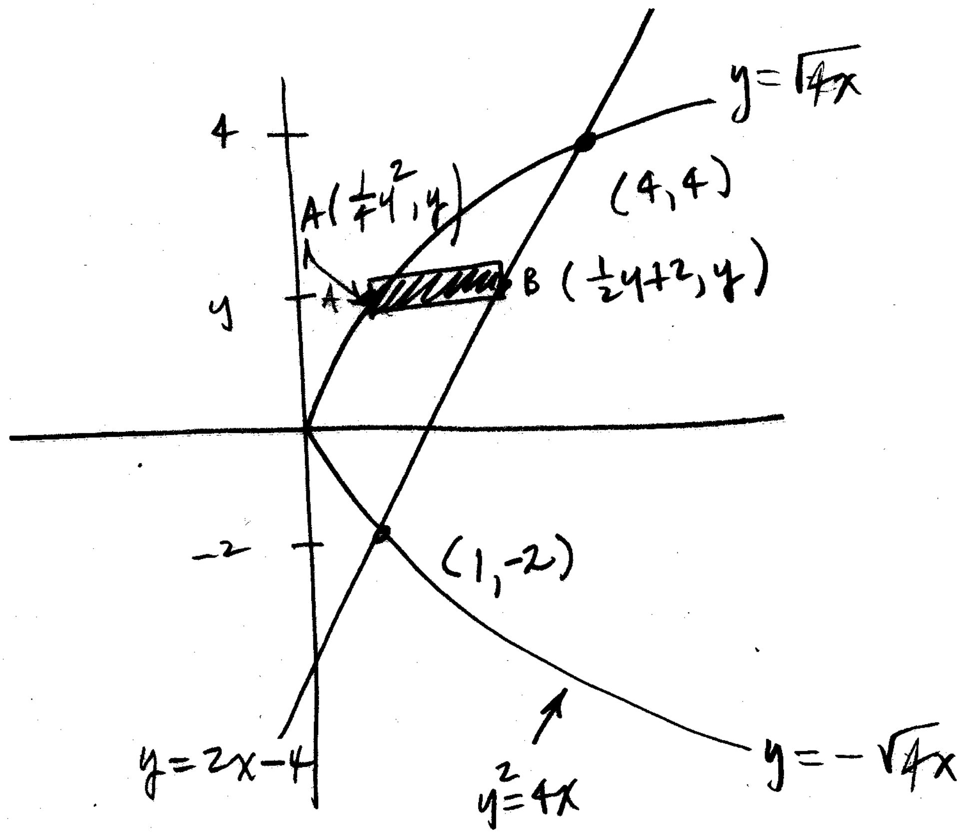
① 10pts



$$\boxed{\begin{aligned} y^2 &= 4x \\ \Rightarrow x &= \tfrac{1}{4}y^2 \end{aligned}}$$

$$\boxed{\begin{aligned} y &= 2x-4 \\ 2x &= y+4 \\ x &= \tfrac{1}{2}y+2 \end{aligned}}$$

$y^2 = 4x$ ①

$y = 2x-4$ ②

Multiply ① by $\frac{1}{2}$: $\frac{1}{2}y^2 = 2x$ ③

Replace $2x$ in ② with $\frac{1}{2}y^2$:

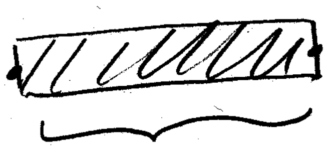
$$y = \tfrac{1}{2}y^2 - 4$$

$$2y = y^2 - 8$$

$$0 = y^2 - 2y - 8$$

$$0 = (y-4)(y+2)$$

$$y = -2, 4$$

dy 

$(\frac{1}{2}y+2) - \frac{1}{4}y^2$

$$dA = \left[\left(\tfrac{1}{2}y+2\right) - \tfrac{1}{4}y^2\right] dy$$

$$dA = \left(-\tfrac{1}{4}y^2 + \tfrac{1}{2}y + 2\right) dy$$

$$A = \int_{-2}^{4} \left(-\frac{1}{4}y^2 + \frac{1}{2}y + 2\right) dy$$

$$= -\frac{1}{12}y^3 + \frac{1}{4}y^2 + 2y \Big|_{-2}^{4}$$

$$= \left[-\frac{1}{12}(4)^3 + \frac{1}{4}(4)^2 + 2(4)\right] - \left[-\frac{1}{12}(-2)^3 + \frac{1}{4}(-2)^2 + 2(-2)\right]$$

