

College of the Redwoods
Mathematics Department
Math 30–College Algebra

Exam #5
Sequences and Series

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Multiple Choice Questions

Directions: In each of the following exercises, select the “best” answer and darken the corresponding oval on your scantron sheet.

1. Find the n^{th} term of the sequence

$$1, -4, 9, -16, \dots$$

- (a) $a_n = 32$ (b) $a_n = -2n$ (c) $a_n = (-2)^n$
 (d) $a_n = (-1)^{n+1}(n)^2$ (e) $a_n = n^2$

2. Find the 5^{th} term of the sequence

$$a_n = 2(n - 1)^2 - 3.$$

- (a) -3 (b) 47 (c) 29 (d) 5 (e) 53

3. Find the 5^{th} term of the recursive sequence

$$a_{k+1} = 2a_k - 1$$

where $a_1 = 2$.

- (a) 17 (b) 10 (c) 9 (d) 5 (e) 2

4. Find the 8^{th} term of the recursive sequence

$$a_{k+1} = a_k + a_{k-1}$$

where $a_1 = 2$ and $a_2 = 3$.

- (a) 34 (b) 55 (c) 99 (d) 102 (e) 19

5. Compute the sum

$$\sum_{k=1}^3 \frac{1}{k}.$$

- (a) $\frac{1}{3}$ (b) $\frac{11}{6}$ (c) $\frac{3}{5}$ (d) $\frac{1}{2}$ (e) 6

6. Find the n^{th} term of the *arithmetic* sequence

$$-8, -5, -2, 1, 4, \dots$$

- (a) $a_n = -8 + 3(n - 1)$ (b) $a_n = -8(3)^{n-1}$ (c) $a_n = -8 - 3n$
 (d) $a_n = -8^n$ (e) $a_n = 3 - 8(n - 1)$

7. Find the two-hundredth term, a_{200} , of the sequence

$$2, 5, 8, 11, \dots$$

- (a) 399 (b) 499 (c) 599 (d) 603 (e) 583

8. The first term of an *arithmetic* sequence is $a_1 = 3$ and the eleventh term is $a_{11} = 23$. Find the n^{th} term, a_n , of the sequence.

- (a) $a_n = 3 + 23(n - 1)$ (b) $a_n = 23 + 10(n - 1)$ (c) $a_n = 3 + 2(n - 1)$
 (d) $a_n = 3 + 10(n - 1)$ (e) $a_n = 23 + 2(n - 1)$

9. Calculate the sum

$$\sum_{k=1}^{200} 2k + 5.$$

- (a) $18,500$ (b) $22,700$ (c) $29,200$ (d) $38,500$ (e) $41,200$

Solutions to Multiple Choice Questions

Solution to Question 1: The all of the terms of the sequence are perfect squares and the terms alternate sign. So the n^{th} term of the sequence is $a_n = (-1)^{n+1}(n)^2$. \square

Solution to Question 2: To find the 5th term of the sequence replace n with 5 in the sequence $a_n = 2(n-1)^2 - 3$.

$$a_5 = 2(5-1)^2 - 3 = 2(4)^2 - 3 = 32 - 3 = 29$$

 \square

Solution to Question 3: The sequence is defined recursively by $a_{k+1} = 2a_k - 1$.

$$\begin{aligned} a_1 &= 2 \\ a_2 &= 2a_1 - 1 = 2(2) - 1 = 3 \\ a_3 &= 2a_2 - 1 = 2(3) - 1 = 5 \\ a_4 &= 2a_3 - 1 = 2(5) - 1 = 9 \\ a_5 &= 2a_4 - 1 = 2(9) - 1 = 17 \end{aligned}$$

 \square

Solution to Question 4: The sequence is defined recursively by $a_{k+1} = a_k + a_{k-1}$ with the first two terms given. Thus

$$\begin{aligned} a_1 &= 2 \\ a_2 &= 3 \\ a_3 &= a_2 + a_1 = 2 + 3 = 5 \\ a_4 &= a_3 + a_2 = 3 + 5 = 8 \\ a_5 &= a_4 + a_3 = 5 + 8 = 13 \\ a_6 &= a_5 + a_4 = 8 + 13 = 21 \\ a_7 &= a_6 + a_5 = 13 + 21 = 34 \\ a_8 &= a_7 + a_6 = 21 + 34 = 55 \end{aligned}$$

 \square

Solution to Question 5:

$$\sum_{k=1}^3 \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{6+3+2}{6} = \frac{11}{6}$$

