$f(x,y) = 8x^3 + y^3 + 6xy$

$f_x(x,y) = 24x^2 + 6y$

$f_y(x,y) = 3y^2 + 6x$

Critical Values:

$24x^2 + 6y = 0$ & $3y^2 + 6x = 0$

$4x^2 + y = 0 \quad (1)$ $\qquad y^2 + 2x = 0 \quad (2)$

Solve (1) for y: $y = -4x^2 \quad (3)$

Sub (3) in (2): $(-4x^2)^2 + 2x = 0$

$16x^4 + 2x = 0$

$8x^4 + x = 0$

$x(8x^3 + 1) = 0$

$x = 0$ or $8x^3 = -1$

$x^3 = -\frac{1}{8}$

$x = -\frac{1}{2}$

Sub $x = 0$ in (1)

$4(0)^2 + y = 0$

$y = 0$

Sub $x = -\frac{1}{2}$ in (1):

$4(-\frac{1}{2})^2 + y = 0$

$y = -1$

Thus, $(0,0)$ and $(-\frac{1}{2},-1)$ are critical points.

<u>More derivatives</u>

$$f_{xx}(x,y) = 48x$$
$$f_{yy}(x,y) = 6y$$
$$f_{xy}(x,y) = 6$$

(x,y)	$f(x,y)$	$f_{xx}(x,y)$	$f_{xx}(x,y)f_{yy}(x,y)-f_{xy}^2(x,y)$	Class.
$(0,0)$	0	0	$(0)(0)-6^2=-36$	Saddle
$(-\frac{1}{2},-1)$	-1	-24	$(-24)(-6)-6^2=108$	Local Max