Level Curves in Matlab

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Math 50C—Multivariable Calculus

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Abstract

Matlab is used to explore the level curve concept of functions mapping R^2 into R.

Prerequisites. Some familiarity with Matlab's `meshgrid` command is required. Rudimentary knowledge of Matlab's element wise operators (`.*`, `./`, `.^`).

Introduction

Let's begin with a definition.

Definition 1 *Let $f : R^2 \to R$. The set $\{(x, y) : f(x, y) = c\}$, where c is an arbitrary constant, is called a level set of the function f.*

Consider the function $f : R^2 \to R$ defined by $f(x, y) = x^2 + y^2$. If you choose $c = 1$, then the set of points

$$\begin{aligned} & \{(x, y) : f(x, y) = c\} \\ = & \{(x, y) : f(x, y) = 1\} \\ = & \left\{(x, y) : x^2 + y^2 = 1\right\} \end{aligned}$$

is a circle of radius 1, centered at the origin (Figure 1), and is called a level set of the function f.

The usual practice is to sketch several level sets by selecting different values for the constant c. For example, if you let $c = 1, 2, 3, 4, 5$, then the following level sets are obtained.

$$\begin{gathered} \left\{(x, y) : x^2 + y^2 = 1\right\} \\ \left\{(x, y) : x^2 + y^2 = 2\right\} \\ \left\{(x, y) : x^2 + y^2 = 3\right\} \\ \left\{(x, y) : x^2 + y^2 = 4\right\} \\ \left\{(x, y) : x^2 + y^2 = 5\right\} \end{gathered}$$

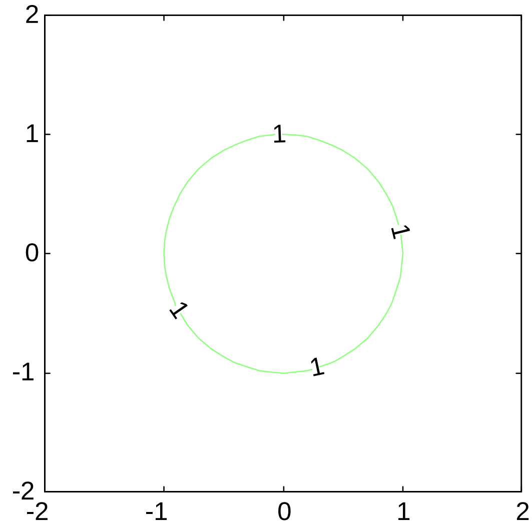

Figure 1: The level set $f(x, y) = 1$.

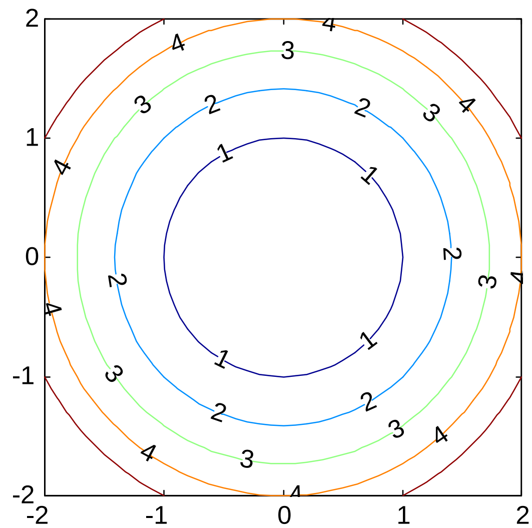

Figure 2: Level curves for $c = 1, 2, 3, 4, 5$.

Each of these level sets is a circle, centered at the origin, with radius 1, $\sqrt{2}$, $\sqrt{3}$, 2, and $\sqrt{5}$, as shown in Figure 2.Note that it is customary to label each level curve with its c-value, as shown in Figure 2.

The level sets pictured in Figure 2 offer a wealth of visual information about the function f. For example, as you move away from the point $(0,0)$, located at the center of Figure 2, the function values increase. As you move toward the point $(0,0)$, the function values decrease. It is no coincidence that the level sets in Figure 2 closely resemble a *topographical* map, where each contour represents a constant height.

There are numerous applications where level curves can be very useful. For example, suppose that the function $f(x,y) = x^2 + y^2$ used to generate the level curves in Figure 2 represents the temperature (in degrees Fahrenheit) at the position (x,y). Any point selected from the curve $x^2 + y^2 = 1$ will have temperature 1 °F, points selected from the curve $x^2 + y^2 = 2$ will have temperature 2 °F, and so on.

Matlab's Contour Command

Matlab simplifies the process of constructing level curves, even for the most difficult of functions.

