5pts

①

	D	I
$+$	x^2	$\sin x$
$-$	$2x$	$-\cos x$
$+$	2	$-\sin x$
$-$	0	$\cos x$

$$\int x^2 \sin x\,dx$$

$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

5pts

② $\displaystyle\int \frac{\sqrt{4-x^2}}{x^2}\,dx$

$x = 2\sin\theta,\ -\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$dx = 2\cos\theta\,d\theta$

$$= \int \frac{\sqrt{4-4\sin^2\theta}}{4\sin^2\theta}(2\cos\theta\,d\theta)$$

$$= \int \frac{2\sqrt{1-\sin^2\theta}}{4\sin^2\theta}(2\cos\theta\,d\theta)$$

$$= \int \frac{\sqrt{\cos^2\theta}}{\sin^2\theta}(\cos\theta\,d\theta)$$

$$= \int \frac{|\cos\theta|}{\sin^2\theta}(\cos\theta\,d\theta)$$, but $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ where $\cos\theta > 0$

$$= \int \frac{\cos\theta}{\sin^2\theta}(\cos\theta\,d\theta)$$

$$= \int \cot^2\theta\,d\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

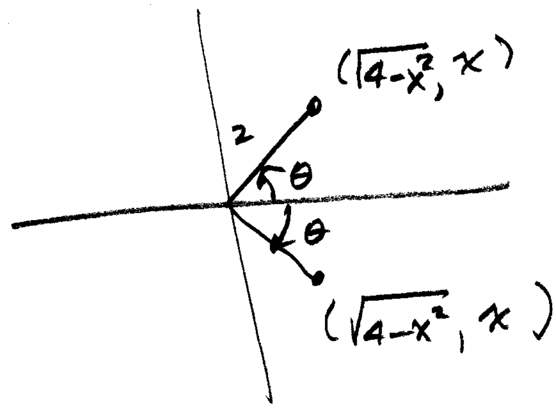
$$\cot^2\theta + 1 = \csc^2\theta$$

$$= \int [\csc^2\theta - 1]\, d\theta$$

$$= -\cot\theta - \theta + C$$

$$= -\frac{\sqrt{4-x^2}}{x} - \sin^{-1}\frac{x}{2} + C$$

$\sin\theta = x/2$



5pts

(3) $$\int \frac{dx}{4-3x-x^2}$$

$$= -\int \frac{dx}{x^2+3x-4}$$

$$\frac{1}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$$

$$= -\int \frac{dx}{(x+4)(x-1)}$$

$$1 = A(x-1) + B(x+4)$$

$x = 1 \Rightarrow B = 1/5$

$x = -4 \Rightarrow A = -1/5$

$$= -\int \left[\frac{-1/5}{x+4} + \frac{1/5}{x-1}\right] dx$$

$$= \frac{1}{5}\int \left[\frac{1}{x+4} - \frac{1}{x-1}\right] dx$$

$$= \frac{1}{5}\left[\ln|x+4| - \ln|x-1|\right] + C$$

$$= \frac{1}{5}\ln\frac{|x+4|}{|x-1|} = \frac{1}{5}\ln\left|\frac{x+4}{x-1}\right| = \ln\left|\frac{x+4}{x-1}\right|^{1/5}$$

5pts

(4) $$\int_e^\infty \frac{dx}{x(\ln x)^p} = \lim_{T\to\infty} \int_e^T \frac{dx}{x(\ln x)^p}$$

$$\int \frac{dx}{x(\ln x)^p} \qquad \begin{aligned} u &= \ln x \\ du &= \frac{1}{x}\,dx \end{aligned}$$

$$= \int \frac{1}{(\ln x)^p} \cdot \frac{1}{x}\,dx$$

$$= \int \frac{1}{u^p}\,du$$

$$= \int u^{-p}\,du$$

$$= \frac{u^{-p+1}}{-p+1}$$

$$= \frac{u^{1-p}}{1-p}$$

$$= \frac{(\ln x)^{1-p}}{1-p}$$

Hence,

$$\int_e^\infty \frac{dx}{x(\ln x)^p}$$

$$= \lim_{T\to\infty} \left[\frac{1}{1-p} \cdot \frac{1}{(\ln x)^{p-1}}\right]_e^T$$

$$= \lim_{T\to\infty} \frac{1}{1-p}\left[\frac{1}{(\ln T)^{p-1}} - \frac{1}{(\ln e)^{p-1}}\right]$$

$$= \lim_{T\to\infty} \frac{1}{1-p}\left[\frac{1}{(\ln T)^{p-1}} - 1\right]$$

$$= \frac{1}{p-1}, \text{ provided } p-1>0$$

$$p>1$$

5pts

(5) $f(x) = \frac{x^3}{6} + \frac{1}{2x} = \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$

$$f'(x) = \frac{1}{2}x^2 - \frac{1}{2}x^{-2}$$

$$1 + [f'(x)]^2 = 1 + \left[\frac{1}{2}x^2 - \frac{1}{2}x^{-2}\right]^2$$

$$= 1 + \frac{1}{4}x^4 - \frac{1}{2} + \frac{1}{4}x^{-4}$$

$$= \frac{1}{4}x^4 + \frac{1}{2} + \frac{1}{4}x^{-4}$$

$$= \left[\frac{1}{2}x^2 + \frac{1}{2}x^{-2}\right]^2$$

Hence, $L = \int_{1/2}^{1} \sqrt{1 + [f'(x)]^2}\, dx$

$$= \int_{1/2}^{1} \sqrt{\left[\frac{1}{2}x^2 + \frac{1}{2}x^{-2}\right]^2}\, dx$$

$$= \int_{1/2}^{1} \left|\frac{1}{2}x^2 + \frac{1}{2}x^{-2}\right| dx$$

$$= \int_{1/2}^{1} \left(\frac{1}{2}x^2 + \frac{1}{2}x^{-2}\right) dx,$$

because $1/2 \le x \le 1$.

Hence,

$$L = \frac{1}{6}x^3 - \frac{1}{2}x^{-1} \Big|_{1/2}^{1}$$

$$= \left[\frac{1}{6}(1)^3 - \frac{1}{2}(1)^{-1}\right] - \left[\frac{1}{6}\left(\frac{1}{2}\right)^3 - \frac{1}{2}\left(\frac{1}{2}\right)^{-1}\right]$$

$$= \left[\frac{1}{6} - \frac{1}{2}\right] - \left[\frac{1}{48} - 1\right]$$

$$= \frac{1}{6} - \frac{1}{2} - \frac{1}{48} + 1$$

$$= \frac{8 - 24 - 1 + 48}{48}$$

$$= \frac{31}{48}$$

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(6) $f(x) = \sqrt{4-x^2}$

$$f'(x) = \frac{1}{2}(4-x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{4-x^2}}$$

$$1+[f'(x)]^2 = 1 + \frac{x^2}{4-x^2} = \frac{4-x^2+x^2}{4-x^2} = \frac{4}{4-x^2}$$

$$A = \int_{-2}^{2} 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$= \int_{-2}^{2} 2\pi \sqrt{4-x^2}\sqrt{\frac{4}{4-x^2}}\,dx$$

$$= \int_{-2}^{2} 2\pi \sqrt{4-x^2} \cdot \frac{2}{\sqrt{4-x^2}}\,dx$$

$$= 4\pi \int_{-2}^{2} dx$$

$$= 4\pi x \Big|_{-2}^{2}$$

$$= 4\pi[2-(-2)]$$

$$= 16\pi$$