1 The Trapezoid Approximation

We first partition the interval $[a, b]$ so that $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$, then we draw a trapezoid, on each subinterval $[x_{k-1}, x_k]$. The endpoints at the end of each subinterval are used to calculate the height of the bases, as shown in **Figure 1**.

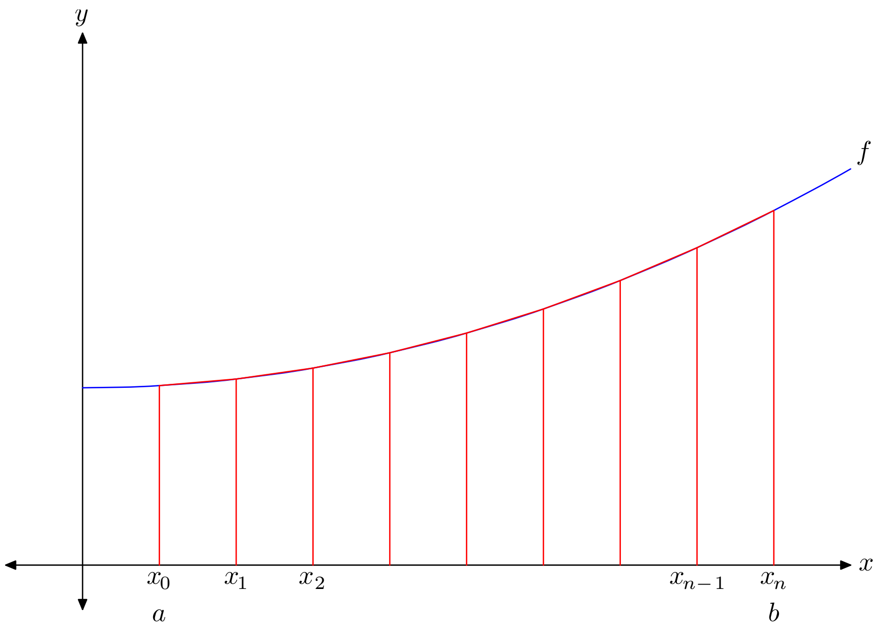

Figure 1 Using the trapezoids to approximate the integral.

The area of the trapezoid is half the bases added together multiplied by its width. Thus, the area of the first trapezoid in **Figure 1** (drawn on the interval $[x_0, x_1]$) is

$$\frac{1}{2}\Big[f(x_0) + f(x_1)\Big]\Delta x.$$

In a similar manner, the area of the second trapezoid is

$$\frac{1}{2}\Big[f(x_1) + f(x_2)\Big]\Delta x.$$

Continuing in this manner, the sum of the areas of the n trapezoids drawn in **Figure 1** is

$$\frac{1}{2}\Big[f(x_0) + f(x_1)\Big]\Delta x + \frac{1}{2}\Big[f(x_1) + f(x_2)\Big]\Delta x + \cdots + \frac{1}{2}\Big[f(x_{n-1}) + f(x_n)\Big]\Delta x. \quad (1)$$

where Δx is the width of each rectangle and is calculated by dividing the length of the interval $[a, b]$ by n,

$$\Delta x = \frac{b-a}{n} \tag{2}$$

The width of each trapezoid, Δx, is a common factor, as is $1/2$, so we can write

$$\frac{1}{2}\left[\Big(f(x_0)+f(x_1)\Big)+\Big(f(x_1)+f(x_2)\Big)+\cdots+\Big(f(x_{n-1})+f(x_n)\Big)\right]\Delta x. \tag{3}$$

This will simplify the computation somewhat and ease the construction of the programming task.

We begin by providing the calculator with some initial data. We assume that the integrand of

$$\int_a^b f(x)dx \tag{4}$$

is currently loaded in the `Y=` Menu. For example if we wish to integrate

$$\int_1^2 \frac{1}{x}dx, \tag{5}$$

then we load $y = 1/x$ in `Y1`.

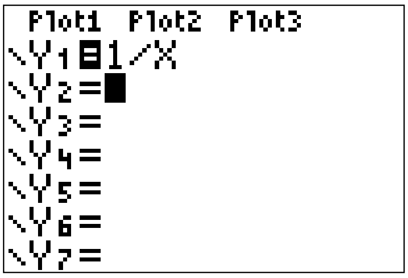

Figure 2 Entering the integrand in the `Y=` Menu.

The calculator will need to know the limits of integration and the number of subintervals in the partition. We also clear the home screen before prompting the user for this data.

```
Clr Home
Prompt A
Prompt B
Prompt N
```

We immediately calculate the width Δx of each subinterval and store it in the variable `H`.

```
(B-A)/N->H
```

Because we begin with the endpoint of the first subinterval, we store the contents of the variable A in X.

```
A->X
```

We will store the sum in the variable S. It is important that we initialize this variable to zero before we begin.

```
0->S
```

We now add the function values in Equation (3). This is accomplished by proceeding one step at a time, moving from left to right. This calls for the use of a For loop.

```
For(K,1,N,1)
  S+Y1(X)+Y1(X+H)->S
  X+H->X
End
```

This loop construct warrants some further explanation. The command For(K,1,N,1) initializes K to 1, then executes the loop. It increments the value of K by 1, then checks to see if K is less than or equal to N. If K is less than or equal to N, the loop is executed again. If K is greater than N, execution of the loop is halted and the program continues by executing the line following the End command, which terminates the loop. In the body of the loop, two things happen. First, the integrand, which was stored in Y1, is evaluated at X and X+H, added to the current value of S, then S is updated to contain this current sum. This is accomplished with the command S+Y1(X)+Y1(X+H)->S. Next, X is incremented by H by executing the command X+H->X. This ensures that the next time through the loop, evaluation of the integrand takes place at the beginning of the *next* subinterval.

Once the loop terminates, S contains the sum $(f(x_0)+f(x_1))+(f(x_1)+f(x_2))+\cdots+(f(x_{n-1})+f(x_n))$. By **Equation 3**, we must still multiply this sum by Δx and divide by 2. The command

```
S*H/2->S
```

multiplies the contents of S by H/2, then updates S to contain this product. The variable S now contains the sum of all N trapezoids pictured in **Figure 1**. All that's left to do is display the answer for the user of the program. We indicate the integration method being used.

```
Disp "METHOD: TRAP"
```

Then we display the sum.

```
Disp "SUM:"
Disp S
```

A complete listing of the program follows in **Appendix 2**.

We need to test our program. Here is some data you can use to determine whether or not your program is running properly.

No. of Trapezoids	Sum
5	0.695635
10	0.693771
20	0.693303

Table 1 Integrating $1/x$ on the interval $[1, 2]$.

Enter the integrand of

$$\int_1^2 \frac{1}{x} dx$$

in the `Y=` Menu, as shown in **Figure 2**. Execute the program. Enter the upper and lower bounds of the integral when prompted. Enter $n = 5$ for the number of trapezoids or subintervals.

Figure 3 Entering the interval $[1, 2]$ and the number of subintervals ($n = 5$).

Note that this result agrees with the first row of **Table 1**. Use the program to verify the results in the next two rows of **Table 1**.

2 Appendix

Here is a complete program listing.

```
Clr Home
Prompt A
Prompt B
Prompt N
(B-A)/N->H
A->X
0->S
For(K,1,N,1)
  S+Y1(X)+Y1(X+H)->S
  X+H->X
End
S*H/2->S
Disp "METHOD: TRAP"
Disp "SUM:"
Disp S
```