

College of the Redwoods
Mathematics Department
Multivariable Calculus

Level Curves in Matlab

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1. Introduction and Prerequisites

In this activity, Matlab is used to explore the level curve concept of functions mapping R^2 into R . Some familiarity with Matlab's `meshgrid` command is required, as well as rudimentary knowledge of Matlab's element wise operators (`.*`, `./`, `.^`).

2. Level Sets and Contours

Let's begin with a definition.

Definition 1 *Let $f : R^2 \rightarrow R$. The set $\{(x, y) : f(x, y) = c\}$, where c is an arbitrary constant, is called a level set of the function f .*

Consider the function $f : R^2 \rightarrow R$ defined by $f(x, y) = x^2 + y^2$. The level sets of f are then defined by

$$\{(x, y) : f(x, y) = c\} \quad \text{or} \quad \{(x, y) : x^2 + y^2 = c\} \quad (1)$$

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If you choose $c = 1$ in [Equation 1](#), then the set of points

$$\{(x, y) : x^2 + y^2 = 1\} \quad (2)$$

is a circle of radius 1, centered at the origin (See [Figure 1](#)), and is called a level set of the function f .

The usual practice is to sketch several level sets by selecting different values for the constant c . For example, if you let $c = 1, 2, 3, 4, 5$ in [Equation 1](#), then the following level sets are obtained.

$$\begin{aligned} \{(x, y) : x^2 + y^2 = 1\} \\ \{(x, y) : x^2 + y^2 = 2\} \\ \{(x, y) : x^2 + y^2 = 3\} \\ \{(x, y) : x^2 + y^2 = 4\} \\ \{(x, y) : x^2 + y^2 = 5\} \end{aligned}$$

Each of these level sets is a circle, centered at the origin, with radius 1, $\sqrt{2}$, $\sqrt{3}$, 2, and $\sqrt{5}$. Note that it is customary to label each level

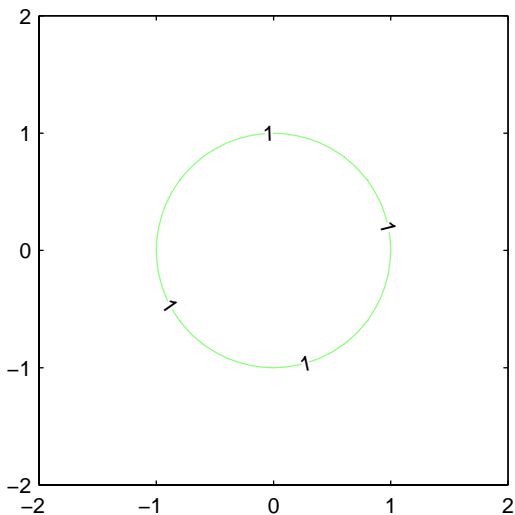
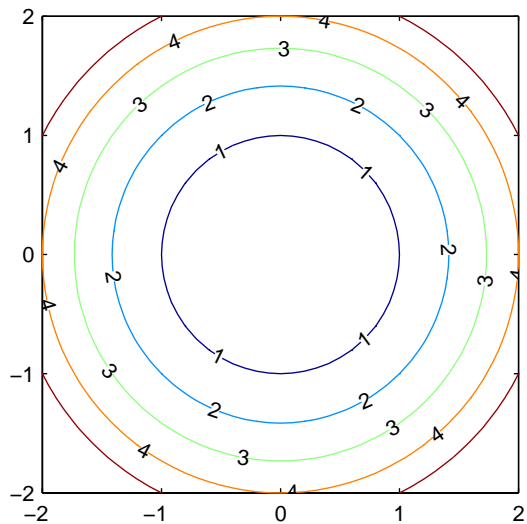


Figure 1: The level set $f(x, y) = 1$.

Figure 2: Level curves for $c = 1, 2, 3, 4, 5$.

curve with its c -value, as shown in [Figure 2](#).

The level sets pictured in [Figure 2](#) offer a wealth of visual information about the function f . For example, as you move away from the point $(0,0)$, located at the center of [Figure 2](#), the function values increase. As you move toward the point $(0,0)$, the function values decrease. It is no coincidence that the level sets in [Figure 2](#) closely resemble a *topographical* map, where each contour represents a constant height.

There are numerous applications where level curves can be very useful. For example, suppose that the function $f(x, y) = x^2 + y^2$ used to generate the level curves in [Figure 2](#) represents the temperature (in degrees Fahrenheit) at the position (x, y) . Any point selected from the curve $x^2 + y^2 = 1$ will have temperature 1°F , points selected from the curve $x^2 + y^2 = 2$ will have temperature 2°F , and so on.

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2.1. Matlab's Contour Command

Matlab simplifies the process of constructing level curves, even for the most difficult of functions.

