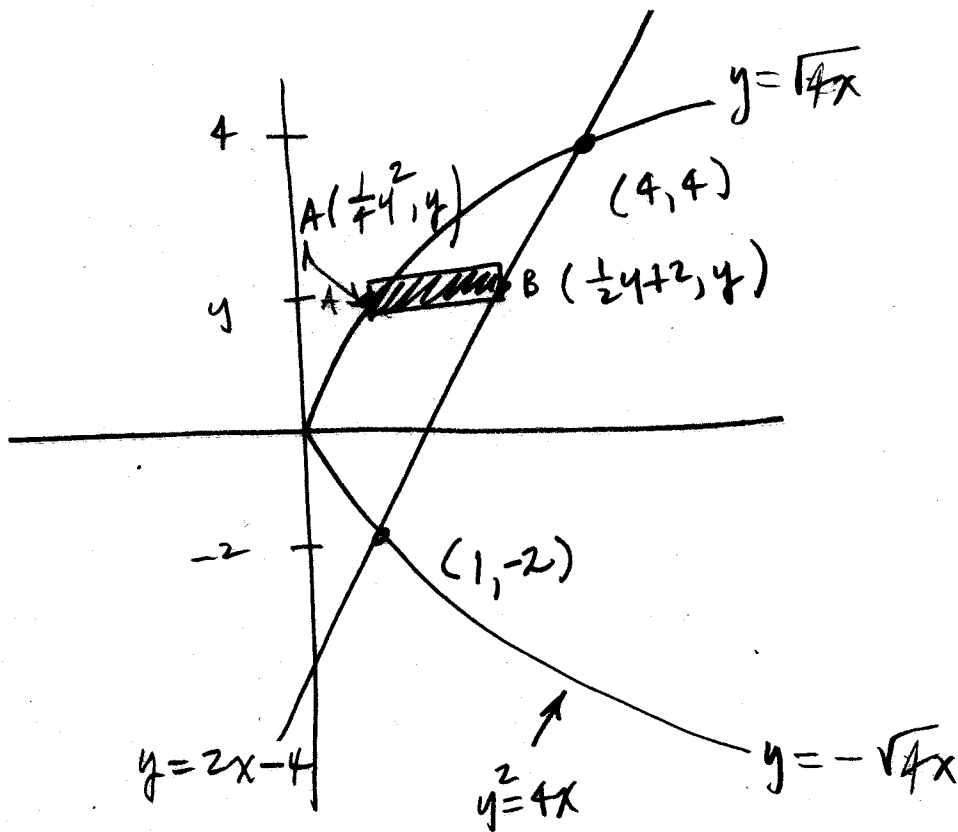


①  
10pts



$$y^2 = 4x \\ \Rightarrow x = \frac{1}{4}y^2$$

$$y = 2x - 4 \\ 2x = y + 4 \\ x = \frac{1}{2}y + 2$$

$$y^2 = 4x \quad \textcircled{1}$$

$$y = 2x - 4 \quad \textcircled{2}$$

Multiply ① by  $\frac{1}{2}$ :  $\frac{1}{2}y^2 = 2x \quad \textcircled{3}$

Replace  $2x$  in ② with  $\frac{1}{2}y^2$ :

$$y = \frac{1}{2}y^2 - 4$$

$$2y = y^2 - 8$$

$$0 = y^2 - 2y - 8$$

$$0 = (y - 4)(y + 2)$$

$$y = -2, 4$$

$$dy \left\{ \int \left[ \frac{1}{2}y + 2 - \frac{1}{4}y^2 \right] dy \right.$$

$$\left. \left( \frac{1}{2}y + 2 \right) - \frac{1}{4}y^2 \right.$$

$$dA = \left[ \left( \frac{1}{2}y + 2 \right) - \frac{1}{4}y^2 \right] dy$$

$$dA = \left( -\frac{1}{4}y^2 + \frac{1}{2}y + 2 \right) dy$$

$$A = \int_{-2}^4 \left( -\frac{1}{4}y^2 + \frac{1}{2}y + 2 \right) dy$$

$$= -\frac{1}{12}y^3 + \frac{1}{4}y^2 + 2y \Big|_{-2}^4$$

$$= \left[ -\frac{1}{12}(4)^3 + \frac{1}{4}(4)^2 + 2(4) \right] - \left[ -\frac{1}{12}(-2)^3 + \frac{1}{4}(-2)^2 + 2(-2) \right]$$

