College Algebra Pretest #4 Solutions

1. Place the system of equations in augmented matrix form, reduce to *row echelon form*, and use back substitution to solve.

a.
$$x + 2y = 4$$
$$3x - y = 6$$

$$\begin{pmatrix} 1 & 2 & 4 \\ 3 & -1 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 4 \\ 0 & -7 & -6 \end{pmatrix} \; -3R_1 + R_2$$

$$= \begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & \frac{6}{7} \end{pmatrix} \; (-1/7)R_1$$

$$x + 2y = 4$$
$$y = \tfrac{6}{7}$$

$$x + 2\left(\frac{6}{7}\right) = 4$$
$$x = \frac{16}{7}$$

Solution is : $\{x = \frac{16}{7}, y = \frac{6}{7}, \}$

b.
$$x + 2y - z = 4$$
$$y + 2z = 8$$
$$2x - y - 3z = 9$$

$$\begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 2 & 8 \\ 2 & -1 & -3 & 9 \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 2 & 8 \\ 0 & -5 & -1 & 1 \end{pmatrix} \; -2R_1 + R_3$$

$$= \begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 9 & 41 \end{pmatrix} \; 5R_2 + R_3$$

$$= \begin{pmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & \frac{41}{9} \end{pmatrix} \; (1/9)R_3$$

$$x + 2y - z = 4$$
$$y + 2z = 8$$
$$z = \tfrac{41}{9}$$

$$y = 8 - 2\left(\frac{41}{9}\right) = -\frac{10}{9}$$
$$x = 4 - 2\left(-\frac{10}{9}\right) + \frac{41}{9} = \frac{97}{9}$$

Solution is : $\left\{x = \frac{97}{9}, y = -\frac{10}{9}, z = \frac{41}{9}\right\}$

c.
$$\begin{aligned} x + y - 2z &= 4 \\ 2x - y + z &= 8 \\ 3x \qquad - z &= 12 \end{aligned}$$

$$\begin{pmatrix} 1 & 1 & -2 & 4 \\ 2 & -1 & 1 & 8 \\ 3 & 0 & -1 & 12 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -2 & 4 \\ 0 & -3 & 5 & 0 \\ 0 & -3 & 5 & 0 \end{pmatrix} \begin{matrix} \\ -2R_1 + R_2 \\ -3R_1 + R_3 \end{matrix}$$

$$= \begin{pmatrix} 1 & 1 & -2 & 4 \\ 0 & 1 & -\frac{5}{3} & 0 \\ 0 & -3 & 5 & 0 \end{pmatrix} (-1/3)R_2$$

$$= \begin{pmatrix} 1 & 1 & -2 & 4 \\ 0 & 1 & -\frac{5}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} R_2 + R_3$$

$x + y - 2z = 4$

$y - \frac{5}{3}z = 0$

z is the free variable. Let $z = t$.

$$y = \frac{5}{3}t$$
$$x = 4 - \left(\frac{5}{3}t\right) + 2(t) = \frac{1}{3}t + 4$$

Solution is : $\left\{x = \frac{1}{3}t + 4,\ y = \frac{5}{3}t,\ z = t\right\}$

2.

	Peanuts	Cashews	Mixed
Price/pound	\$1.75	\$3.25	\$2.05

Let P = the number of pounds of peanuts and C = the number of pound cashews.

$P + C = 25$

$1.75P + 3.25C = 2.05 * 25$

$$\begin{pmatrix} 1 & 1 & 25 \\ 1.75 & 3.25 & 51.25 \end{pmatrix}$$

Using the calculator to obtain *reduced row echelon form* gives

$$\begin{pmatrix} 1 & 0 & 20 \\ 0 & 1 & 5 \end{pmatrix}$$

The mix contains 20 pounds of peanuts and 5 pounds of cashews.

3.

	Mutual Fund	C.D.
Interest rate	5%	6%

Let M = the amount of the money in the mutual fund and C = the amount of the money in the certificate of deposit.