Example 1 *Sketch several level curves of the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by*

$$f(x, y) = \frac{-3y}{x^2 + y^2 + 1} \quad (3)$$

over the region $\{(x, y) : -2 \leq x \leq 2, -2 \leq y \leq 2\}$ and label each level curve with its constant function value.

Solution. First use the `meshgrid` command to create a grid of x and y -values on the given domain. Calculate the function value at each point and use Matlab's `contour` command to draw the level curves. The following commands should produce an image similar to that in [Figure 3](#).

```
>> [x,y]=meshgrid(-2:.1:2);  
>> z=-3*y./(x.^2+y.^2+1);  
>> contour(x,y,z)
```

If you are not satisfied by the number of level curves produced, it is a simple matter to add more. The following command should produce 10 level curves, similar to those in [Figure 4](#).

```
>> contour(x,y,z,10)
```

2.2. Labeling the Contours

It is a simple task to label each level curves with its constant function value. The following commands were used to produce the image in [Figure 5](#).

```
>> [c,h]=contour(x,y,z);  
>> clabel(c,h);
```



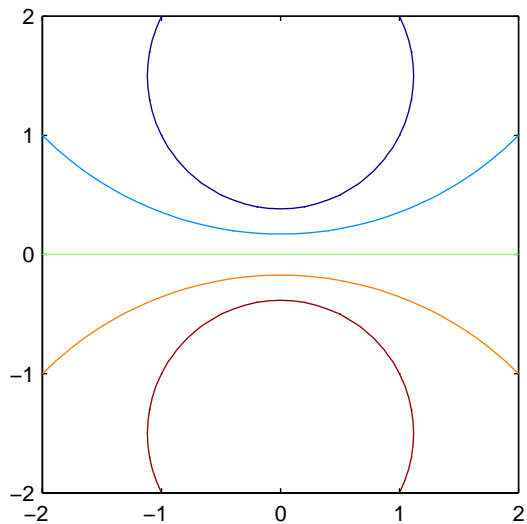


Figure 3: Level curves of $f(x, y) = -3y/(x^2 + y^2 + 1)$.

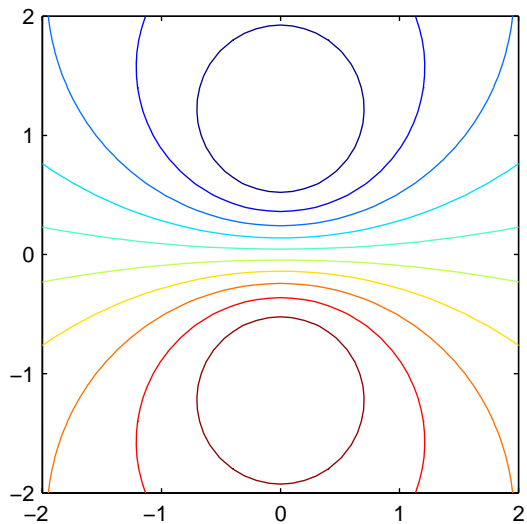


Figure 4: Ten level curves.

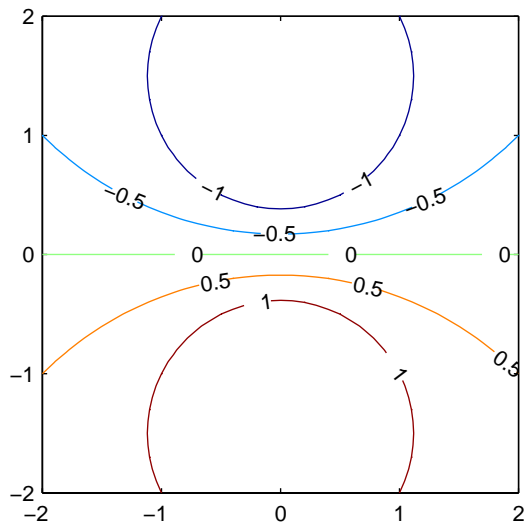


Figure 5: Labeling the level curves.

Note: If you are unhappy with the placement of the labels, then you might want to try `clabel(c,h,'manual')` instead. This command will allow you to place the labels with your mouse.