$$= \left[ -\frac{16}{3} + 4 + 8 \right] - \left[ \frac{2}{3} + 1 - 4 \right]$$

$$= -\frac{16}{3} + 12 + \frac{2}{3} + 3$$

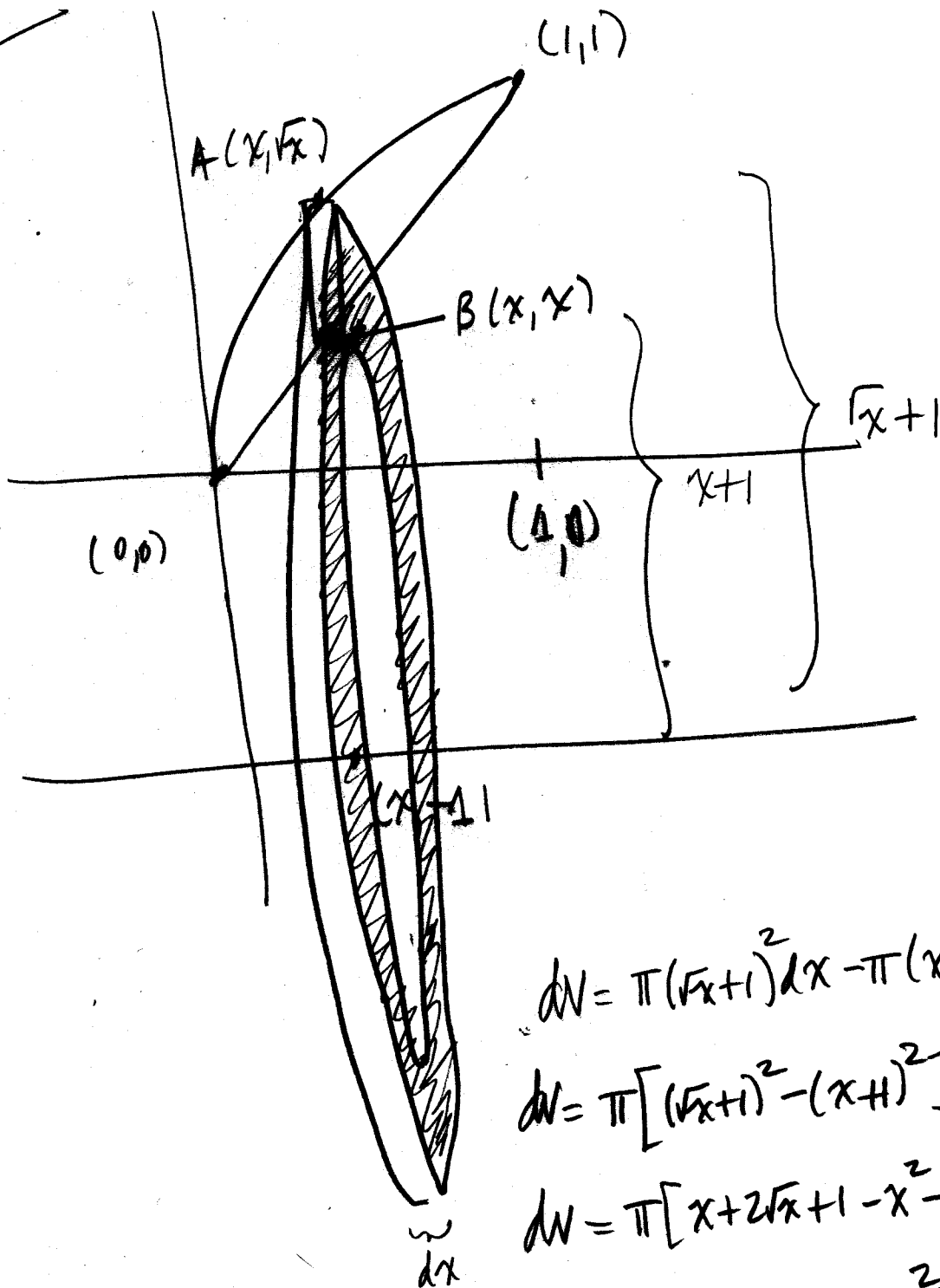
$$= 15 - \frac{14}{3}$$

$$= 15 - 4\frac{2}{3}$$

$$= 9$$

②

10 points



$$dV = \pi (\sqrt{x+1})^2 dx - \pi (x+1)^2 dx$$

