Quiz # 4

5 pts

(a)

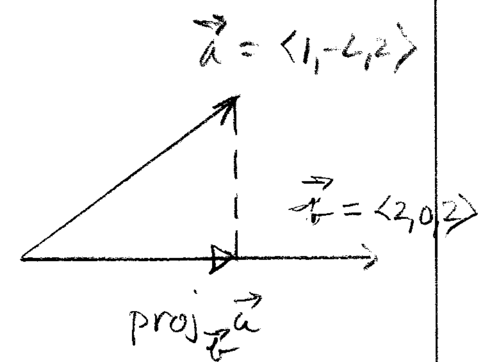
$$\text{proj}_{\vec{b}}\,\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\,\vec{b}$$

$$= \frac{\langle 1,-2,2\rangle \cdot \langle 2,0,2\rangle}{\langle 2,0,2\rangle \cdot \langle 2,0,2\rangle}\,\langle 2,0,2\rangle$$

$$= \frac{6}{8}\,\langle 2,0,2\rangle$$

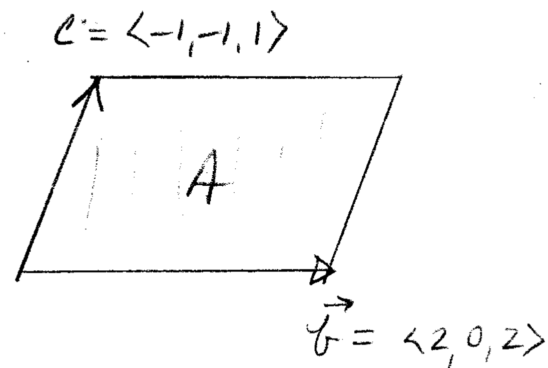
$$= \frac{3}{4}\,\langle 2,0,2\rangle$$

$$= \left\langle \frac{3}{2}, 0, \frac{3}{2} \right\rangle$$



5 pts

(b)



$$A = \|\vec{b}\times\vec{c}\|$$

$$= \|\langle 2,-4,-2\rangle\|$$

$$= \|2\langle 1,-2,-1\rangle\|$$

$$= 2\|\langle 1,2,-1\rangle\|$$

$$= 2\sqrt{1+4+1}$$

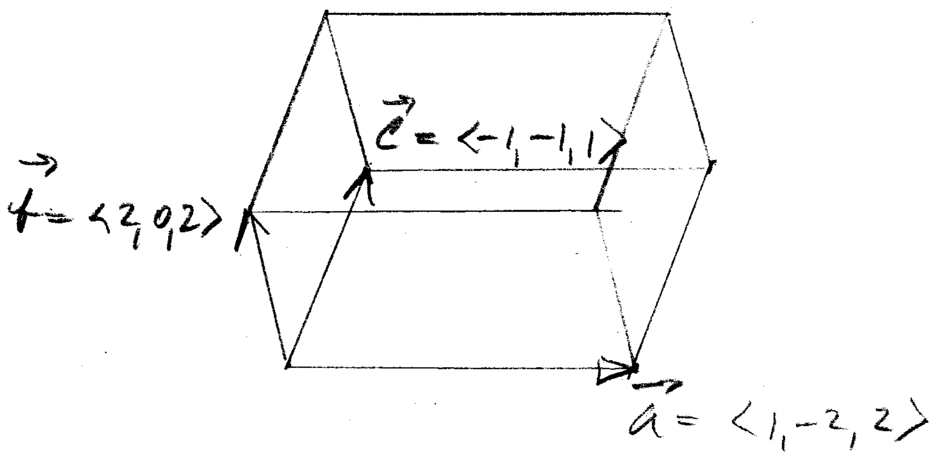
$$= 2\sqrt{6}$$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 2 \\ -1 & -1 & 1 \end{vmatrix}$$

$$= \langle 2,-4,-2\rangle$$

5 pts

1c



$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & -2 & 2 \\ 2 & 0 & 2 \\ -1 & -1 & 1 \end{vmatrix}$$

