College of the Redwoods
Mathematics Department

Math 50C — Multivariable Calculus
Quiz #7

David Arnold

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Quiz Questions

Read Carefully! *You have until Monday (11/19/07) to complete the quiz. The quiz is due at the beginning of class on Monday (11/19/07). Late quizzes are not accepted.*

This quiz is open notes, open book. This includes any supplementary texts or online documents. You must answer all of the exercises on your own. You are not allowed to work in groups or pairs on the quiz. You are not allowed to enlist the aid of a tutor or friend to help with the quiz. You are not allowed to read the exercises in the quiz, then seek help on similar questions. Once you open the quiz and read the questions, you may not seek any outside help of any kind.

I am not interested in reading pages and pages of calculations without accompanying narrative. It is essential that you include sound mathematical writing that both explains and justifies your solution or proof. Grammar and punctuation are important, as is the organization of your solution on the written page.

When working in the lab, please do not work at a terminal next to any other student who is also working on the quiz. For the sake of propriety, please separate yourselves when working on the quiz in the lab. Know that PS110 is also available for computer work. You can log onto the computers in PS110 and obtain files in your account in exactly the same manner as in PS116.

For information on transferring files from home, see the links for connecting to MSEMac from both PC's and Macs on the syllabus.

`http://online.redwoods.edu/instruct/darnold/MULTCALC/Syl.htm`

Place the solution to each exercise on a separate *sheet of paper. On a good sheet of paper, write out (longhand) and sign the following honor pledge.*

> I promise that all work found herein is my own. I have received no help from tutors, colleagues, or other teachers. I also promise that I have refrained from sharing my work and ideas with other students in the class. I have also honored all of the quiz constraints listed in the directions.

Arrange your solutions in order, place these quiz page(s) on top of your solutions, then place the honor pledge on top of the quiz as a cover sheet. Staple. Good luck!

EXERCISE 1. Given

$$z = x^2 - 2xy - 3y^2,$$

where

$$x = u^2 - v^2 \qquad \text{and} \qquad y = uv,$$

use the chain rule to determine the partial derivatives $\partial z/\partial u$ and $\partial z/\partial v$. In each case, clearly perform each of the steps:

- State the form of the chain rule appropriate for the partial derivative your are computing. Include a "derivative tree" with your result.
- Obtain the partial derivative in terms of x, y, u, and v.
- State the final answer in terms of u and v only.

No credit will be given if the steps above are not followed exactly. *No cheatin' dog!*

EXERCISE 2. Consider the function

$$f(x, y) = \frac{3x}{x^2 + y^2 + 1}.$$

Perform each of the following tasks.

(a) Use Matlab to sketch contours of f on the domain $\{(x, y) : -4 \leq x, y \leq 4\}$. Use Matlab's `clabel` command to label contours in 'manual' mode.

(b) Use pencil-and-paper calculations to calculate the gradient of f.

(c) Use the result of part (b) to superimpose the gradient field on your contour plot in part (a). Do **not** use Matlab's `gradient` command for this purpose.

(d) Use `axis equal` to show the orthogonality of the gradient field to the contours. Label the axes and provide an appropriate title. Include printouts of the result and your source code with your examination papers.

EXERCISE 3. Consider the function

$$f(x, y) = 4 - x^2 - 2xy - 3y^2.$$

Perform each of the following tasks.

(a) Sketch the graph of f on the domain $\{(x, y) : 0 \leq x, y \leq 2\}$.

(b) Find a unit vector $\hat{\mathbf{u}}$ that points in the direction of the vector $\overrightarrow{PQ}$, where $P = (1, 1)$ and $Q = (2, 2)$.

(c) Find the directional derivative of f at the point $(1, 1)$ in the direction of $\hat{\mathbf{u}}$, the unit vector found in part (b).

(d) Use the result of part (c) to determine the equation of the line that is tangent to the graph of f at the point $(1, 1, f(1, 1))$ in the direction of $\hat{\mathbf{u}}$.

(e) Use Matlab to superimpose the graph of the tangent line in part (d) on the surface drawn in part (a). Include printouts of the result and the source code with your examination papers.

EXERCISE 4. Consider the equation

$$x^2 - 2xy + 3y^2 = 2.$$

Perform each of the following tasks.

(a) Construct a function $f(x, y)$ so that $x^2 - 2xy + 3y^2 = 2$ is a level curve of the function $f(x, y)$.

(b) Calculate the gradient of the function f constructed in part (a). Use the gradient to find the equations of the tangent and normal lines to the level curve $x^2 - 2xy + 3y^2 = 2$ at the point $(1, 1)$.

(c) Use Matlab's `contour` command to draw the single level curve $x^2 - 2xy + 3y^2 = 2$. **Hint:** Use the form `contour(x,y,z,[v,v])` to accomplish this task. Type `help contour` for more information.

(d) Use Matlab to superimpose the plots of the tangent and normal lines found in part (b) on the plot of the level curve in part (c). Include printouts of the result and the source code with your examination papers.