2.3. Choosing Particular Contours

You might have noted by now that Matlab automatically decides on the optimum c -values when plotting the level curves $f(x, y) = c$. You can easily override this automatic selection and plot contours for particular c -values. For example, suppose that you want level curves for $c = -1.25, -1.00, -0.75, \dots, 1.5$. Recall that the Matlab code `-1.25:.25:1.25` will produce this vector of c -values. The following command was used to create the image in [Figure 6](#).

```
>> [c,h]=contour(x,y,z,-1.25:.25:1.25);  
>> clabel(c,h)
```

Note: Again, if you do not care for the crowded appearance of the labels in [Figure 6](#), try the command `clabel(c,h,'manual')`, which

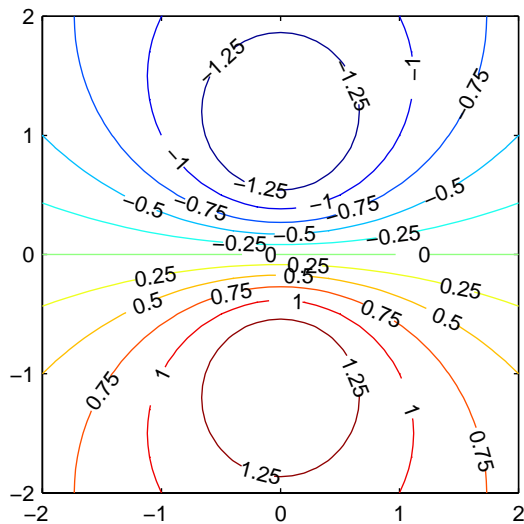


Figure 6: Specifying particular contours.

will allow you to set the labels individually with the mouse.

2.4. Implicit Function Plotting

Often, an equation such as

$$x^3 + y^3 = 3xy \quad (4)$$

is difficult (or impossible) to solve for y in terms of x . However, you can use Matlab's `contour` command as an implicit function plotter, eliminating the need to explicitly solve the equation for y in terms of x before plotting. Begin by making one side of [Equation 4](#) equal to zero.

$$x^3 + y^3 - 3xy = 0 \quad (5)$$

Next, define a function $f : R^2 \rightarrow R$ by $f(x, y) = x^3 + y^3 - 3xy$. [Equation 5](#) now reads

$$f(x, y) = 0, \quad (6)$$

where $f(x, y) = x^3 + y^3 - 3xy$. Consequently, [Equation 6](#) is the level curve $f(x, y) = 0$ of the function $f(x, y) = x^3 + y^3 - 3xy$. You can plot a single level curve of a function by using Matlab's `contour` command in the form `contour(x,y,z,[c c])`. The following commands should produce an image similar to that in [Figure 7](#). Note how a finer mesh is used in this example to improve the accuracy of the plot.

```
>> [x,y]=meshgrid(-2:.05:2);  
>> z=x.^3+y.^3-3*x.*y;  
>> contour(x,y,z,[0,0])
```

2.5. Homework Exercises

1. Consider the function defined by the equation

$$f(x, y) = -xye^{-x^2-y^2}$$

- (a) Use Matlab's `contour` command to plot twenty (20) contours over the domain $\{(x, y) : -2 \leq x, y \leq 2\}$.

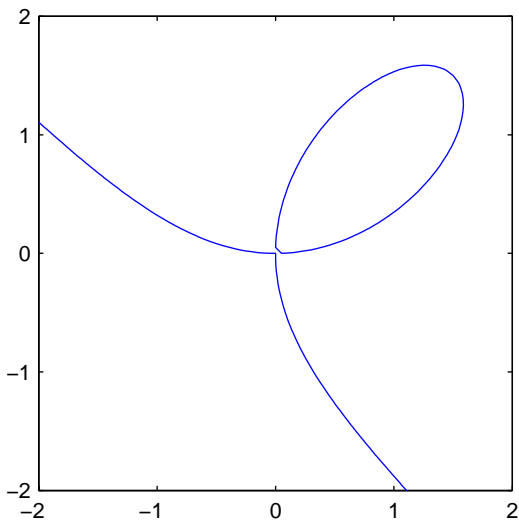


Figure 7: Plotting a single contour.

- (b) Use the form `clabel(c,h,'manual')` to selectively label several contours with the mouse.
- (c) Obtain a printout of your result.

2. Consider the function defined by the equation

$$f(x, y) = -xye^{-x^2-y^2}$$

- (a) Use Matlab's `contour3` command to plot twenty (20) contours over the domain $\{(x, y) : -2 \leq x, y \leq 2\}$. *Hint: Type `help contour3` to obtain help on the `contour3` command.*
- (b) Turn off the grid with the command `grid off`.
- (c) Obtain a printout of your result.

3. Consider the function defined by the equation

$$f(x, y) = -xye^{-x^2-y^2}$$

- (a) Use Matlab's `meshc` command to obtain a simultaneous plot of the surface and the level curves of the function over the domain $\{(x, y) : -2 \leq x, y \leq 2\}$. *Hint: Type `help meshc` to obtain help on the `meshc` command.*
 - (b) Remove hidden line removal with the command `hidden off`.
 - (c) Obtain a printout of your result.
4. Sketch the graph of the famous *knot curve* whose points satisfy the equation

$$(x^2 - 1)^2 = y^2(3 + 2y)$$

Obtain a printout of your result.