$$dV = \pi [(\sqrt{x+1})^2 - (x+1)^2] dx$$

$$dV = \pi [x + 2\sqrt{x} + 1 - x^2 - 2x - 1] dx$$

$$dV = \pi [2\sqrt{x} - x - x^2] dx$$

$$V = \int_0^1 \pi [2x^{1/2} - x - x^2] dx$$

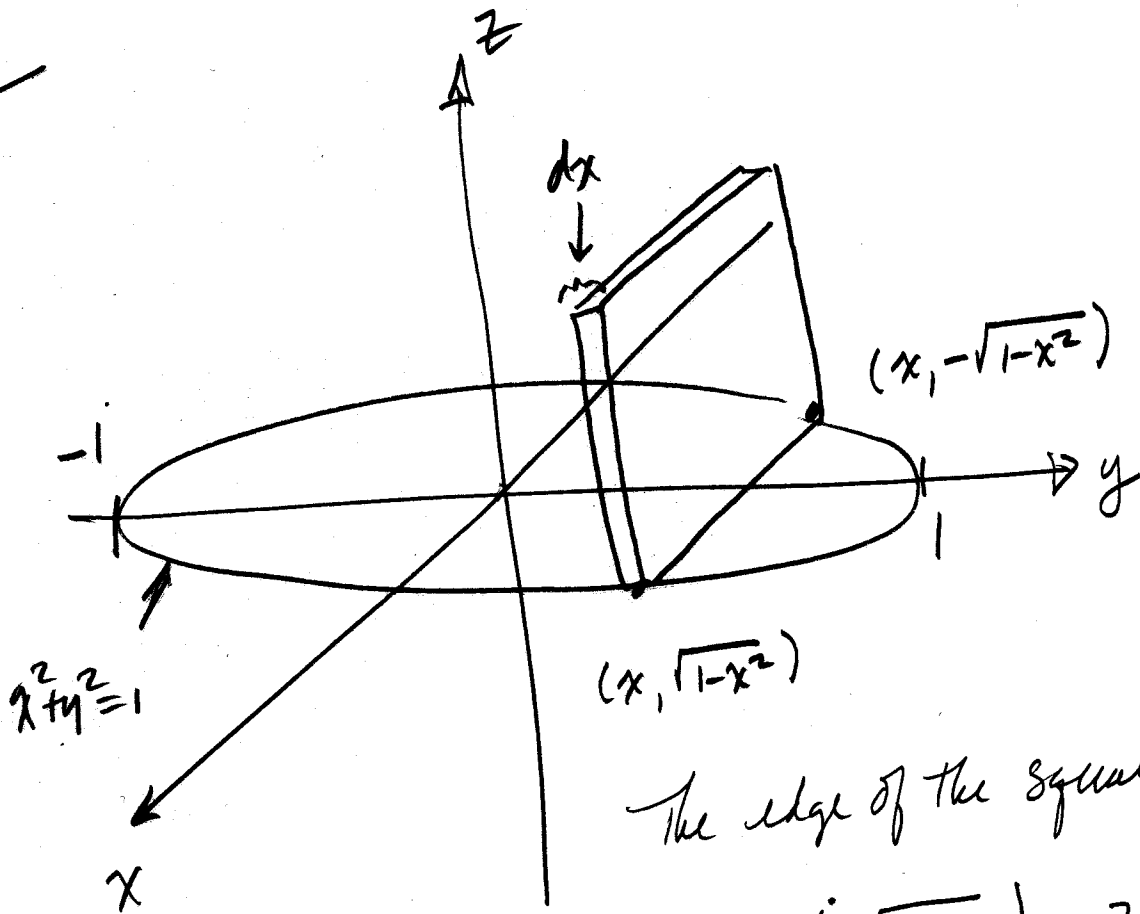
$$= \pi \left[ \frac{4}{3} x^{3/2} - \frac{1}{2} x^2 - \frac{1}{3} x^3 \right]_0^1$$