$$= \left[-\frac{16}{3} + 4 + 8\right] - \left[\frac{2}{3} + 1 - 4\right]$$

$$= -\frac{16}{3} + 12 - \frac{2}{3} + 3$$

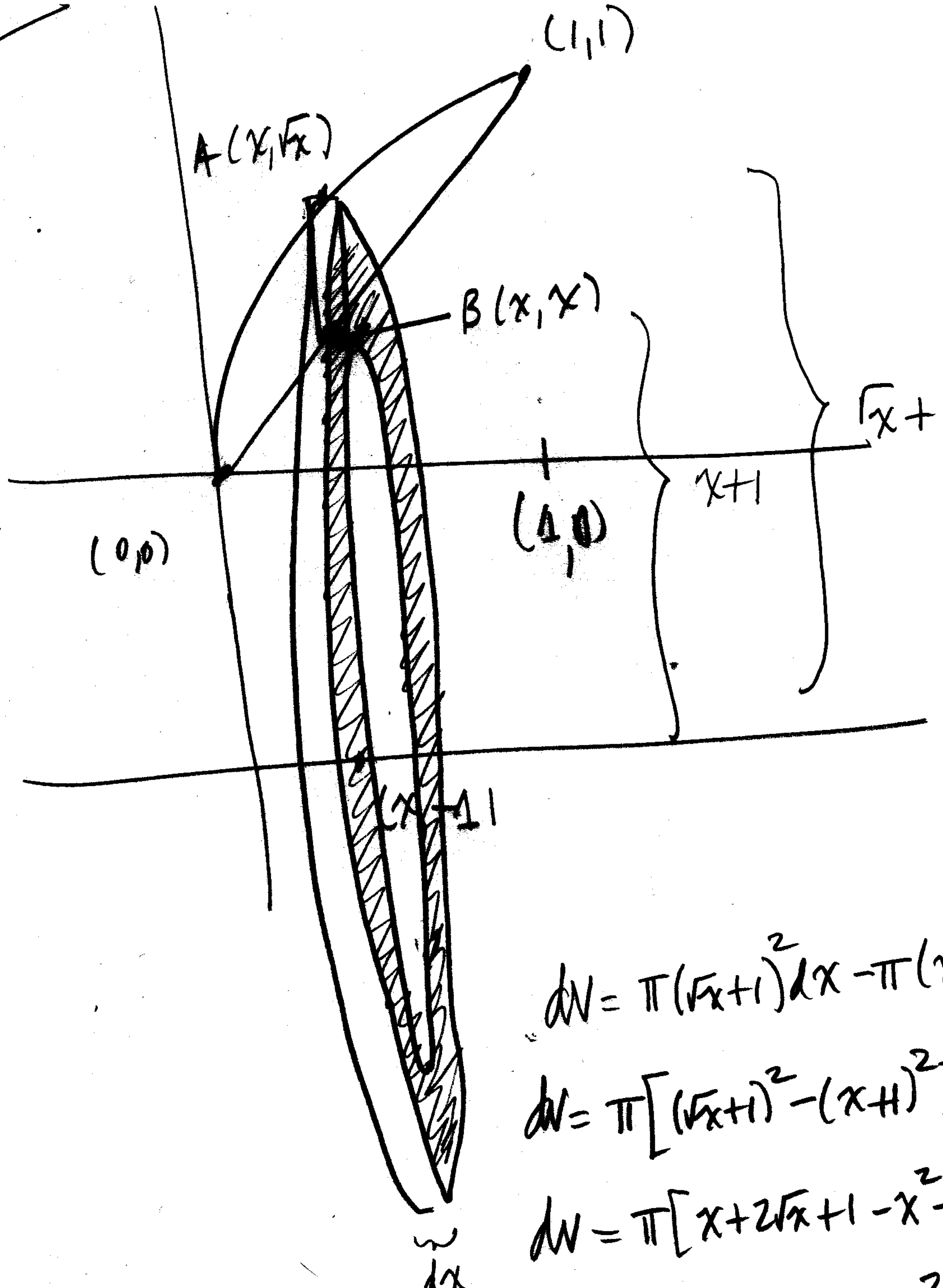
$$= 15 - \frac{18}{3}$$

$$= 15 - 6$$

$$= 9$$

(2)

10 points



$dV = \pi(\sqrt{x}+1)^2 dx - \pi(x+1)^2 dx$

$dV = \pi\left[(\sqrt{x}+1)^2 - (x+1)^2\right] dx$

$dV = \pi\left[x + 2\sqrt{x} + 1 - x^2 - 2x - 1\right] dx$

$dV = \pi\left[2\sqrt{x} - x - x^2\right] dx$

$$V = \int_0^1 \pi \left[2x^{1/2} - x - x^2 \right] dx$$

$$= \pi \left[\frac{4}{3}x^{3/2} - \frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1$$