$M + C = 11,000$

$0.05M + 0.06C = 623$

$$\begin{pmatrix} 1 & 1 & 11,000 \\ 0.05 & 0.06 & 623 \end{pmatrix}$$

Using the calculator to obtain *reduced row echelon form* gives

$$\begin{pmatrix} 1 & 0 & 3700 \\ 0 & 1 & 7300 \end{pmatrix}$$

Jamal invested $3700 in the mutual fund and $7300 in the certificate of deposit.

4. Let N = the number of nickels, D = the number of dimes, and Q = the number of quarters.

$N + D + Q = 41$

$0.05N + 0.1D + 0.25Q = 6.10$

$Q = D + 4$

$$\begin{pmatrix} 1 & 1 & 1 & 41 \\ .05 & .1 & .25 & 6.10 \\ 0 & -1 & 1 & 4 \end{pmatrix}$$

Using the calculator to obtain *reduced row echelon form* gives

$$\begin{pmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 1 & 17 \end{pmatrix}$$

Jamie has 11 nickels, 13 dimes, and 17 quarters in her pocket. deposit.

5. Let T = the amount invested at 12%, H = the amount invested at 13%, and F = the amount invested at 14.5%.

$T + H + F = 40,000$

$0.12T + 0.13H + 0.145F = 5370$

$0.145F = 0.13H + 1050$

$$\begin{pmatrix} 1 & 1 & 1 & 40,000 \\ .12 & .13 & .145 & 5370 \\ 0 & -.13 & .145 & 1050 \end{pmatrix}$$

Using the calculator to obtain *reduced row echelon form* gives

$$\begin{pmatrix} 1 & 0 & 0 & 10,000 \\ 0 & 1 & 0 & 12,000 \\ 0 & 0 & 1 & 18,000 \end{pmatrix}$$

Don invested \$10,000 at 12%, \$12,000 at 13%, and \$18,000 at 14.5%.

6. Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 \\ 0 & 5 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} -1 & -1 \\ 2 & 6 \end{pmatrix}$$

a.

$$AB = \begin{pmatrix} -2 & 13 \\ 6 & 1 \end{pmatrix}$$

b.

$$BA = \begin{pmatrix} -11 & 2 \\ -15 & 10 \end{pmatrix}$$

c.

$$(B+C)(B-C) = \begin{pmatrix} -3 & 2 \\ 2 & 11 \end{pmatrix}\begin{pmatrix} -1 & 4 \\ -2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -14 \\ -24 & -3 \end{pmatrix}$$

d.

$$B^2 - C^2 = \begin{pmatrix} 4 & 9 \\ 0 & 25 \end{pmatrix} - \begin{pmatrix} -1 & -5 \\ 10 & 34 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 14 \\ -10 & -9 \end{pmatrix}$$

e.

$$A(2B) = \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} -4 & 6 \\ 0 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} -4 & 26 \\ 12 & 2 \end{pmatrix}$$

f.

$$2(AB) = 2\begin{pmatrix} -2 & 13 \\ 6 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} -4 & 26 \\ 12 & 2 \end{pmatrix}$$

g.

$$A(B+C) = \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} -3 & 2 \\ 2 & 11 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 24 \\ 13 & 16 \end{pmatrix}$$

h.

$$AB + AC = \begin{pmatrix} -2 & 13 \\ 6 & 1 \end{pmatrix} + \begin{pmatrix} 3 & 11 \\ 7 & 15 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 24 \\ 13 & 16 \end{pmatrix}$$

7.

$$AB = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} w & x \\ y & z \end{pmatrix}$$
$$= \begin{pmatrix} w+2y & x+2z \\ 2w+4y & 2x+4z \end{pmatrix}$$

To solve $AB = 0$, where 0 is the 2×2 zero matrix, we must solve

$$w + 2y = 0$$
$$x + 2z = 0$$
$$2w + 4y = 0$$
$$2x + 4z = 0$$

This system can be placed in the augmented matrix

$$\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 2 & 0 & 4 & 0 & 0 \\ 0 & 2 & 0 & 4 & 0 \end{pmatrix}$$

