

1 Difference Quotients in Review

Here is a little more practice on the difference quotient. First, another example.

Example 1

If $f(x) = x/(x + 5)$, then simplify

$$\frac{f(x) - f(2)}{x - 2}.$$

Solution. If

$$f(x) = \frac{x}{x + 5},$$

then

$$\begin{aligned} \frac{f(x) - f(2)}{x - 2} &= \frac{\frac{x}{x+5} - \frac{2}{7}}{x - 2} \\ &= \frac{\frac{x}{x+5} - \frac{2}{7}}{x - 2} \cdot \frac{7(x + 5)}{7(x + 5)} \\ &= \frac{\left(\frac{x}{x+5}\right) 7(x + 5) - \left(\frac{2}{7}\right) 7(x + 5)}{7(x - 2)(x + 5)} \\ &= \frac{7x - 2(x + 5)}{7(x - 2)(x + 5)} \\ &= \frac{7x - 2x - 10}{7(x - 2)(x + 5)} \\ &= \frac{5x - 10}{7(x - 2)(x + 5)} \\ &= \frac{5(x - 2)}{7(x - 2)(x + 5)} \\ &= \frac{5}{7(x + 5)} \end{aligned}$$

Of course, this argument is valid only if $x \neq 2, -5$.

2 Exercises

For each of the following functions, find

$$\frac{f(x) - f(2)}{x - 2}.$$

In each case, state the values of x for which your argument is not valid.

1. $f(x) = 2x^2 + 3x + 4$

Solution: $2x + 7$

2. $f(x) = 3x^2 - 5x - 8$

Solution: $3x + 1$

3. $f(x) = \frac{x}{x+3}$

Solution: $\frac{3}{5(x+3)}$

4. $f(x) = \frac{x+2}{x-5}$

Solution: $\frac{7}{3(x-5)}$