College of the Redwoods
Mathematics Department
Math 30—College Algebra

Exam #2—Chapter 3

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Multiple Choice Questions

Directions: *In each of the following exercises, select the "best" answer and darken the corresponding oval on your scantron sheet. Calculators are **not** allowed on this portion of the exam.*

1. $\sqrt{-8}$ equals

(a) $-2\sqrt{2}$ (b) $-2\sqrt{2}i$ (c) $4\sqrt{2}i$

(d) $3\sqrt{2}i$ (e) None of these

2. $(2+3i)^2$ equals

(a) $4-9i$ (b) $-5+12i$ (c) $5-6i$

(d) $-5+6i$ (e) $5-12i$

3. One of the solutions of $2x^2+2=x$ is

(a) $-1+\sqrt{3}i$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{2}+\frac{\sqrt{15}}{2}i$

(d) $\frac{1}{2}-\frac{\sqrt{15}}{2}i$ (e) None of these

4. What is the x-value of the vertex of the parabola $y = mx^2 + 2cx + \omega^2$?

(a) $-\dfrac{c}{m}$ (b) $\dfrac{c}{m}$ (c) $-\dfrac{m}{c}$

(d) $\dfrac{2c}{w^2}$ (e) $-\dfrac{2c}{w^2}$

5. Jim has 60 board feet available to fence a rectangular garden on three sides, the remaining side bordered by one of the walls of his house. He has determined that the area of the garden is given by $A = w(60 - 2w)$, where w is the width of the garden. What is the maximum area possible for Jim's garden?

(a) $400\,\mathrm{ft}^2$ (b) $450\,\mathrm{ft}^2$ (c) $500\,\mathrm{ft}^2$

(d) $550\,\mathrm{ft}^2$ (e) $600\,\mathrm{ft}^2$

6. Find k so that $x + 1$ is a factor of $p(x) = kx^3 - x^2 + x - 2$.

(a) -4 (b) 4 (c) 2

(d) -3 (e) -2

7. The rational function

$$f(x) = \frac{x^2 + 3}{x + 1}$$

has a slant asymptote. What is it's equation?

(a) $y = x + 1$ (b) $y = x - 1$ (c) $y = 4$

(d) $y = 1 - x$ (e) None of these

Essay Questions

Directions: *Place the solution to each of the following exercises on your own paper. You must follow directions explicitly and show all work to receive full credit. Calculators are **not** allowed on this portion of the exam.*

Exercise 1. The graph of the equation

$$y = -2x^2 + 4x + 6$$

is a parabola.

(a) Place the equation in vertex form,

$$y = a(x - h)^2 + k,$$

and identify the coordinates of the vertex.

(b) Find the x- and y-intercepts.

(c) Sketch the parabola on graph paper. Include the axis of symmetry and label it with its equation. Label the vertex and the x- and y-intercepts with their coordinates.

Exercise 2. Consider the polynomial

$$p(x) = x^3 + 4x^2 - 7x - 10.$$

(a) Use the rational root theorem to list all possible rational zeros.

(b) Find the zeros of p and express $p(x)$ as a product of linear factors.

(c) Sketch the graph of p.

(d) Use interval notation to describe the solution of $p(x) < 0$.

Exercise 3. Consider the rational function

$$f(x) = \frac{2x+3}{x-2}.$$

(a) Identify the x-intercept of the graph of f.

(b) Identify the vertical asymptote of the graph of f.

(c) Use limit notation, as shown in class, to identify the horizontal asymptote of f.

(d) Sketch the graph of f on graph paper. Label the asymptotes with their equations. Label the x-intercept with its coordinates.

(e) Use interval notation to describe the solution of $f(x) \geq 0$.

Solutions to Quizzes

Solution to Question 1:

$$\begin{aligned}\sqrt{-8} &= \sqrt{8}\ i \\ &= \sqrt{4}\sqrt{2}\ i \\ &= 2\sqrt{2}\ i\end{aligned}$$

□

Solution to Question 2:

$$\begin{aligned}(2+3\ i)^2 &= (2)^2+2(2)(3i)+(3\ i)^2\\ &= 4+12\ i-9\\ &= -5+12\ i\end{aligned}$$

□

Solution to Question 3: Begin by setting the equation equal to 0.

$$2x^2 - x + 2 = 0$$

Use quadratic formula to solve the equation.

$$\begin{aligned} x &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(2)}}{2(2)} \\ &= \frac{1 \pm \sqrt{1-16}}{4} \\ &= \frac{1 \pm \sqrt{15}\, i}{4} \\ &= \frac{1}{4} \pm \frac{\sqrt{15}}{4}\, i \end{aligned}$$

□

Solution to Question 4: The x-coordinate of the vertex of the quadratic equation $y = ax^2 + bx + c$ is given by $x = \frac{-b}{2a}$.