Placing the matrix in *reduced row echelon form* gives

$$\begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

This dependent system has 2 free variables, y and z. Letting $y = t$ and $z = s$ gives solution : $\{w = -2t,\ x = -2s,\ y = t,\ z = s\}$. Finally, if $t = 1$ and $s = 1$, a solution to the problem is

$$B = \begin{pmatrix} -2 & -2 \\ 1 & 1 \end{pmatrix}$$

8.

$$\begin{pmatrix} 1 & -1 & 0 & 2 \\ 2 & -2 & 4 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & 0 \\ 2 & 2 & 0 \\ -1 & -2 & 4 \\ 4 & 5 & -2 \end{pmatrix} = \begin{pmatrix} 7 & 10 & -4 \\ -6 & -8 & 16 \end{pmatrix}$$

9. Given the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$, use the augmented Gauss method to find the A^{-1}.

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 2 & 3 & 1 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{pmatrix} \; -2R_1 + R_3$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & -3 & -2 & -1 & 1 \end{pmatrix} \; -R_2 + R_3$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2/3 & 1/3 & -1/3 \end{pmatrix} \; (-1/3)R_3$$

$$= \begin{pmatrix} 1 & 1 & 0 & 1/3 & -1/3 & 1/3 \\ 0 & 1 & 0 & -4/3 & 1/3 & 2/3 \\ 0 & 0 & 1 & 2/3 & 1/3 & -1/3 \end{pmatrix} \; \begin{matrix} -R_3 + R_1 \\ -2R_3 + R_2 \end{matrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 5/3 & -2/3 & -1/3 \\ 0 & 1 & 0 & -4/3 & 1/3 & 2/3 \\ 0 & 0 & 1 & 2/3 & 1/3 & -1/3 \end{pmatrix} \; -R_2 + R_1$$

$$A^{-1} = \begin{pmatrix} 5/3 & -2/3 & -1/3 \\ -4/3 & 1/3 & 2/3 \\ 2/3 & 1/3 & -1/3 \end{pmatrix}$$

10. Use the inverse of the coefficient matrix to solve the following systems of equations. Recall

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ IX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

a. $\begin{aligned} x + 2y &= 4 \\ 2x + 5y &= 10 \end{aligned}$

$$\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 10 \end{pmatrix}$$

The inverse of A is found using the Gauss Method.

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{pmatrix} \; -2R_1 + R_2$$

$$= \begin{pmatrix} 1 & 0 & 5 & -2 \\ 0 & 1 & -2 & 1 \end{pmatrix} \; -2R_2 + R_1$$

$$A^{-1} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$$

Now the solution to the system is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 10 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

Solution is : $\{x = 0,\ y = 2\}$.

b. $\begin{aligned} x - y - z &= 1 \\ x \qquad + 2z &= 4 \\ 2x - 3y + z &= 0 \end{aligned}$

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 2 \\ 2 & -3 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$$

The inverse of A is found using the Gauss Method.

$$\begin{pmatrix} 1 & -1 & -1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 2 & -3 & 1 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & -1 & 3 & -2 & 0 & 1 \end{pmatrix} \begin{matrix} \\ -R_1 + R_2 \\ -2R_1 + R_3 \end{matrix}$$

$$= \begin{pmatrix} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 6 & -3 & 1 & 1 \end{pmatrix} \begin{matrix} \\ \\ R_2 + R_3 \end{matrix}$$

$$= \begin{pmatrix} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & 1/6 & 1/6 \end{pmatrix} \begin{matrix} \\ \\ (1/6)R_3 \end{matrix}$$

$$= \begin{pmatrix} 1 & -1 & 0 & 1/2 & 1/6 & 1/6 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1/6 & 1/6 \end{pmatrix} \begin{matrix} R_3 + R_1 \\ -3R_3 + R_2 \\ \end{matrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 1 & 2/3 & -1/3 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1/2 & 1/6 & 1/6 \end{pmatrix} \quad R_2 + R_1$$

$$A^{-1} = \begin{pmatrix} 1 & 2/3 & -1/3 \\ 1/2 & 1/2 & -1/2 \\ -1/2 & 1/6 & 1/6 \end{pmatrix}$$