$$= \pi \left[ \frac{4}{3} - \frac{1}{2} - \frac{1}{3} \right]$$

$$= \pi \left[ \frac{1}{2} \right]$$

$$= \frac{\pi}{2}$$

③  
10 points



The edge of the square is

$$\sqrt{1-x^2} - (-\sqrt{1-x^2}) = 2\sqrt{1-x^2}$$

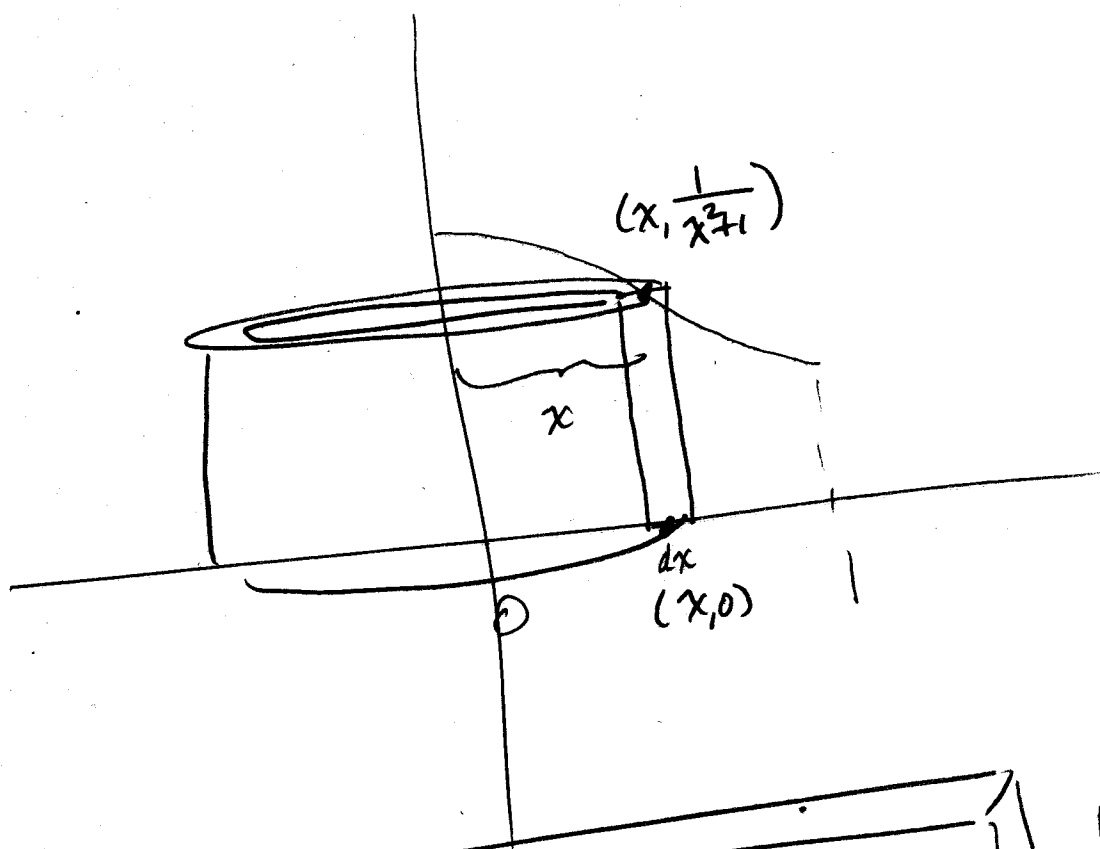
Therefore, the area of the face is  $(2\sqrt{1-x^2})^2$   
and because the thickness is  $dx$ ,

$$dV = (2\sqrt{1-x^2})^2 dx = 4(1-x^2) dx$$

$$\text{Hence, } V = \int_{-1}^1 4(1-x^2) dx$$
$$= 4 \left( x - \frac{1}{3}x^3 \right) \Big|_{-1}^1$$

$$= 4 \left[ \left( 1 - \frac{1}{3} \right) - \left( -1 + \frac{1}{3} \right) \right] = 4 \left[ \frac{2}{3} - \left( -\frac{2}{3} \right) \right] = \frac{16}{3}$$

④  
10 points



A hand-drawn diagram of a rectangular volume element. The length of the rectangle is labeled  $2\pi x$ . The width is labeled  $dx$ . The height is labeled  $\frac{1}{x^2+1}$ . The volume element is labeled with the equation  $dV = 2\pi x \cdot \frac{1}{x^2+1} dx$ .

$$V = \int_0^1 \frac{2\pi x}{1+x^2} dx$$

$$= 2\pi \int_0^1 \frac{1}{x^2+1} (x dx)$$

$$u = x^2 + 1 \Rightarrow du = 2x dx$$

$$\begin{aligned} V &= \pi \int_1^2 \frac{1}{u} du \\ &= \pi \ln|u| \Big|_1^2 \\ &= \pi (\ln 2 - \ln 1) \\ &= \pi \ln 2 \end{aligned}$$