 \square

Solution to Question 6: The first term of the arithmetic series is $a_1 = -8$ and the common difference is $d = 3$. Using the fact that for an arithmetic series the n^{th} term is $a_n = a_1 + (n-1)d$,

$$a_n = -8 + 3(n-1)$$

 \square

Solution to Question 7: The sequence is arithmetic with first term $a_1 = 23$ and common difference $d = 3$. Then

$$\begin{aligned} a_n &= 2 + (n-1)3 \\ a_n &= 2 + 3n - 3 \\ a_n &= 3n - 1 \\ a_{200} &= 3(200) - 1 \\ a_{200} &= 599 \end{aligned}$$

□

Solution to Question 8: Use the fact that for an arithmetic series the n^{th} term is $a_n = a_1 + (n - 1)d$ to solve for d with $a_1 = 3$ and $a_{11} = 23$.

$$\begin{aligned} 23 &= 3 + (11 - 1)d \\ 20 &= 10d \\ 2 &= d \end{aligned}$$

Therefore,

$$a_n = 3 + 2(n - 1).$$

□

Solution to Question 9: Write the series to notice the sum is an arithmetic series.

$$\sum_{k=1}^{200} 2k + 5 = 7 + 9 + 11 + \cdots + 405$$

Indeed the sum is arithmetic with first term $a_1 = 7$ and last term $a_{200} = 405$. Now use the arithmetic series formula $S_n = \frac{n}{2}a_1 + a_n$.

$$\sum_{k=1}^{200} 2k + 5 = \frac{200}{2}(7 + 405) = 100(412) = 412,000$$

□

Solution to Question 10: The problem gives the first and last terms of the arithmetic series. To use the arithmetic series formula, $S_n = \frac{n}{2}(a_1 + a_n)$, the number of terms needs to be determined. The first term is $a_1 = 3$ and the common difference is $d = 4$, so $a_n = 3 + 4(n - 1)$. Thus

$$\begin{aligned} 123 &= 3 + 4(n - 1) \\ 120 &= 4(n - 1) \\ 30 &= n - 1 \\ 31 &= n \end{aligned}$$

Now it is possible to compute the sum.

$$S_{31} = \frac{31}{2}(3 + 123) = \frac{31}{2}(126) = 31(63) = 1953$$

□

Solution to Question 11: The common ratio is

$$r = \frac{a_{k+1}}{a_k} = \frac{2}{4} = \frac{1}{2} = \frac{1}{2}$$

□

Solution to Question 12: Use the n^{th} term formula $a_n = a_1 r^{n-1}$ with a first term $a_1 = 3$ and common ratio $r = \frac{-6}{3} = -2$ to get

$$a_n = 3(-2)^{n-1}.$$

□

Solution to Question 13: The finite sum for a geometric series is $S_n = \frac{a_1(1 - r^n)}{1 - r}$ where a_1 is the first term and r is the common ratio. The first term is $a_1 = 1$ and the common ratio is $r = \frac{2}{1} = \frac{4}{2} = 2$. The sum of the first 8 terms is

$$S_8 = \frac{1(1 - 2^8)}{1 - 2} = \frac{1 - 256}{-1} = 255$$

□

Solution to Question 14: The finite sum for a geometric series is $S_n = \frac{a_1(1 - r^n)}{1 - r}$ where the first term is $a_1 = 1$ and the common ratio is $r = \frac{a}{1} = a$. The sum of the 6 terms is

$$S_6 = \frac{1(1 - a^6)}{1 - a}$$

□

Solution to Question 15: The infinite sum for a geometric series is $S_\infty = \frac{a_1}{1 - r}$ where a_1 is the first term and r is the common ratio such that $|r| < 1$. The first term is $a_1 = 5$ and the common ratio is $r = \frac{2}{3}$. Since the common ratio is less than 1 the infinite sum is

$$S_\infty = \frac{5}{1 - 2/3} = \frac{5}{1/3} = 15$$

□

Solution to Question 16: The infinite sum for a geometric series is $S_\infty = \frac{a_1}{1 - r}$ where a_1 is the first term and r is the common ratio such that $|r| < 1$. The first term is $a_1 = 1$ and the common ratio is $r = \frac{x}{1} = x$. Since $-1 < x < 1$ the infinite sum is

$$S_\infty = \frac{1}{1 - x}$$

□

Solution to Question 17: Use the Fundamental Counting Principle to solve the problem. To have boy/girl pairs as dance partners there are 8 boys and 11 girls. The total number of dance partners is $8 \times 11 = 88$ different dance partners.