Example 2 *Sketch several level curves of the function $f : R^2 \to R$ defined by*

$$f(x,y) = \frac{-3y}{x^2 + y^2 + 1}$$

over the region $\{(x,y) : -2 \leq x \leq 2,\ -2 \leq y \leq 2\}$ and label each level curve with its constant function value.

Solution. First use the meshgrid command to create a grid of x- and y-values on the given domain. Calculate the function value at each point and use Matlab's `contour` command to draw the level curves. The following commands should produce an image similar to that in Figure 3.

```
>> [x,y]=meshgrid(-2:.1:2);
>> z=-3*y./(x.^2+y.^2+1);
>> contour(x,y,z)
```

If you are not satisfied by the number of level curves produced, it is a simple matter to add more. The following command should produce 10 level curves, similar to those in Figure 4.

```
>> contour(x,y,z,10)
```

Labeling the Contours

It is a simple task to label each level curves with its constant function value. The following commands were used to produce the image in Figure 5.

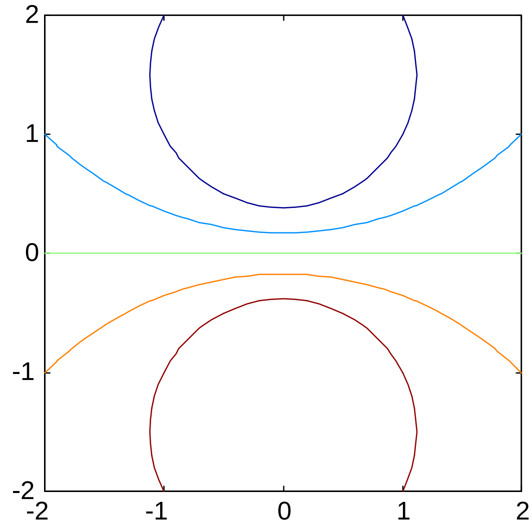

Figure 3: Level curves of $f(x, y) = -3y/(x^2 + y^2 + 1)$.

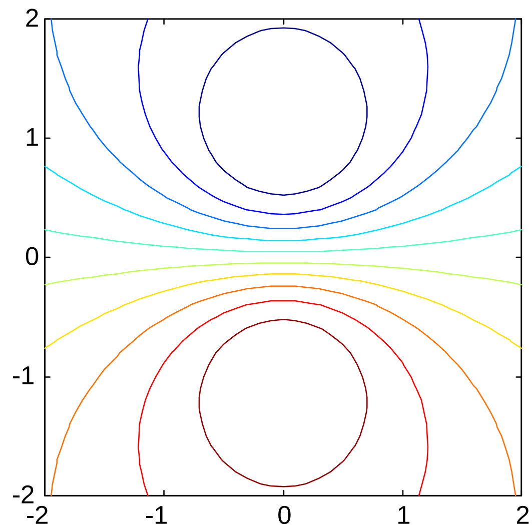

Figure 4: Ten level curves.

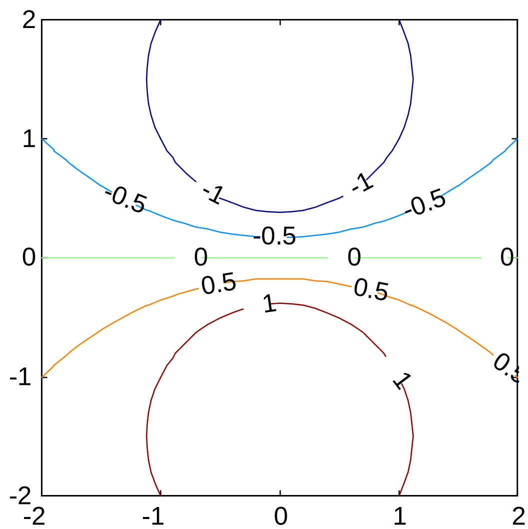

Figure 5: Labeling the level curves.

```
>> [c,h]=contour(x,y,z);
>> clabel(c,h);
```

Note: If you are unhappy with the placement of the labels, then you might want to try `clabel(c,h,'manual')` *instead. This command will allow you to place the labels with your mouse.*

Choosing Particular Contours

You might have noted by now that Matlab automatically decides on the optimum c-values when plotting the level curves $f(x, y) = c$. You can easily override this automatic selection and plot contours for particular c-values. For example, suppose that you want level curves for $c = -1.25, -1.00, -0.75, \ldots, 1.5$. Recall that the Matlab code `-1.25:.25:1.25` will produce this vector of c-values. The following command was used to create the image in Figure 6.

```
>> [c,h]=contour(x,y,z,-1.25:.25:1.25);
>> clabel(c,h)
```

