

College of the Redwoods
Mathematics Department

Math 50C — Multivariable Calculus
Quiz #3

David Arnold

Quiz Questions

Read Carefully! You have until Wednesday (9/26/07) to complete the quiz. The quiz is due at the beginning of class on Wednesday (9/26/07). Late quizzes are not accepted.

This quiz is open notes, open book. This includes any supplementary texts or online documents. You must answer all of the exercises on your own. You are not allowed to work in groups or pairs on the quiz. You are not allowed to enlist the aid of a tutor or friend to help with the quiz. You are not allowed to read the exercises in the quiz, then seek help on similar questions. Once you open the quiz and read the questions, you may not seek any outside help of any kind.

I am not interested in reading pages and pages of calculations without accompanying narrative. It is essential that you include sound mathematical writing that both explains and justifies your solution or proof. Grammar and punctuation are important, as is the organization of your solution on the written page.

When working in the lab, please do not work at a terminal next to any other student who is also working on the quiz. For the sake of propriety, please separate yourselves when working on the quiz in the lab. Know that PS110 is also available for computer work. You can log onto the computers in PS110 and obtain files in your account in exactly the same manner as in PS116.

For information on transferring files from home, see the links for connecting to MSEM Mac from both PC's and Macs on the syllabus.

<http://online.redwoods.edu/instruct/darnold/MULTCALC/Syl.htm>

Place the solution to each exercise on a separate sheet of paper. On a good sheet of paper, write out (longhand) and sign the following honor pledge.

I promise that all work found herein is my own. I have received no help from tutors, colleagues, or other teachers. I also promise that I have refrained from sharing my work and ideas with other students in the class. I have also honored all of the quiz constraints listed in the directions.

Arrange your solutions in order, place these quiz page(s) on top of your solutions, then place the honor pledge on top of the quiz as a cover sheet. Staple. Good luck!

EXERCISE 1. Consider the polar equation

$$r = 1 - \cos \theta. \quad (1)$$

- (a) Use Matlab's `plot` command and the polar to rectangular transformation equations to sketch the plot the the polar equation (1). Include a printout of the plot and the script file code that produced the plot.

Solution: See:

<http://online.redwoods.edu/instruct/darnold/multcalc/fall2007/quizzes/quiz3/quiz3.html>

Exercise 1

- (b) Use integration (by hand, no numerics) to determine the length of the curve created in part (a).

Solution: The length is given by the formula

$$L = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

Set $r = 1 - \cos \theta$ and $dr/d\theta = \sin \theta$ in the formula and compute

$$\begin{aligned} L &= \int_0^{2\pi} \sqrt{(1 - \cos \theta)^2 + (\sin \theta)^2} d\theta \\ &= \int_0^{2\pi} \sqrt{1 - 2 \cos \theta + \cos^2 \theta + \sin^2 \theta} d\theta \\ &= \int_0^{2\pi} \sqrt{2 - 2 \cos \theta} d\theta \\ &= \sqrt{2} \int_0^{2\pi} \sqrt{1 - \cos \theta} d\theta \\ &= 2 \int_0^{2\pi} \sqrt{\frac{1 - \cos \theta}{2}} d\theta. \end{aligned}$$

Use the half angle identity for cosine to write

$$\begin{aligned} L &= 2 \int_0^{2\pi} \sqrt{\sin^2 \frac{\theta}{2}} d\theta \\ &= 2 \int_0^{2\pi} \left| \sin \frac{\theta}{2} \right| d\theta \\ &= 2 \int_0^{2\pi} \sin \frac{\theta}{2} d\theta. \end{aligned}$$

Integrate.

$$\begin{aligned} L &= 2 \left[-\cos \frac{\theta}{2} \cdot 2 \right]_0^{2\pi} \\ &= -4 [\cos \pi - \cos 0] \\ &= -4[-1 - 1] \\ &= 8. \end{aligned}$$

Exercise 1

- (c) Use integration (by hand, no numerics) to determine the area contained inside the curve created in part (a).

Solution: The area is given by the formula

$$A = \int_0^{2\pi} \frac{1}{2} r^2 d\theta.$$

Substituting $r = 1 - \cos \theta$,

$$\begin{aligned} A &= \int_0^{2\pi} \frac{1}{2} (1 - \cos \theta)^2 d\theta \\ &= \frac{1}{2} \int_0^{2\pi} (1 - 2 \cos \theta + \cos^2 \theta) d\theta. \end{aligned}$$

Using the half angle identity for cosine,

$$\begin{aligned} A &= \frac{1}{2} \int_0^{2\pi} \left(1 - 2 \cos \theta + \frac{1 + \cos 2\theta}{2} \right) d\theta \\ &= \frac{1}{4} \int_0^{2\pi} (3 - 4 \cos \theta + \cos 2\theta) d\theta. \end{aligned}$$

Integrating,

$$\begin{aligned} A &= \frac{1}{4} \left[3\theta - 4 \sin \theta + \frac{1}{2} \sin 2\theta \right]_0^{2\pi} \\ &= \frac{1}{4} [6\pi] \\ &= \frac{3\pi}{2}. \end{aligned}$$

Exercise 1