5 pts

1

	D	I
+	x^3	e^{-x}
−	$3x^2$	$-e^{-x}$
+	$6x$	e^{-x}
−	6	$-e^{-x}$
+	0	e^{-x}

$$\int x^3 e^{-x}\,dx$$

$$= -x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x}$$

$$= -e^{-x}(x^3+3x^2+6x+6)$$

5 pts

2

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$x = 2\sin\theta$, $-\pi/2 \le \theta \le \pi/2$

$dx = 2\cos\theta\,d\theta$

$$= \int \frac{2\cos\theta\,d\theta}{4\sin^2\theta\sqrt{4-4\sin^2\theta}}$$

$$= \int \frac{2\cos\theta\,d\theta}{(4\sin^2\theta)\sqrt{4}\sqrt{1-\sin^2\theta}}$$

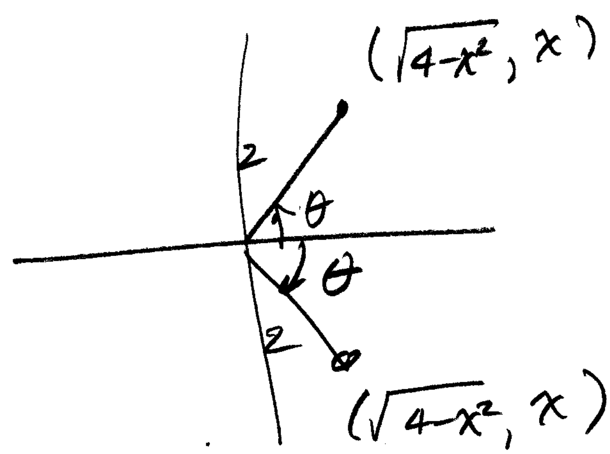
$$= \frac{2}{8}\int \frac{\cos\theta\,d\theta}{\sin^2\theta\sqrt{\cos^2\theta}}$$

$$= \frac{1}{4}\int \frac{\cos\theta\,d\theta}{\sin^2\theta\,|\cos\theta|},$$

Since $-\pi/2 \le \theta \le \pi/2$,
$\cos\theta > 0$ & $|\cos\theta| = \cos\theta$

$$= \frac{1}{4}\int \frac{\cos\theta\,d\theta}{(\sin^2\theta)(\cos\theta)}$$

$$= \frac{1}{4}\int \frac{d\theta}{\sin^2\theta}$$



$$= \frac{1}{4}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{4}\cot\theta + C$$

