

Absolute Value and Piecewise Functions
College Algebra

Exercise. Consider the function

$$f(x) = |x + 2| + |x - 4|. \tag{1}$$

Write a piecewise definition for the function f and sketch the graph.

Solution. Recall the basic definition of absolute value.

$$|x| = \begin{cases} x, & \text{if } x \geq 0; \\ -x, & \text{if } x < 0. \end{cases}$$

It is useful to summarize this definition on a number line.

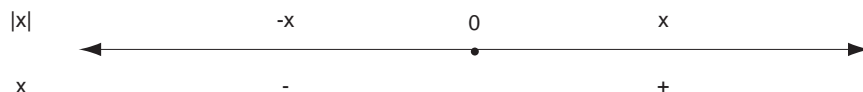


Figure 1

Now, back to equation (1). If x is a number less than -2 , then $x + 2$ is a negative number and $|x + 2| = -(x + 2)$. If x is a number greater than or equal to -2 , then $x + 2$ is positive or zero (non-negative) and $|x + 2| = x + 2$. We can summarize these statements on a number line.

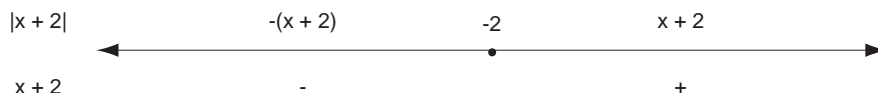


Figure 2

If x is a number less than 4 , then $x - 4$ is a negative number and $|x - 4| = -(x - 4)$. If x is a number greater than or equal to 4 , then $x - 4$ is positive or zero (non-negative) and $|x - 4| = x - 4$. We can summarize these statements on a number line.

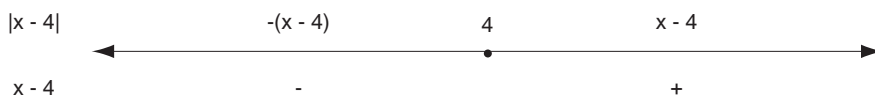


Figure 3

In fact, we can save space by combining the two previous number lines.

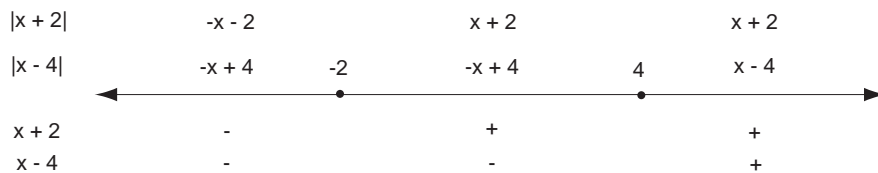


Figure 4

We can now make a piecewise definition for equation (1) by simply adding the the equations in each region in Figure 4, as follows:

$$f(x) = \begin{cases} -2x + 2 & \text{if } x < -2; \\ 6 & \text{if } -2 \leq x < 4; \\ 2x - 2 & \text{if } x \geq 4. \end{cases} \tag{2}$$

We can now graph each piece defined in equation (2).

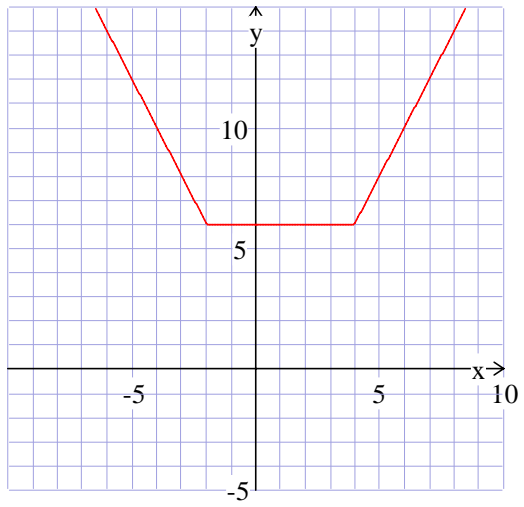


Figure 5