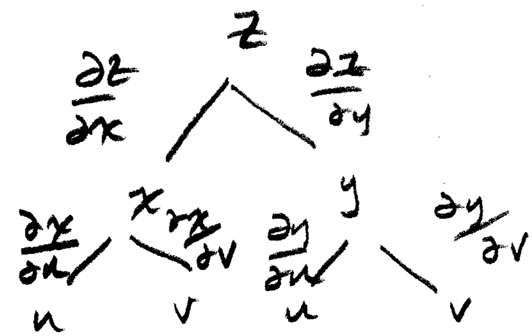
6pts

① $z = x^2 - 2xy - 3y^2$

$x = u^2 - v^2$

$y = uv$



$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$= (2x - 2y)(2u) + (-2x - 6y)(v)$$

$$= 4xu - 4yu - 2xv - 6yv$$

$$= 4(u^2 - v^2)u - 4(uv)u - 2(u^2 - v^2)v - 6(uv)v$$

$$= 4u^3 - 4uv^2 - 4u^2v - 2u^2v + 2v^3 - 6uv^2$$

$$= 4u^3 - 6u^2v - 10uv^2 + 2v^3$$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$= (2x-2y)(-2v) + (-2x-6y)(u)$$

$$= -4xv + 4yv - 2xu - 6yu$$

$$= -4(u^2-v^2)v + 4(uv)v - 2(u^2-v^2)u - 6(uv)u$$

$$= -4u^2v + 4v^3 + \underline{4uv^2} - 2u^3 + \underline{2uv^2} - 6u^2v$$

$$= -2u^3 - 10u^2v + 6uv^2 + 4v^3$$

10 pts

(2) $f(x,y) = \dfrac{3x}{x^2+y^2+1}$

$$f_x(x,y) = \frac{(x^2+y^2+1)(3) - 3x(2x)}{(x^2+y^2+1)^2}$$

$$= \frac{3x^2+3y^2+3-6x^2}{(x^2+y^2+1)^2}$$

$$= \frac{3y^2-3x^2+3}{(x^2+y^2+1)^2}$$

$$f_y(x,y) = \frac{(x^2+y^2+1)(0)-(3x)(2y)}{(x^2+y^2+1)^2}$$

$$= \frac{-6xy}{(x^2+y^2+1)^2}$$

10pts

③ $f(x,y) = 4 - x^2 - 2xy - 3y^2$

$$\hat{u} = \frac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} = \frac{\langle 1,1\rangle}{\sqrt{2}} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

$$D_{\hat{u}} f(x,y) = \nabla f(x,y)\cdot \hat{u}$$

$$= \langle -2x-2y, -2x-6y\rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$$

