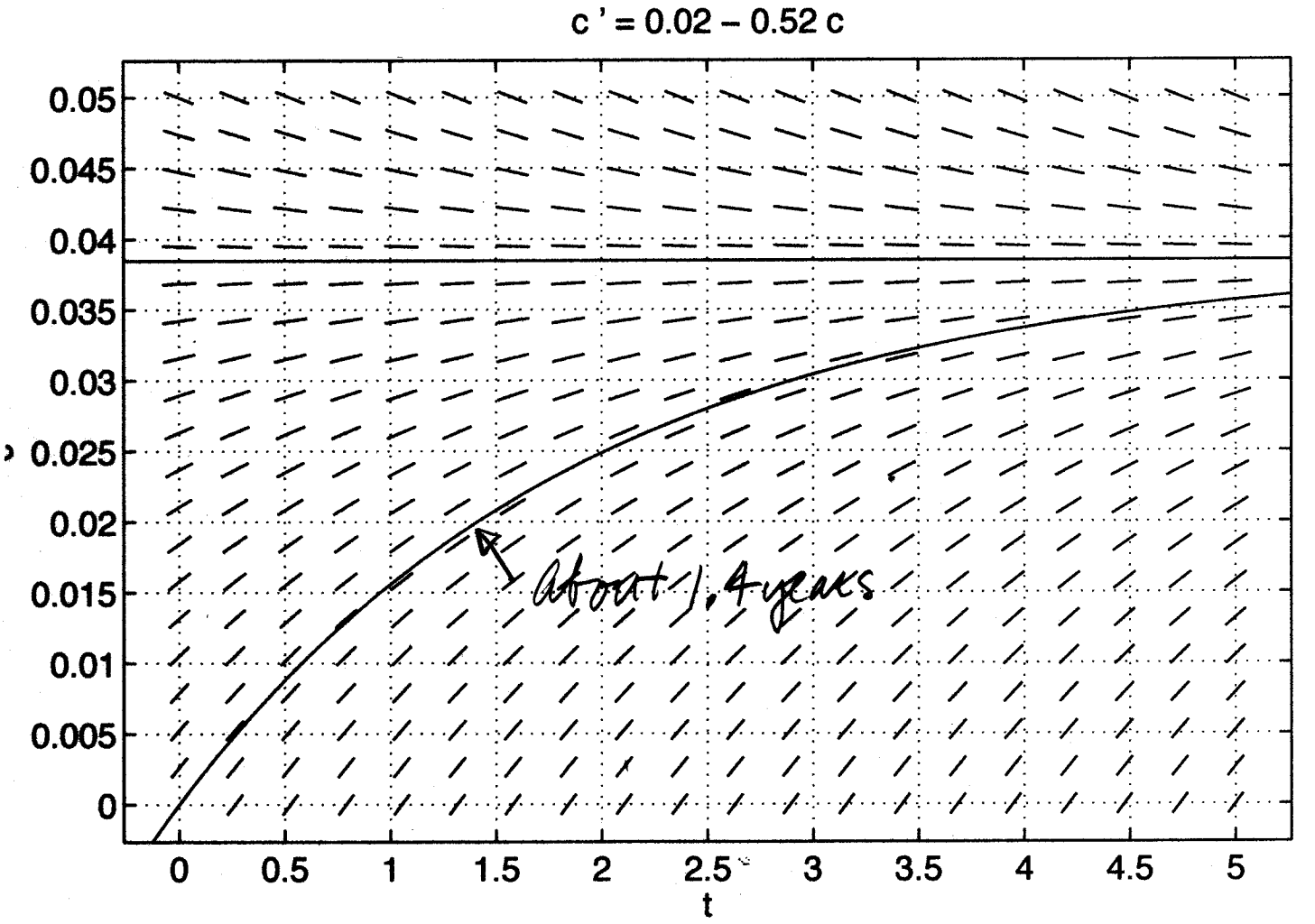
$c = \frac{2}{52}$

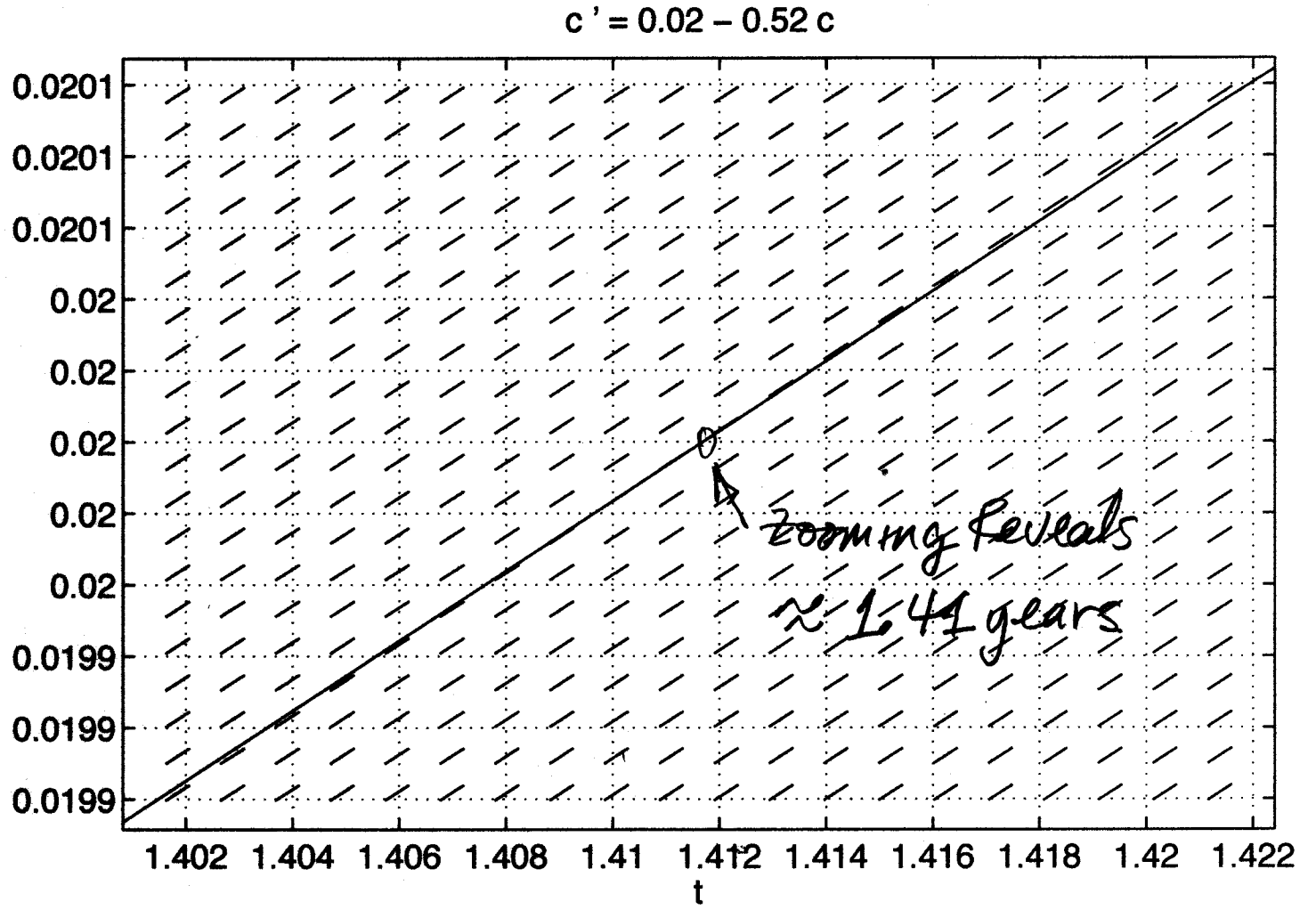
$c = \frac{1}{26}$

$c \approx 0.0385$






c ' = 0.02 – 0.52 c
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t
about 1.4 years


c ' = 0.02 – 0.52 c
0.0201
0.0201
0.0201
0.02
0.02
0.02
0.02
0.02
0.0199
0.0199
0.0199
1.402
1.404
1.406
1.408
1.41
1.412
1.414
1.416
1.418
1.42
1.422
t
zooming reveals
≈ 1.41 years