Upward Bound: A Summer Class ... Physical Geometry

Rex Sinclair, Chair, Math. Dept. U. B

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0.1 Preface

Use this document as a guide. This is less than a text book, but more than a course outline. It's designed to be used as a guide for Upward Bound Program classes of physical geometry meeting twice a week for 90 minutes, for five weeks. Students should rely on two prime sources for material, lecture notes and the enclosed 'Activities.' These were developed for C/R's Basic Algebra Course by Mike Butler and Michele Olsen, with a few contributions of my own. The instructor should freely call upon personal knowledge.

Chapter 1

Upward Bound, First Lesson

1.1 Miscellaneous First Day Items

1.1.1 The Syllabus

All college course teachers hand out a syllabus which is a contract between the student and school. Like all contracts it's based on good will; i. e. not every single possibility or eventuality can be anticipated. General terms of expectations and performance are described and subject to change.

1.1.2 The Calculator

The TI-83 is a tiny but powerful computer that can draw graphs or pictures.

The basic functions

The off/on, Clear, Zoom, and y = menus are basic functions to master.

1.1.3 Angular Measurement

An angle is a measurement of the amount of rotation it would take to cause one line to fall exactly on another In text this symbol, $\angle$, is very descriptive, the horizontal line needs a certain amount of rotation to align with the other line, or vice versa.

Humans generally have difficulty judging angular measurement, compared to say, distance.

Often Greek symbols are used for angles...$\alpha, \beta, \theta, \phi$, like these: Called alpha, beta, theta, phi et. cet.

Units...are arbitrary

- Gons...the European measurement, has easy numbers.
- Mils...the Military measurement, approximates 2Π
- Degrees(minutes, seconds)...a common method, derived from an ancient civilization, probably with a more efficient number system than ours.
- Radians...a natural measurement, based upon the ratio of the circumference of a circle to its diameter...i. e. Π.

The Calculator Set to Degrees or Radians

The **Mode** setting Radians or Degrees determines how the calculator interprets numbers for angular measurement.

Converting From one to the other

Set the calculator to **Radians**, now enter 90, but tell the calculator this is degrees by 2nd-angle-degree, then push return and see 1.5707...This corresponds to the number of radians in ninety degrees.

Setting the calculator to **Degrees** via **Mode** as above allows conversion in the other direction.

1.1.4 Activity 1

Break into groups, gather the material, listen to the instructions, and solve the problem.

1.2 Homework 1

1.2.1 Part One: Angular Conversion

Convert Degrees to Radians			
Degrees	Radians	**degrees**	radians
30		45	
60		90	
120		150	
180		225	
225		270	
540		720	

Convert Radians to Degrees			
Radians	degrees	**Radians**	degrees
.5		1	
1.5		2	
3		4	
5		10	

1.2.2 Part Two: Essay

Enter 30.5 on your calculator main screen, using 2nd-angle, DMS, enter, observe the output. Describe what you think the output is.

Chapter 2

Upward Bound, Lesson Two

2.1 The Radian

The radian is the name given to angular measurement relating arc length to the radius of a circle. If a radius of one inch is swung through $\frac{\Pi}{4}$ radians, (45^o), then an arc of about 0.78 inches in length is drawn, arc-distance, s is given by the formula:

$$s = r\theta$$

If a one inch radius is swung through Π radians then the arc would make a half circle, and its length would be about 3.14 inches. If the same radius swung through 2Π then whole circle would be drawn with a circumference of about 6.28 inches.

2.1.1 Approximations to PI

Through the ages mankind tried to approximate PI: a simple example is $\frac{22}{7}$ What kind of a number is Π ?

- Natural Numbers(=positive numbers)
- Zero
- Integers (positive and negative whole numbers)
- Rational numbers (numbers that can be expressed as ratio of two integers)
- irrational numbers (numbers that can't be expressed as the ratio of two integers) Π is one of these.

2.1.2 The PI Activity

Consider other approximations to Π by completing the first page of 'The PI LAB' Using jar lids, and measuring tapes find...

PI Lab, Part I

- An average PI
- Measuring circumference and diameter of objects.
- Use Calculator to average

PI Lab, Part II

Besides Arc length and Diameter Π is involved with circular area and volume (cylindrical volume and spherical volume)

- Relating circular area to PI
- Relating Cylindrical volume to PI

PI Lab, Part IV, Followup

Graphical analysis of data: Circumference plotted versus Diameter can be used to find an approximation to Π

2.2 Homework 2

Part 1. Calculate the area of a circle with a radius of 2 inches, compare it to the area of a circle with a radius of 4 inches, and answer this question: How many times as large is the bigger circle to the smaller one?

Part 2. How long would the radius of circle have to be to have just twice the area of the smaller (2 inch radius) circle?

Part 3. What is the exact ratio of the radii of any two circles where the area of one is twice that of the other?

Chapter 3

Upward Bound, Lesson 3

3.1 Lesson Three

3.1.1 Other Areas

Last lesson Π was discussed: one other application of Π includes area of a sphere. Areas of polygons (many sided figure) can be found by breaking them into smaller, more understandable parts, and then summing the parts.

3.1.2 The Dots and Area Activiity

This activity leads you to discover formulas relating what you observe to general formulas for finding areas of peculiar shaped objects, at least ones that can be broken up the way of this activity.

The Dot Methods: part 1

Look at sheet I of this activity: let dots on the boundary of the areas (square, triangle, etc.) be called 'b'. Let interior dots be called 'i'. Do similar to parts 1a on blackboard together.

The 'Rule'

Let's consider the methods used to reach the 'Rule'

- Take Data: i. e. count the dots.
- Record the data: i. e. Make chart
- Look for pattern
- Try formulas for predictability.
- Test (part 1c)
- Use (part 1d)

Modified ... to include an interior dot: part 2

- Figure the areas
- Make the table
- Invent a formula
- Test it

Modified ... two interior dots: part3

Do as before, and come up with a 'Rule'

Pick's Rule: part 4

You are to formulate a general rule for any number of dots on the inside, and boundary using the hint. The hint shows that Pick's Rule will always have the fraction $\frac{b}{2}$, the minus one ...

$$0 + \frac{b}{2} - 1$$

and for the case where i is zero, there is a zero. You have worked out two other cases. See if you can invent a formula using what you are given in the hint, the value for i, and your results.

Using Pick's Rule: Parts 6, 7

Complete this section.

3.1.3 Homework

Complete 'An Experiment with addition, multiplication, rectangles and triangles.

Chapter 4

Upward Bound Lesson 4

4.1 Similar Triangles

If angles of a triangle are equal, sides are proportional.

Consider chalk on blackboard examples. If two angles of a triangle are equal, what about the third? If three sides are equal, what about the angles? What if two sides and one angle are equal, how would the triangles compare?

Using the Priciples

Consider the problem of measuring the height of a tree or flag pole, using the sun, shadow and known height of an object or person.

Using an instrument for angles

Consider the Mark II Astro compass for obtaining accurate angles to measure inaccessible point distances.

i. Its controls ii. Its setup.

Technical Definitions

Alternate interior angles. Parallel Lines. Proportions.

4.2 Outside Work: Different than Inside

Applying these principles outside using, string, sticks, mirror, Astro compass, Sun, and luck.

The Mirror as an aid

Angle of Incidence and reflection.

4.3 Homework

Complete the fill in parts of 'Out Door Geometry'