Note: Again, if you do not care for the crowded appearance of the labels in Figure 6, try the command `clabel(c,h,'manual')`*, which will allow you to set the labels individually with the mouse.*

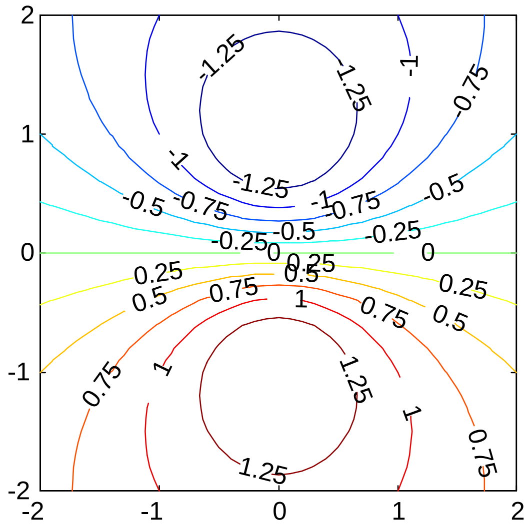

Figure 6: Specifying particular contours.

Implicit Function Plotting

Often, an equation such as

$$x^3 + y^3 = 3xy \tag{1}$$

is difficult (or impossible) to solve for y in terms of x. However, you can use Matlab's `contour` command as an implicit function plotter, eliminating the need to explicitly solve the equation for y in terms of x before plotting. Begin by making one side of the equation (1) equal to zero.

$$x^3 + y^3 - 3xy = 0 \tag{2}$$

Next, define a function $f : R^2 \to R$ by $f(x, y) = x^3 + y^3 - 3xy$. Equation (2) now reads

$$f(x, y) = 0 \tag{3}$$

where $f(x, y) = x^3 + y^3 - 3xy$. Consequently, equation (3) is the level curve $f(x, y) = 0$ of the function $f(x, y) = x^3 + y^3 - 3xy$. You can plot a single level curve of a function by using Matlab's `contour` command in the form `contour(x,y,z,[c c])`. The following commands should produce an image similar to that in Figure 7. Note how a finer mesh is used in this example to improve the accuracy of the plot.

```
>> [x,y]=meshgrid(-2:.05:2);
>> z=x.^3+y.^3-3*x.*y;
>> contour(x,y,z,[0,0])
```

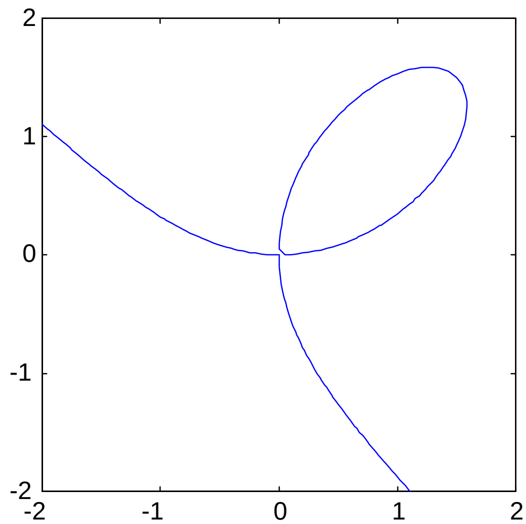

Figure 7. Plotting a single contour.

Homework Exercises

1. Consider the function defined by the equation

$$f(x,y) = -xye^{-x^2-y^2}$$

   (a) Use Matlab's `contour` command to plot twenty (20) contours over the domain $\{(x,y) : -2 \le x, y \le 2\}$.

   (b) Use the form `clabel(c,h,'manual')` to selectively label several contours with the mouse.

   (c) Obtain a printout of your result.

2. Consider the function defined by the equation

$$f(x,y) = -xye^{-x^2-y^2}$$

   (a) Use Matlab's `contour3` command to plot twenty (20) contours over the domain $\{(x,y) : -2 \le x, y \le 2\}$. *Hint: Type* `help contour3` *to obtain help on the* `contour3` *command.*

   (b) Turn off the grid with the command `grid off`.

   (c) Obtain a printout of your result.

3. Consider the function defined by the equation

$$f(x,y) = -xye^{-x^2-y^2}$$

(a) Use Matlab's `meshc` command to obtain a simultaneous plot of the surface and the level curves of the function over the domain $\{(x,y) : -2 \leq x, y \leq 2\}$. *Hint: Type* `help meshc` *to obtain help on the* `meshc` *command.*

(b) Remove hidden line removal with the command `hidden off`.

(c) Obtain a printout of your result.

4. Sketch the graph of the famous *knot curve* whose points satisfy the equation

$$(x^2 - 1)^2 = y^2(3 + 2y)$$

Obtain a printout of your result.