$$= \frac{1}{\sqrt{2}} \langle -2x-2y, -2x-6y\rangle \cdot \langle 1,1\rangle$$

$$= \frac{1}{\sqrt{2}}(-2x-2y-2x-6y)$$

$$= \frac{1}{\sqrt{2}}(-4x-8y)$$

$$D_{\hat{u}} f(1,1) = \frac{1}{\sqrt{2}}\left(-4(1) - 8(1)\right)$$

$$= -\frac{12}{\sqrt{2}}$$

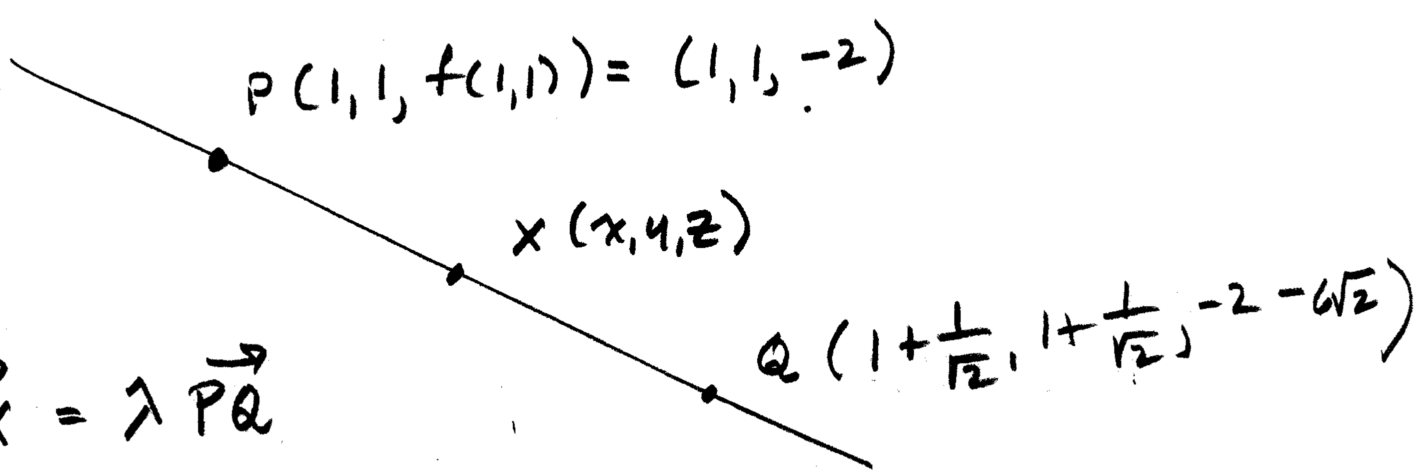
$$= -\frac{12\sqrt{2}}{2}$$

$$= -6\sqrt{2}$$

$f(1,1)$

$= 4 - 1 - 2 - 3$

$= -2$



$$\vec{PX} = \lambda \vec{PQ}$$

$$\langle x-1, y-1, z+2 \rangle = \lambda \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -6\sqrt{2} \right\rangle$$

$$x = 1 + \lambda/\sqrt{2}$$

$$y = 1 + \lambda/\sqrt{2}$$

$$z = -2 - 6\sqrt{2}\,\lambda$$

(4) $x^2 - 2xy + 3y^2 = 2$

Let $f(x,y) = x^2 - 2xy + 3y^2$. Then,

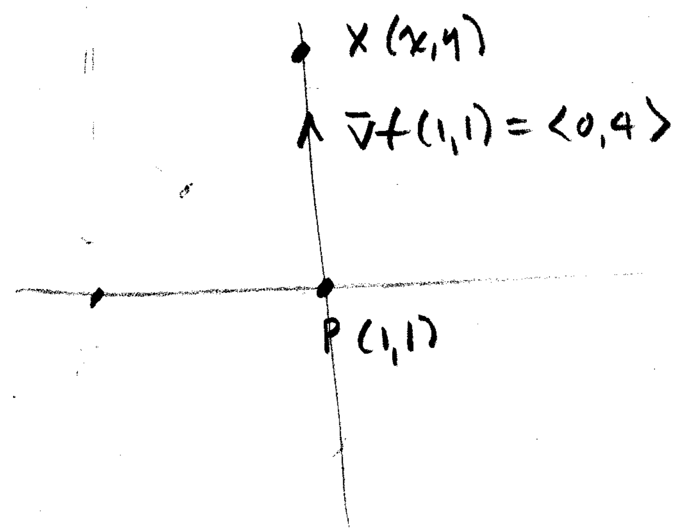
$$f(x,y) = 2$$

is the level curve of f and identical to

$$x^2 - 2xy + 3y^2 = 2.$$

$\nabla f(x,y) = \langle 2x-2y, -2x+6y \rangle$

$\nabla f(1,1) = \langle 0, 4 \rangle$ [orthogonal to level curve].



<u>Normal Line</u>:

$\overrightarrow{PX} = \lambda \nabla f(1,1)$

$\langle x-1, y-1 \rangle = \lambda \langle 0,4 \rangle$

$x = 1$

$y = 1 + 4\lambda$

Tangent line:

$$\overrightarrow{PX} = \lambda \langle -4, 0 \rangle$$

$$\langle x-1, y-1 \rangle = \lambda \langle -4, 0 \rangle$$

$$x = 1 - 4\lambda$$

$$y = 1.$$