□

Solution to Question 18: Without the two letters 'P' and 'I' there are 24 letters to choose for the first two positions on the license plate. Without the digits '3', '1', and '4' there are 7 digits remaining to choose from. Using the Fundamental Counting Principle the number of license plates is $24 \times 24 \times 7 \times 7 \times 7 \times 7 = 1,382,976$.

□

Solution to Question 19: To arrange six students in six chairs there are 6 possibilities for the first chair. After the first person is seated there are 5 possibilities for the second chair and so on until all of the chairs are full. So the total number of ways 6 students can be seated in 6 chairs is

$$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = 720.$$

□

Solution to Question 20: The number of permutations of n objects taken k at a time is

$${}_n P_k = \frac{n!}{(n - k)!}.$$

Thus

$${}_n P_{n-2} = \frac{n!}{(n - (n - 2))!} = \frac{n!}{2!} = \frac{n!}{2}.$$

□

Solution to Question 21: The selection of 3 officers from the 12 members is an ordered arrangement of the members. So the number of different ways the officers can be chosen is a permutation problem.

$${}_{12} P_3 = \frac{12!}{(12 - 3)!} = \frac{12!}{9!} = \frac{12 \times 11 \times 10 \times 9!}{9!} = 12 \times 11 \times 10 = 1320$$

□

Solution to Question 22: Since the 3 members asked to serve on the committee are not ranked or ordered the problem is a combination problem. The number of combinations of n objects taken k at a time is

$${}_nC_k = \frac{n!}{(n-k)!k!}.$$

So,

$${}_{12}C_3 = \frac{12!}{9!3!} = \frac{12 \times 11 \times 10 \times 9!}{3!9!} = \frac{1320}{6} = 220$$

□

Solution to Question 23: The number of combinations of n objects taken k at a time is

$${}_nC_k = \frac{n!}{(n-k)!k!}.$$

So,

$${}_nC_3 = \frac{n!}{(n-3)!3!} = \frac{n(n-1)(n-2)(n-3)!}{3!(n-3)!} = \frac{n(n-1)(n-2)}{6}$$

□

Solution to Question 24:

$$\begin{aligned} \binom{n+1}{4} &= 5 \binom{n}{3} \\ \frac{(n+1)!}{4!(n+1-4)!} &= \frac{5n!}{3!(n-3)!} \\ \frac{(n+1)n!}{24(n-3)!} &= \frac{5n!}{6(n-3)!} \\ \frac{n+1}{24} &= \frac{5}{6} \\ n+1 &= 20 \\ n &= 19 \end{aligned}$$

□

Solution to Question 25: The next row of Pascal's Triangle is

$$1 \ 4 \ 6 \ 4 \ 1$$

□

Solution to Question 26: The binomial expansion for $(a+b)^4$ is

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4.$$

To evaluate $(2x-y)^4$ let $a = 2x$ and $b = -y$ in the equation above.

$$\begin{aligned} (2x-y)^4 &= (2x)^4 + 4(2x)^3(-y) + 6(2x)^2(-y)^2 + 4(2x)(-y)^3 + (-y)^4 \\ &= 16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4 \end{aligned}$$

□

Solution to Question 27: The $(k+1)^{\text{st}}$ term in the binomial expansion of $(a+b)^n$ is $\binom{n}{k}a^{n-k}b^k$. So the 11^{th} term of the binomial expansion of $(3x+y)^{11}$ is

$$\binom{11}{10}(3x)^1(y)^{10} = 11(3x)y^{10} = 33xy^{10}$$

□

Solution to Question 28: To solve $\sum_{k=0}^4 \binom{4}{k} (x)^{4-k} (-2)^k = 1$ notice the left hand side of the equation is the fourth degree binomial expansion for $(x - 2)$. That is

$$\begin{aligned} \sum_{k=0}^4 \binom{4}{k} (x)^{4-k} (-2)^k &= 1 \\ (x - 2)^4 &= 1 \\ (x - 2) &= \pm 1 \\ x &= 3 \\ x &= 1 \end{aligned}$$

□

Solution to Question 29: The sum resembles the sigma notation for the binomial expansion, $(a + b)^n = \sum_{k=0}^n \binom{n}{k} (a)^{n-k} (b)^k$ with $n = 5$. If $a = 1$ and $b = 1$, then

$$\begin{aligned} \binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \cdots + \binom{5}{5} &= \sum_{k=0}^5 \binom{5}{k} (1)^{n-k} (1)^k \\ &= (1 + 1)^5 \\ &= 32 \end{aligned}$$

□