Now the solution to the system is

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2/3 & -1/3 \\ 1/2 & 1/2 & -1/2 \\ -1/2 & 1/6 & 1/6 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 11/3 \\ 5/2 \\ 1/6 \end{pmatrix}$$

Solution is : $\left\{x = \frac{11}{3},\ y = \frac{5}{2},\ z = \frac{1}{6}\right\}$.

11. Evaluate the determinants.

a.

$$\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = 1(-2) - 3(2)$$

$$= -8$$

b. Using expansion along the first row yields

$$\begin{vmatrix} 1 & 2 & -3 \\ 1 & 4 & -1 \\ -2 & 2 & 2 \end{vmatrix} = 1(-1)^2 M_{11} + 2(-1)^3 M_{12} + -3(-1)^4 M_{13}$$

$$= 1\begin{vmatrix} 4 & -1 \\ 2 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 4 \\ -2 & 2 \end{vmatrix}$$

$$= 1(8-(-2)) - 2(2-2) - 3(2-(-8))$$

$$= 10 - 0 - 30$$

$$= -20$$

c. Using expansion along the third column yields

$$\begin{vmatrix} 1 & 1 & -1 & 1 \\ 2 & -2 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 2 & -2 & 0 & 1 \end{vmatrix} = -1(-1)^4 M_{13} + 0(-1)^5 M_{23} + 0(-1)^6 M_{33} + 0(-1)^7 M_{34}$$

$$= -1\begin{vmatrix} 2 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & -2 & 1 \end{vmatrix} + 0 + 0 + 0$$

Now expanding the 3×3 determinant along the second row gives

$$-1\begin{vmatrix} 2 & -2 & 3 \\ 0 & 1 & -1 \\ 2 & -2 & 1 \end{vmatrix} = -1\left[0(-1)^3 M_{21} + 1(-1)^4 M_{22} - 1(-1)^5 M_{23}\right]$$

$$= -1\left[0 + 1\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -2 \\ 2 & -2 \end{vmatrix}\right]$$

$$= -1[1(2-6) + 1(-4-(-4))]$$

$$= -1[-4+0]$$

$$= 4$$

12. Use Cramer's rule to solve

$$3x - 5y = 12$$

$$2x + 9y = 18$$

The solution is of the form $x = \dfrac{D_x}{D}$ and $y = \dfrac{D_y}{D}$ where

$$D = \begin{vmatrix} 3 & -5 \\ 2 & 9 \end{vmatrix}$$
$$= 27 - (-10)$$
$$= 37$$
$$D_x = \begin{vmatrix} 12 & -5 \\ 18 & 9 \end{vmatrix}$$
$$= 108 - (-90)$$
$$= 198$$
$$D_y = \begin{vmatrix} 3 & 12 \\ 2 & 18 \end{vmatrix}$$
$$= 54 - 24$$
$$= 30$$

Solution is : $\left\{x = \frac{198}{37}, y = \frac{30}{37}\right\}$

13. Use Cramer's rule to solve

$$\begin{aligned} x - y - z &= 1 \\ x \quad + 2z &= 4 \\ 2x - 3y + z &= 0 \end{aligned}$$

The solution is of the form $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, and $z = \frac{D_z}{D}$ where

$$D = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 0 & 2 \\ 2 & -3 & 1 \end{vmatrix}$$