So for the equation $y = mx^2 + 2cx + \omega^2$, the x-value of the vertex is

$$\begin{aligned} x_v &= -\frac{2c}{2m} \\ x_v &= -\frac{c}{m} \end{aligned}$$

□

Solution to Question 5: The area is given by $A = -2w^2 + 60w$. The graph of the quadratic equation is a parabola that opens down. The maximum area will occur at the vertex of the parabola. First find the w-coordinate of the parabola,

$$w_v = -\frac{60}{2(-2)} = 15.$$

When $w = 15$ feet the area will be at a maximum. To find the maximum area, substitute $w = 15$ into the equation for the area.

$$A = 15(60 - 2(15)) = 15(30) = 450.$$

The maximum possible area for the garden is 450 ft^2. □

Solution to Question 6: If $x+1$ is a factor of $p(x) = kx^3 - x^2 + x - 2$, then the remainder when $p(x)$ is divided by $(x+1)$ is 0. Instead of using long division, use synthetic division.

$$\begin{array}{c|cccc} \boxed{\text{-1}} & k & -1 & 1 & -1 \\ & & -k & k+1 & -k-2 \\ \hline & k & -k-1 & k+2 & -k-4 \end{array}$$

Finally set the remainder to zero.

$$\begin{aligned} -k-4 &= 0 \\ -k &= 4 \\ k &= -4 \end{aligned}$$

□

Solution to Question 7: Since the degree of the numerator is larger than the degree of the denominator use long division to rewrite the rational function.

$$\begin{array}{r|l} & x - 1 \\ \hline x + 1 \;) & x^2 + 0x + 3 \\ & x^2 + x \\ \hline & \quad -x + 3 \\ & \quad -x - 1 \\ \hline & \qquad\quad 4 \end{array}$$

Now

$$f(x) = \frac{x^2+3}{x+1} = x - 1 + \frac{4}{x+1}.$$

For large x-values,

$$\lim_{x\to\infty} \frac{4}{x+1} = 0.$$

So as $x \to \infty$, $f(x) \to x - 1$. Therefore the equation for the slant asymptote is $y = x - 1$ □

Solutions to Exercises

Exercise 1(a) First put the quadratic in vertex form.

$$\begin{aligned} y &= -2x^2 + 4x + 6 \\ y &= -2(x^2 - 2x - 3) \\ y &= -2(x^2 - 2x + 1 - 1 - 3) \\ y &= -2((x-1)^2 - 4) \\ y &= -2(x-1)^2 + 8 \end{aligned}$$

The parabola has vertex at $(1, 8)$. □

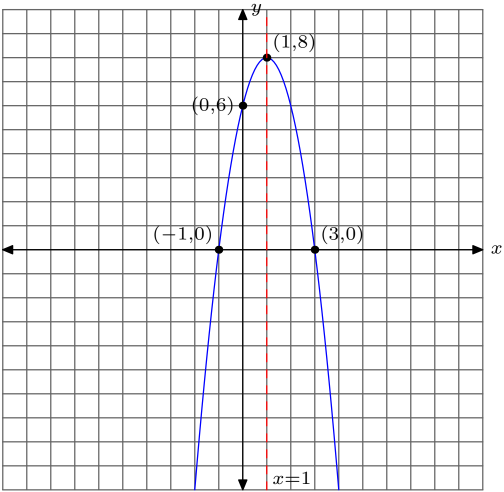
Exercise 1(b) When $x = 0$, $y = 6$ to give the y-intercept at $(0, 6)$. To find the x-intercept, let $y = 0$ and solve for x.

$$\begin{aligned} 0 &= -2x^2 + 4x + 6 \\ 0 &= -2(x^2 - 2x - 3) \\ 0 &= (x - 3)(x + 1) \\ x &= 3, -1 \end{aligned}$$

□

Exercise 1(c) Since the leading coefficient of the quadratic is negative the graph of the parabola opens down. The axis of symmetry is the vertical line through the vertex. So the equation of the axis of symmetry is $x = 1$.

Combining the information above gives the following graph.


y
(1,8)
(0,6)
(−1,0)
(3,0)
x
x=1

□

Exercise 2(a) The factors of the constant are

$$p = \pm 1,\ \pm 2,\ \pm 5,\ \pm 10.$$

The factors of the leading coefficient are

$$q = \pm 1.$$

The possible rational zeros are

$$\frac{p}{q} = \pm 1,\ \pm 2,\ \pm 5,\ \pm 10.$$

□

Exercise 2(b) Now use synthetic division to find a zero, and hence a factor, of the polynomial.

-1	1	4	-7	-10
		-1	-3	10
	1	3	-10	0

Since $x = -1$ is a zero, $x + 1$ is a factor of $p(x)$. So

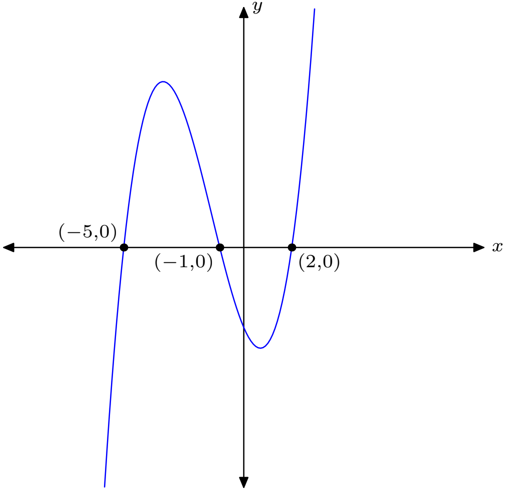
$$\begin{aligned} p(x) &= (x+1)(x^2+3x-10) \\ p(x) &= (x+1)(x+5)(x-2) \end{aligned}$$

The remaining factors give zeros at $x = -5$ and $x = 2$. □

Exercise 2(c) From the original equation the y-intercept is $p(0) = -10$. With the zeros and the y-intercept it is now possible to determine the end behaviour.

