

College of the Redwoods  
Mathematics Department

Math 50B — Integral Calculus  
Exam #5

David Arnold

## Quiz Questions

**Read Carefully!** You have until Wednesday (5/7/08) to complete the exam. The exam is due at the beginning of class on Wednesday (5/7/08). Late exams are not accepted.

This exam is open notes, open book. This includes any supplementary texts or online documents. You must answer all of the exercises on your own. You are not allowed to work in groups or pairs on the exam. You are not allowed to enlist the aid of a tutor or friend to help with the exam. You are not allowed to read the exercises in the exam, then seek help on similar questions. Once you open the exam and read the questions, you may not seek any outside help of any kind.

I am not interested in reading pages and pages of calculations without accompanying narrative. It is essential that you include sound mathematical writing that both explains and justifies your solution or proof. Grammar and punctuation are important, as is the organization of your solution on the written page.

When working in the Mathlab or PS116, please do not work next to any other student who is also working on the exam. For the sake of propriety, please separate yourselves when working on the exam in these areas.

Place the solution to each exercise on a separate sheet of paper. On a good sheet of paper, write out (longhand) and sign the following honor pledge.

I promise that all work found herein is my own. I have received no help from tutors, colleagues, or other teachers. I also promise that I have refrained from sharing my work and ideas with other students in the class. I have also honored all of the exam constraints listed in the directions.

Arrange your solutions in order, place these exam page(s) on top of your solutions, then place the honor pledge on top of the exam as a cover sheet. Staple. Good luck!

EXERCISE 1. Consider the sequence whose  $n$ th term is defined by

$$a_n = \left(1 + \frac{2}{n}\right)^n.$$

- Use Winplot to sketch the first 20 terms of the sequence. Estimate the limit based on the resulting plot. Obtain a printout and include it with your examination papers.
- Use l' Hopital's rule to find the exact limit of the sequence.

EXERCISE 2. Find the sum of the series defined by

$$\sum_{n=0}^{\infty} 2 \left(-\frac{1}{3}\right)^n.$$

EXERCISE 3. Find the sum of the series defined by

$$\sum_{n=0}^{\infty} \frac{1}{n^2 + 5n + 6}.$$

*Hint: Partial fraction decomposition.*

EXERCISE 4. Use the Integral Test to determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$$

converges or diverges. Make sure you demonstrate that the hypotheses of the Integral Test Theorem are satisfied.

EXERCISE 5. Use the Limit Comparison Test to determine whether the series define by

$$\sum_{n=1}^{\infty} \frac{3n^2 + n}{n^4 + \sqrt{n}}$$

converges or diverges.

EXERCISE 6. Use the Ratio Test to determine the interval of convergence for the power series defined by

$$\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n+1)}{n!} x^n.$$

Also, state the radius of convergence.

EXERCISE 7. Use differentiation to determine a power series for the function

$$f(x) = \frac{1}{(x-1)^2}.$$

Use summation notation within the body of your calculation and state your final answer using summation notation.

EXERCISE 8. Consider the function defined by

$$f(x) = \sqrt{1+x}.$$

- Find  $T_3(x)$ , the third degree Taylor polynomial. Expand around  $x = 0$ .
- Use Winplot (or Geogebra) to plot both  $f$  and  $T_3$  on the same coordinate system over the interval  $(-1, 1)$ . Obtain a printout and include it with your examination papers.
- Use Taylor's inequality to estimate the accuracy of the approximation  $f(x) \approx T_3(x)$  when  $x$  lies in the interval  $(-1, 1)$ .
- Check your result in part (3) by using Winplot to graph  $|R_3(x)|$  on  $(-1, 1)$ . Include your estimate from part (3) as a horizontal line in your plot. Obtain a printout and include it with your examination papers.