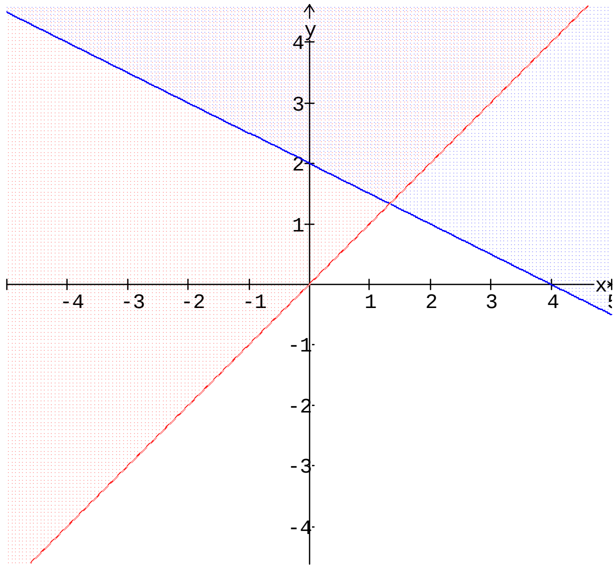
$$D_x = \begin{vmatrix} 1 & -1 & -1 \\ 4 & 0 & 2 \\ 0 & -3 & 1 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 4 & 2 \\ 2 & 0 & 1 \end{vmatrix}$$

$$D_z = \begin{vmatrix} 1 & -1 & 1 \\ 1 & 0 & 4 \\ 2 & -3 & 0 \end{vmatrix}$$

14. Sketch the solution set of the following inequalities.

a. $\begin{aligned} x + 2y &\geq 4 \\ y &> x \end{aligned}$

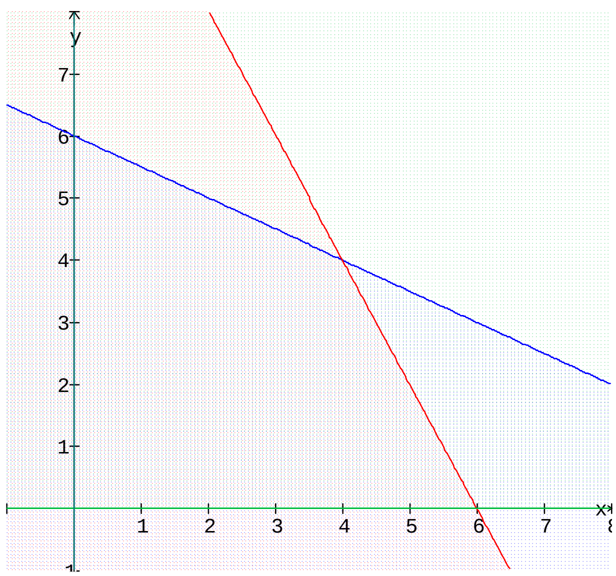


b. $x + 2y \leq 12$

$2x + y \leq 12$

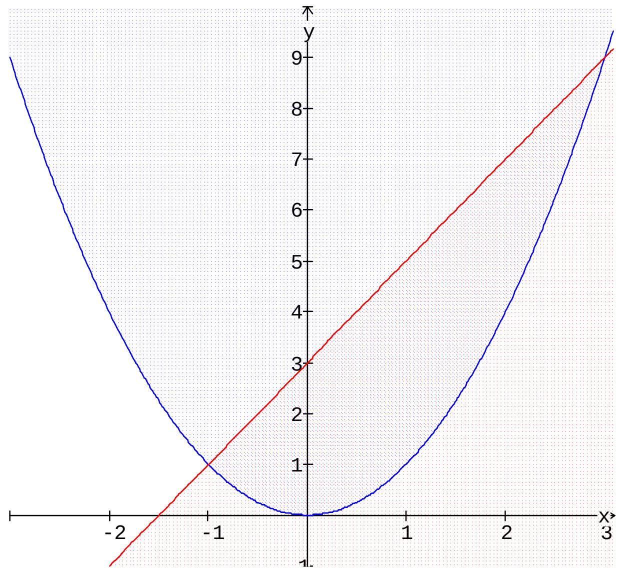
$x \geq 0$

$y \geq 0$



c. $y \geq x^2$

$y \leq 2x + 3$



y
9
8
7
6
5
4
3
2
1
-2
-1
1
2
3
x