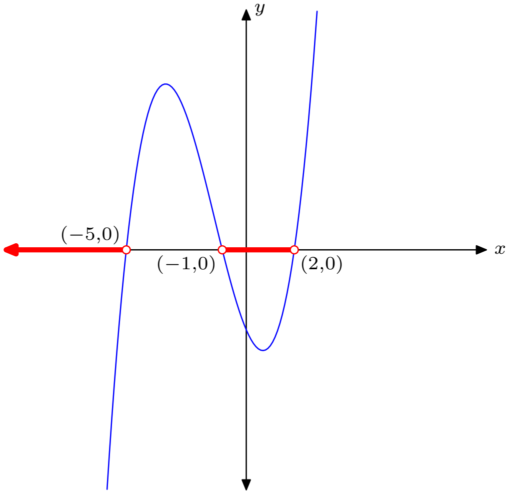
$$\lim_{x\to\infty} p(x) = +\infty, \quad \lim_{x\to-\infty} p(x) = -\infty$$

Combining all the previous information to sketch the graph of $p(x)$.


y
x
(−5,0)
(−1,0)
(2,0)

□

Exercise 2(d) To solve $p(x) < 0$, shade the intervals on the x-axis where the graph of $p(x)$ is below the x-axis.



Finally, expressing the solution in interval notation gives the solution to $p(x) < 0$ as

$$(-\infty, -5) \cup (-1, 2).$$

□

Exercise 3(a) To find the x-intercept, set the numerator equal to zero.

$$\begin{aligned} 2x+3 &= 0 \\ 2x &= -3 \\ x &= \frac{-3}{2} \end{aligned}$$

□

Exercise 3(b) To find the vertical asymptote, set the denominator equal to zero.

$$\begin{aligned} x - 2 &= 0 \\ x &= 2 \end{aligned}$$

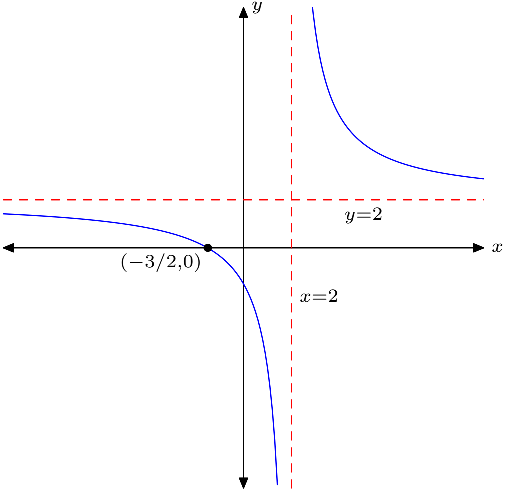
□

Exercise 3(c) To find the horizontal asymptote let x approach $\pm\infty$.

$$\begin{aligned}\lim_{x\to\infty}\frac{2x+3}{x-2} &= \lim_{x\to\infty}\frac{2+\frac{3}{x}}{1-\frac{2}{x}}\\ &= \frac{2+0}{1-0}\\ &= 2\end{aligned}$$

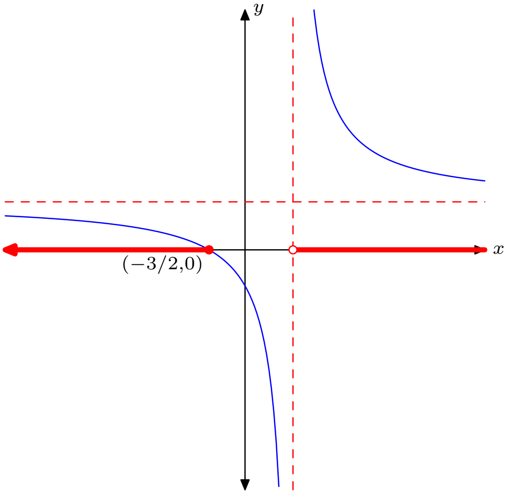
□

Exercise 3(d) Combining the above information along with the y-intercept of $f(0) = -\frac{3}{2}$. to get the graph of $f(x)$.



□

Exercise 3(e) To solve $f(x) \geq 0$ shade the intervals on the x-axis where the graph of $f(x)$ is above or on the x-axis.



Finally, expressing the solution in interval notation gives the solution to $f(x) \geq 0$ as

$$(-\infty, -3/2] \cup (2, \infty).$$

□