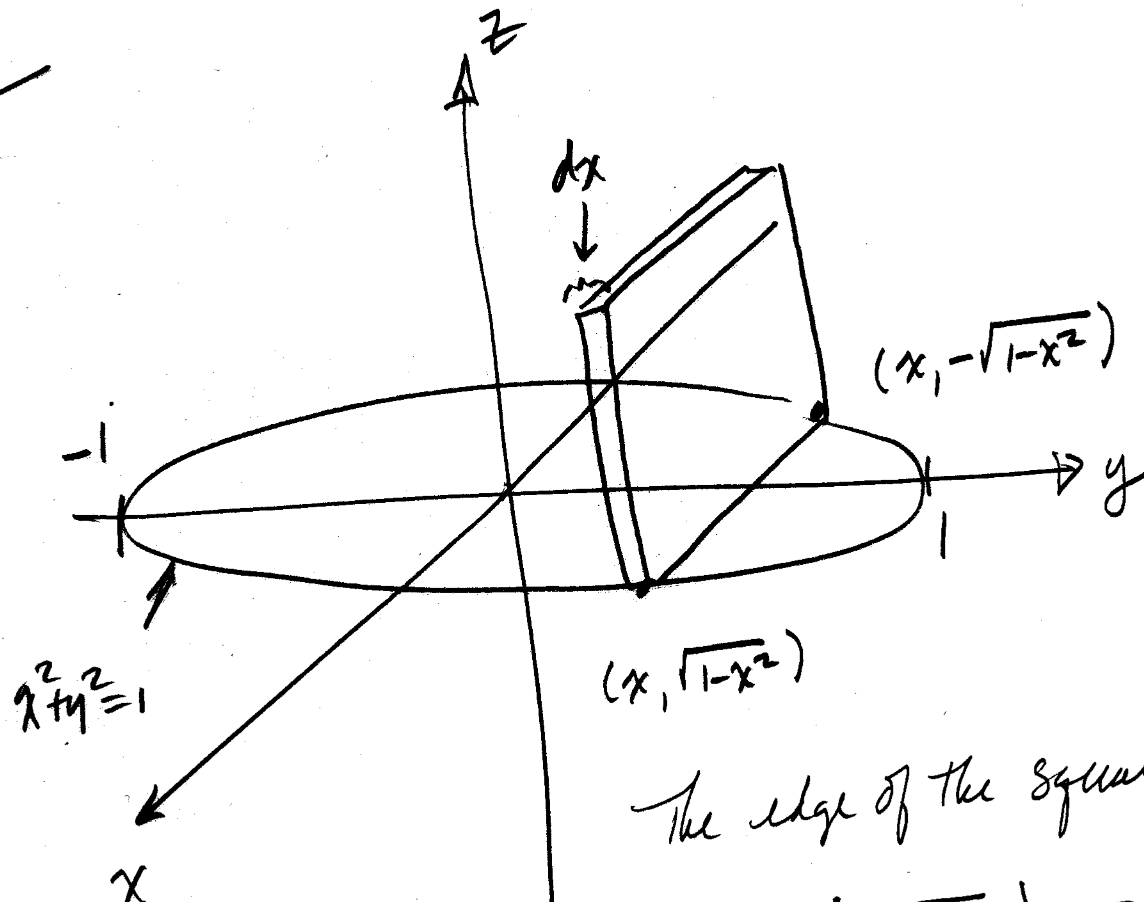
$$= \pi \left[\frac{4}{3} - \frac{1}{2} - \frac{1}{3} \right]$$

$$= \pi \left[\frac{1}{2} \right]$$

$$= \frac{\pi}{2}$$

(3)

10 points



The edge of the square is

$$\sqrt{1-x^2} - (-\sqrt{1-x^2}) = 2\sqrt{1-x^2}$$

Therefore, the area of the face is $(2\sqrt{1-x^2})^2$ and because the thickness is dx,

