6pts

(1a) $\boxed{\text{Prove: } \dfrac{\sin B}{1+\cos B} + \dfrac{1+\cos B}{\sin B} = 2\csc B}$

43

5
5
5
5
5
3
3
3
3
3
3
43

Proof: $\dfrac{\sin B}{1+\cos B} + \dfrac{1+\cos B}{\sin B}$

$$= \frac{\sin^2 B}{(1+\cos B)\sin B} + \frac{(1+\cos B)^2}{(1+\cos B)\sin B}$$

$$= \frac{\sin^2 B}{(1+\cos B)\sin B} + \frac{1+2\cos B+\cos^2 B}{(1+\cos B)\sin B}$$

$$= \frac{\sin^2 B + 1 + 2\cos B + \cos^2 B}{(1+\cos B)\sin B}$$

$$= \frac{2+2\cos B}{(1+\cos B)\sin B}$$

$$= \frac{2(1+\cos B)}{(1+\cos B)\sin B}$$

$$= \frac{2}{\sin B}$$

$$= 2\csc B$$

(14) Prove: $\cos^2\beta - \sin^2\beta = \dfrac{1-\tan^2\beta}{1+\tan^2\beta}$

Proof:
$$\frac{1-\tan^2\beta}{1+\tan^2\beta} = \frac{1-\frac{\sin^2\beta}{\cos^2\beta}}{1+\frac{\sin^2\beta}{\cos^2\beta}}$$

$$= \frac{\left(1-\frac{\sin^2\beta}{\cos^2\beta}\right)\cos^2\beta}{\left(1+\frac{\sin^2\beta}{\cos^2\beta}\right)\cos^2\beta}$$

$$= \frac{\cos^2\beta-\sin^2\beta}{\cos^2\beta+\sin^2\beta}$$

$$= \frac{\cos^2\beta-\sin^2\beta}{1}$$

$$= \cos^2\beta-\sin^2\beta$$

5pts

(2a) $\cos\left(-\frac{11\pi}{12}\right)$

$= \cos\frac{11\pi}{12}$

$= \cos\left(\frac{8\pi}{12}+\frac{3\pi}{12}\right)$

$= \cos\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

5pts

(2b) $\sin\frac{\pi}{8} = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$

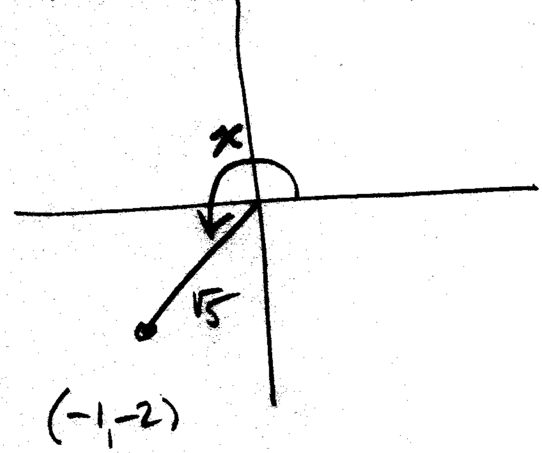
$= \sqrt{\frac{1-\sqrt{2}/2}{2}}$

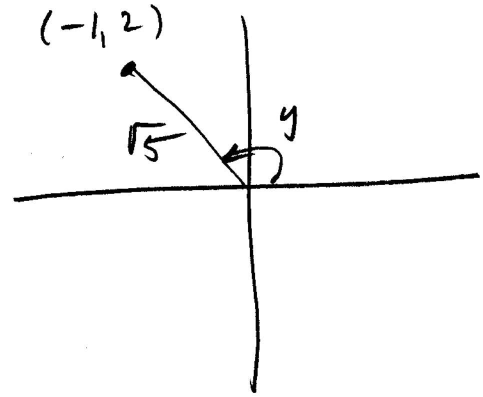
$= \sqrt{\frac{2-\sqrt{2}}{4}}$

$= \frac{\sqrt{2-\sqrt{2}}}{2}$

6 pts

(3) $\tan x = 2, \quad \pi < x < 3\pi/2$ $\qquad$ $\tan y = -2, \quad \pi/2 < y < \pi$





$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$= \left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{1}{\sqrt{5}}\right) - \left(\frac{-2}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$$

$$= \frac{1}{5} + \frac{4}{5}$$

$$= 1$$

3 pts

(4a) $\sin 38^\circ \cos 22^\circ + \sin 22^\circ \cos 38^\circ$

$$= \sin(38^\circ + 22^\circ)$$

$$= \sin 60^\circ$$

$$= \sqrt{3}/2$$

3pts

4f

$$\frac{\tan\frac{\pi}{12} - \tan\frac{\pi}{3}}{1 + \tan\frac{\pi}{12}\tan\frac{\pi}{3}}$$

$$= \tan\left(\frac{\pi}{12} - \frac{\pi}{3}\right)$$

$$= \tan\left(\frac{\pi}{12} - \frac{4\pi}{12}\right)$$

$$= \tan\left(-\frac{3\pi}{12}\right)$$

$$= \tan\left(-\frac{\pi}{4}\right)$$

$$= -\tan\frac{\pi}{4}$$

$$= -1$$

3pts

4e

$$\sin\frac{\pi}{12}\cos\frac{\pi}{12}$$

$$= \frac{1}{2}\sin 2\left(\frac{\pi}{12}\right)$$

$$= \frac{1}{2}\sin\frac{\pi}{6}$$

$$= \frac{1}{2}\left(\frac{1}{2}\right)$$

$$= \frac{1}{4}$$

3pts

4d

$$2\sin^2 75^\circ - 1$$

$$= -\cos 2(75^\circ)$$

$$= -\cos 180^\circ$$

$$= -\left(-\sqrt{3}/2\right)$$

$$= \sqrt{3}/2$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - \sin^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$

3pts

5. $\sin 5x \cos 7x = \frac{1}{2}\left[\sin(5x+7x) + \sin(5x-7x)\right]$

$= \frac{1}{2}\sin 12x + \frac{1}{2}\sin(-2x)$

$= \frac{1}{2}\sin 12x - \frac{1}{2}\sin 2x$

3pts

6. $\cos 5\theta + \cos 3\theta$

$= \cos(4\theta+\theta) + \cos(4\theta-\theta)$

$= \cancel{\cos 4\theta \cos\theta} \sin 4\theta \sin\theta + \cos 4\theta \cos\theta + \sin 4\theta \sin\theta$

$= 2\cos 4\theta \cos\theta$