$$= 1(2) + 2(4) + 2(-2)$$

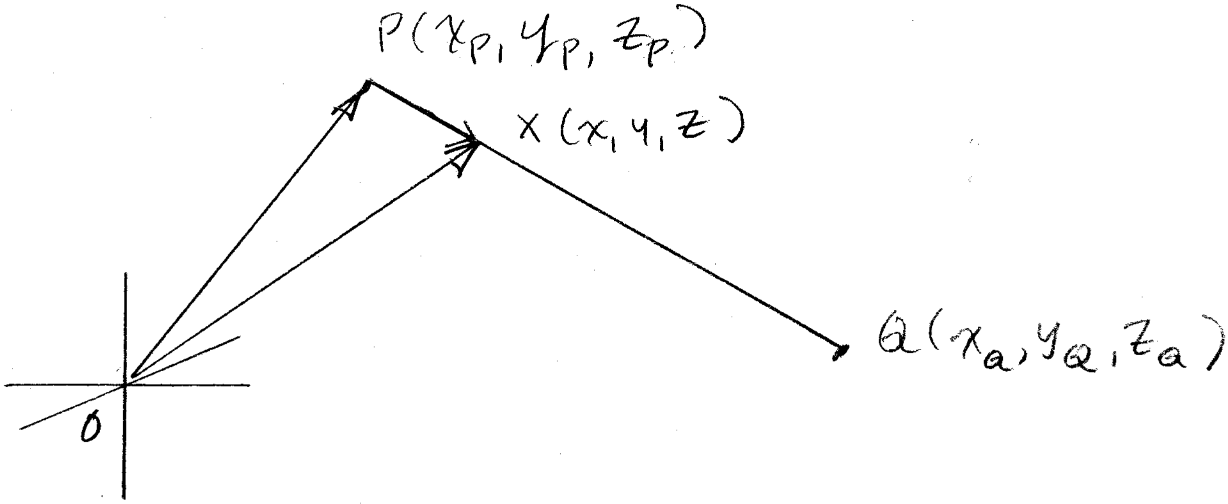
$$= 2 + 8 - 4$$

$$= 6$$

$$\text{Volume} = |\vec{a} \cdot (\vec{b} \times \vec{c})| = |6| = 6$$

5.2#13

(2)



$$\vec{OX} = \vec{OP} + \frac{1}{5}\vec{PQ}$$

$$\langle x, y, z \rangle = \langle x_p, y_p, z_p \rangle + \frac{1}{5}\langle x_Q - x_p, y_Q - y_p, z_Q - z_p \rangle$$

$$\langle x, y, z \rangle = \langle x_p + \frac{1}{5}(x_Q - x_p), y_p + \frac{1}{5}(y_Q - y_p), z_p + \frac{1}{5}(z_Q - z_p) \rangle$$

$$\langle x, y, z \rangle = \langle \frac{4}{5}x_p + \frac{1}{5}x_Q, \frac{4}{5}y_p + \frac{1}{5}y_Q, \frac{4}{5}z_p + \frac{1}{5}z_Q \rangle$$

(3)

First, find a point that lies on both planes.

$$x+2y-z=4$$
$$2x+y-2z=4$$

Set $z=0$.

$$x+2y=4 \quad (1)$$
$$2x+y=4 \quad (2)$$

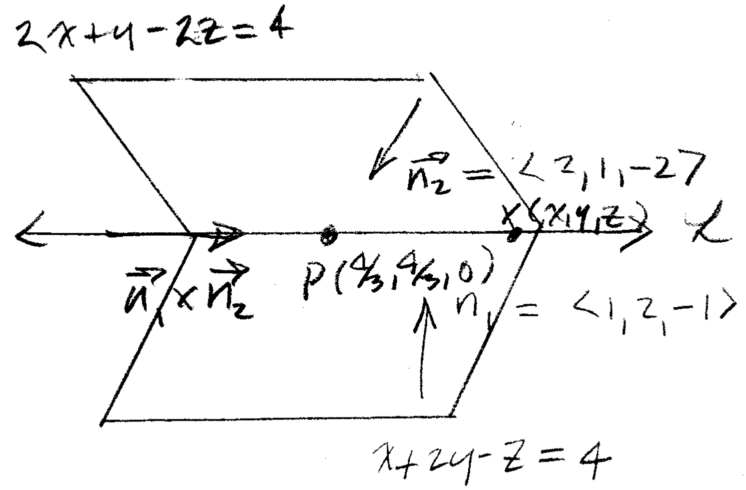
$-2(1)$: $-2x-4y=-8 \quad (3)$
$2x+y=4 \quad (2)$

$(3)+(2)$: $-3y=-4$
$y=4/3$

Sub $y=4/3$ in (1):

$$x+2(4/3)=4$$
$$x+8/3=12/3$$
$$x=4/3$$

Point $P(4/3, 4/3, 0)$ is on line L.



Find the cross product

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & -2 \end{vmatrix}$$

$$= \langle -3, 0, -3 \rangle$$

Eqn of line L

$$\vec{PX} = \lambda(\vec{n}_1 \times \vec{n}_2)$$

$$\langle x-4/3, y-4/3, 0 \rangle = \lambda \langle -3, 0, -3 \rangle$$

$$\boxed{\begin{aligned} x &= 4/3 - 3\lambda \\ y &= 4/3 \\ z &= -3\lambda \end{aligned}}$$