

College of the Redwoods
Mathematics Department

Math 50B — Integral Calculus
Exam #3

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Quiz Questions

Read Carefully! You have until Monday (3/24/08) to complete the exam. The exam is due at the beginning of class on Monday (3/24/08). Late examzes are not accepted.

This exam is open notes, open book. This includes any supplementary texts or online documents. You must answer all of the exercises on your own. You are not allowed to work in groups or pairs on the exam. You are not allowed to enlist the aid of a tutor or friend to help with the exam. You are not allowed to read the exercises in the exam, then seek help on similar questions. Once you open the exam and read the questions, you may not seek any outside help of any kind.

I am not interested in reading pages and pages of calculations without accompanying narrative. It is essential that you include sound mathematical writing that both explains and justifies your solution or proof. Grammar and punctuation are important, as is the organization of your solution on the written page.

When working in the Mathlab or PS116, please do not work next to any other student who is also working on the exam. For the sake of propriety, please separate yourselves when working on the exam in these areas.

Place the solution to each exercise on a separate sheet of paper. On a good sheet of paper, write out (longhand) and sign the following honor pledge.

I promise that all work found herein is my own. I have received no help from tutors, colleagues, or other teachers. I also promise that I have refrained from sharing my work and ideas with other students in the class. I have also honored all of the exam constraints listed in the directions.

Arrange your solutions in order, place these exam page(s) on top of your solutions, then place the honor pledge on top of the exam as a cover sheet. Staple. Good luck!

EXERCISE 1. Evaluate each of the following improper integrals. In each case, provide a printout of the plot using Winplot. You may not use any tables of integrals in determining results. If a substitution of some type is required, that is the route you are expected to follow. You must show all work to receive credit.

$$(a) \int_0^{\infty} \frac{dx}{\sqrt{x}(x+4)} dx$$

$$(b) \int_1^4 \frac{dx}{(x-2)^{2/3}} dx$$

EXERCISE 2. In this exercise, the goal is to find a formula for the circumference of a circle of radius r , centered at the origin, using the arc length formula.

- Note the symmetry of the circle, so an argument can be made to find the arc length of the quarter circle in the first quadrant, then multiply the result by 4 to determine the circumference of the full circle. Set up this integral to determine the arc length of the quarter circle of radius r in the first quadrant.
- Note that the definite integral in part (a) is improper. Find an exact value of the definite integral. No tables or memorized formulae may be quoted or used. If push comes to shove, use trigonometric substitution.
- Use the result from part (b) to determine the circumference of the full circle.

EXERCISE 3. A transform is a formula that converts or "transforms" one function into another. The Laplace Transform of a function $f(t)$, which plays an important role in differential equations, is denoted by $\mathcal{L}\{f(t)\}$ and is defined by

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt.$$

Find the Laplace Transform of $\sin t$. That is, find $\mathcal{L}\{\sin t\}$. *Note: Your answer will be a function of s .*

EXERCISE 4. Determine the surface area of a sphere of radius r by rotating the appropriate curve about the x -axis.