$$= -\frac{1}{4}\cdot\frac{\sqrt{4-x^2}}{x} + C$$

$$= -\frac{\sqrt{4-x^2}}{4x} + C.$$

5 pts

(3) $\displaystyle\int \frac{dx}{x^3+4x}$

$$\frac{1}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$$

$$1 = Ax^2+4A+Bx^2+Cx$$

$$1 = (A+B)x^2+Cx+4A$$

$$= \int\left[\frac{1/4}{x} + \frac{-1/4\, x}{x^2+4}\right]dx$$

$A+B=0$

$C=0$

$4A=1$

$$= \frac{1}{4}\int \frac{1}{x}dx - \frac{1}{8}\int\frac{2x}{x^2+4}dx$$

$$= \frac{1}{4}\ln|x| - \frac{1}{8}\ln|x^2+4| + C$$

$\therefore A=\frac{1}{4}, B=-\frac{1}{4}, C=0$

$$= \frac{2}{8}\ln|x| - \frac{1}{8}\ln|x^2+4| + C$$

$$= \frac{1}{8}\left[2\ln|x| - \ln|x^2+4|\right] + C$$

$$= \frac{1}{8}\left[\ln|x|^2 - \ln|x^2+4|\right] + C$$

$$= \frac{1}{8}\left[\ln|x^2| - \ln|x^2+4|\right] + C$$

$$= \frac{1}{8}\left[\ln x^2 - \ln(x^2+4)\right] + C$$

$$= \frac{1}{8}\ln\frac{x^2}{x^2+4} + C$$

$$= \ln\sqrt[8]{\frac{x^2}{x^2+4}} + C$$

5pts

(4) $F(s) = \int_0^\infty f(t)e^{-st}\,dt$

$$= \int_0^\infty e^{-2t}e^{-st}\,dt$$

$$= \int_0^\infty e^{-(2t+st)}\,dt$$

$$= \int_0^\infty e^{-(s+2)t}\, dt$$

$$= \lim_{T\to\infty} \int_0^T e^{-(s+2)t}\, dt$$

$$= \lim_{T\to\infty} \left. \frac{e^{-(s+2)t}}{-(s+2)} \right|_0^T$$

$$= \lim_{T\to\infty} \frac{-1}{s+2} \left[e^{-(s+2)T} - e^{-(s+2)(0)} \right]$$

$$= \lim_{T\to\infty} \frac{-1}{s+2} \left[e^{-(s+2)T} - 1 \right]$$

$$= -\frac{1}{s+2} \left[0 - 1 \right], \text{ provided } s+2>0$$

$$= \frac{1}{s+2}, \text{ provided } s>-2.$$

(5) $f(x) = \frac{x^5}{6} + \frac{1}{10x^3} = \frac{1}{6}x^5 + \frac{1}{10}x^{-3}$

$f'(x) = \frac{5}{6}x^4 - \frac{3}{10}x^{-4}$

$1 + [f'(x)]^2 = 1 + \frac{25}{36}x^8 - \frac{1}{2} + \frac{9}{100}x^{-8}$

$= \frac{25}{36}x^8 + \frac{1}{2} + \frac{9}{100}x^{-8}$

Hence, $L = \int_1^2 \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_1^2 \sqrt{\frac{25}{36}x^8 + \frac{1}{2} + \frac{9}{100}x^{-8}}\, dx$

$= \int_1^2 \sqrt{\left(\frac{5}{6}x^4 + \frac{3}{10}x^{-4}\right)^2}\, dx$

$= \int_1^2 \left|\frac{5}{6}x^4 + \frac{3}{10}x^{-4}\right| dx$

$= \int_1^2 \left(\frac{5}{6}x^4 + \frac{3}{10}x^{-4}\right) dx$ because $1 \le x \le 2$

$$L = \frac{1}{6}x^{5} - \frac{1}{10}x^{-3} \Big|_{1}^{2}$$

$$L = \left[\frac{1}{6}(2)^{5} - \frac{1}{10}(2)^{-3}\right] - \left[\frac{1}{6}(1)^{5} - \frac{1}{10}(1)^{-3}\right]$$

$$L = \frac{32}{6} - \frac{1}{80} - \frac{1}{6} + \frac{1}{10}$$

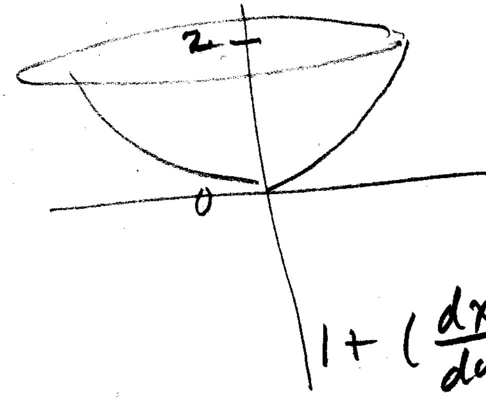
$$L = \frac{31}{6} - \frac{1}{80} + \frac{1}{10}$$

$$L = \frac{1240}{240} - \frac{3}{240} + \frac{24}{240}$$

$$\begin{array}{r} 1240 \\ +\ 24 \\ \hline 1264 \\ -\ 3 \\ \hline 1261 \end{array}$$

$$L = \frac{1261}{240}$$

(6) 5pts



$x = f(y) = y^3$

$\frac{dx}{dy} = f'(y) = 3y^2$

$1 + \left(\frac{dx}{dy}\right)^2 = 1 + [f'(y)]^2 = 1 + 9y^4$

$A = \int 2\pi$

$$\begin{array}{r} 16 \\ 9 \\ \hline 144 \end{array}$$

$$A = \int_0^2 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$u = 1 + 9y^4$

$du = 36y^3\, dy$

$$A = \int_0^2 2\pi y^3 \sqrt{1 + 9y^4}\, dy$$

$$A = \frac{\pi}{18} \int_0^2 \sqrt{1 + 9y^4}\, (36y^3 dy)$$

$$A = \frac{\pi}{18} \int_1^{145} \sqrt{u}\, du$$

$$A = \frac{\pi}{18} \left(\frac{2}{3} u^{3/2}\right)\Bigg|_1^{145}$$

$$A = \frac{\pi}{27} \left[145^{3/2} - 1^{3/2}\right]$$