$$dV = (2\sqrt{1-x^2})^2 dx = 4(1-x^2)dx$$

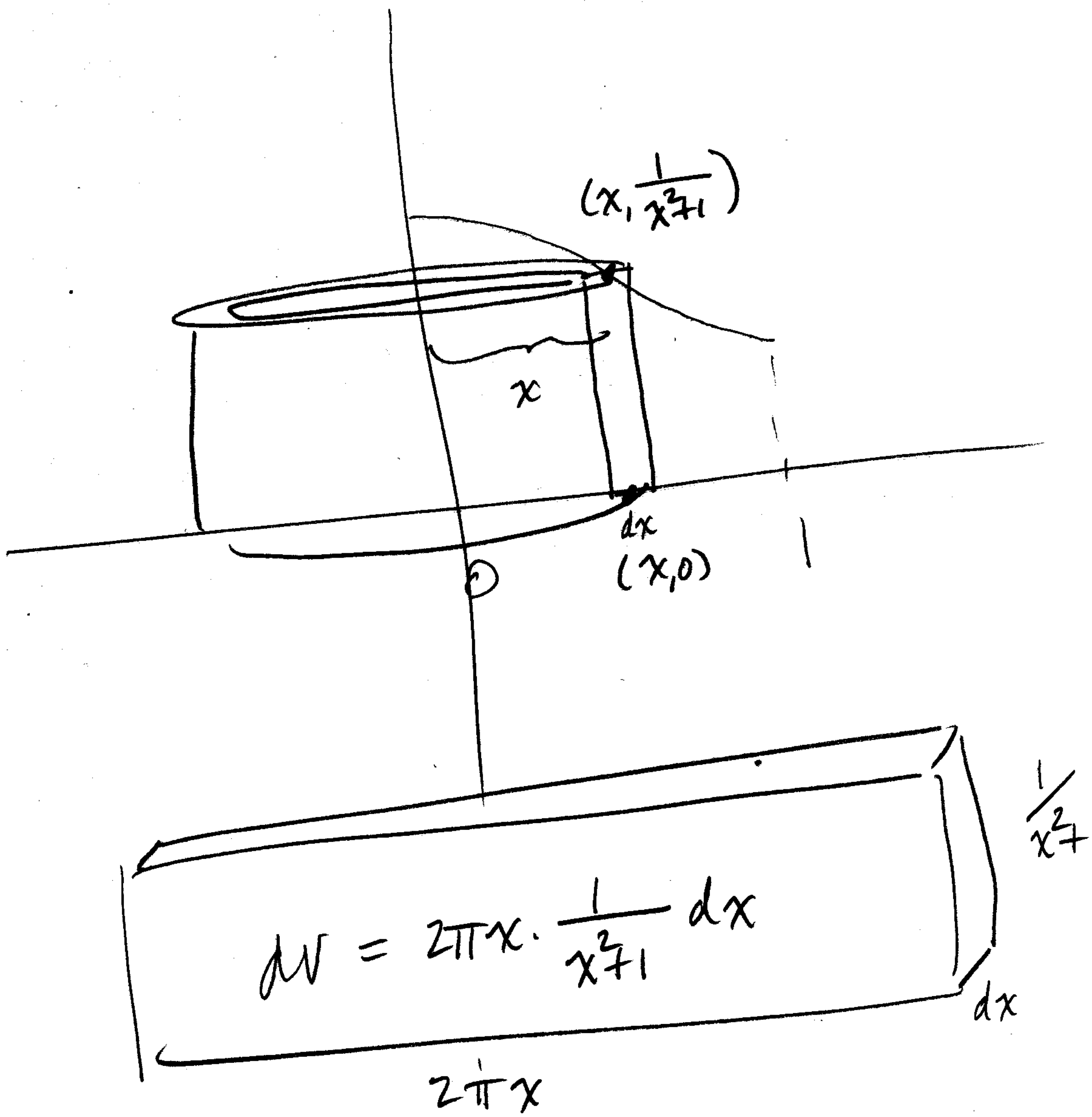
Hence, $V = \int_{-1}^{1} 4(1-x^2)dx$

$$= 4\left(x - \tfrac{1}{3}x^3\right)\Big|_{-1}^{1}$$

$$= 4\left[\left(1-\tfrac{1}{3}\right) - \left(-1+\tfrac{1}{3}\right)\right] = 4\left[\tfrac{2}{3} - \left(-\tfrac{2}{3}\right)\right] = \frac{16}{3}$$

4

10 points



$$dV = 2\pi x \cdot \frac{1}{x^2+1}\, dx$$

$$V = \int_0^1 \frac{2\pi x}{1+x^2}\, dx$$

$$= \pi \int_0^1 \frac{1}{x^2+1}\,(2x\,dx)$$

$$u = x^2+1 \Rightarrow du = 2x\,dx$$

$$V = \pi \int_1^2 \frac{1}{u}\, du$$

$$= \pi \ln|u| \Big|_1^2$$

$$= \pi(\ln 2 - \cancel{\ln 1}^{\,0})$$

$$= \pi \ln 2$$