Matrix Multiplication—Alternate Methods

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Abstract

Matlab is used to demonstrate matrix multiplication by columns and rows.

Prerequisites. A basic understanding of ordinary matrix multiplication.

1 Matrix Multiplication By Columns

In computing the product of matrices A and B, the first column of AB is found by multiplying A times the first column of B, the second column of AB is found by multiplying A times the second column of B, etc. The purpose of this activity is to use Matlab to demonstrate this method of multiplication.

First enter the following matrix A.

```
>> A=[1 2 3;4 5 6;7 8 9]

A =

     1     2     3
     4     5     6
     7     8     9
```

Now enter the following vectors.

```
>> b1=[1;1;-1]

b1 =

     1
     1
    -1
```

```
>> b2=[2;-3;1]

b2 =

     2
    -3
     1
```

Next, compute the matrix products $A\mathbf{b}_1$ and $A\mathbf{b}_2$.

```
>> A*b1

ans =

     0
     3
     6
>> A*b2

ans =

    -1
    -1
    -1
```

Finally, form the matrix $B = [\mathbf{b}_1\,\mathbf{b}_2]$ and compute the matrix product AB.

```
>> B=[b1 b2]

B =

     1     2
     1    -3
    -1     1

>> A*B

ans =

     0    -1
     3    -1
     6    -1
```

What's the point? It's important to note that the first column of AB is $A\mathbf{b}_1$ and the second column of AB is $A\mathbf{b}_2$.

Theorem 1 *Let A and B be matrices with dimensions that allow them to be multiplied. If $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n$ are vectors representing the columns of matrix B, then*

$$\begin{aligned} AB &= A\,[\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n] \\ &= [A\mathbf{b}_1, A\mathbf{b}_2, \ldots, A\mathbf{b}_n] \end{aligned}$$

2 Matrix Multiplication By Rows

In computing the product of matrices A and B, the first row of AB is found by multiplying the first row of A times B, the second row of AB is found by multiplying the second row of A times B, etc. The purpose of this activity is to use Matlab to demonstrate this method of multiplication.

First enter the row vectors of matrix A.

```
>> r1=[1 2 3]

r1 =

     1     2     3

>> r2=[4 5 6]

r2 =

     4     5     6

>> r3=[7 8 9]

r3 =

     7     8     9
```

Next, enter the matrix B.

```
>> B=[1 2;1 -3;-1 1]

B =

     1     2
     1    -3
    -1     1
```

Next, compute the matrix products $\mathbf{r}_1 B$, $\mathbf{r}_2 B$, and $\mathbf{r}_3 B$.

```
>> r1*B
```

```
ans =

     0    -1

>> r2*B

ans =

     3    -1

>> r3*B

ans =

     6    -1
```

Finally, compute the matrix product AB.

```
>> A*B

ans =

     0    -1
     3    -1
     6    -1
```

What's the point? It's important to note that the first row of AB is $\mathbf{r}_1 B$, the second row of AB is $\mathbf{r}_2 B$, and the third row of AB is $\mathbf{r}_3 B$.

Theorem 2 *Let A and B be matrices with dimensions that allow them to be multiplied. If $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$ are vectors representing the rows of matrix A, then*

$$\begin{aligned} AB &= \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_n \end{bmatrix} B \\ &= \begin{bmatrix} \mathbf{r}_1 B \\ \mathbf{r}_2 B \\ \vdots \\ \mathbf{r}_n B \end{bmatrix} \end{aligned}$$

3 The Product of a Matrix and Column Vector

Enter the following column vectors.

```
>> c1=[1;2;3]
```

```
c1 =

     1
     2
     3

>> c2=[4;5;6]

c2 =

     4
     5
     6

>> c3=[7;8;9]

c3 =

     7
     8
     9
```

Form the following linear combination of the vectors $\mathbf{c}_1, \mathbf{c}_2$, and $\mathbf{c}_3$.

```
>> -2*c1+3*c2-5*c3

ans =

   -25
   -29
   -33
```

Form the matrix $C = [\mathbf{c}_1\ \mathbf{c}_2\ \mathbf{c}_3]$, the vector $\mathbf{x} = \begin{bmatrix} -2 \\ 3 \\ -5 \end{bmatrix}$, and find the matrix product $C\mathbf{x}$.

```
>> C=[c1 c2 c3]

C =

     1     4     7
     2     5     8
     3     6     9

>> x=[-2;3;-5]
```

```
x =

    -2
     3
    -5

>> C*x

ans =

   -25
   -29
   -33
```

What's the point? You may compute the matrix-vector product $C\mathbf{x}$ by taking a linear combination of the columns of C using the elements of vector $\mathbf{x}$ for the weights.

Theorem 3 *If* $C = [\mathbf{c}_1\ \mathbf{c}_2\ \cdots\ \mathbf{c}_n]$ *and* $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$, *then* $C\mathbf{x} = \mathbf{x}_1\mathbf{c}_1 + x_2\mathbf{c}_2 + \cdots + x_n\mathbf{c}_n$.

4 The Product of a Row Vector and Matrix

Enter the following row vectors.

```
>> x=[5 -6 4]

x =

     5    -6     4

>> r1=[1 2 3]

r1 =

     1     2     3

>> r2=[4 5 6]

r2 =

     4     5     6
```

```
>> r3=[7 8 9]

r3 =

     7     8     9
```

Form a linear combination of the row vectors $\mathbf{r}_1, \mathbf{r}_2$, and $\mathbf{r}_3$ using the elements of the row vector $\mathbf{x}$ as weights.

```
>> 5*r1-6*r2+4*r3

ans =

     9    12    15
```

Form the matrix C using the row vectors $\mathbf{r}_1, \mathbf{r}_2$, and $\mathbf{r}_3$, then form the matrix product $\mathbf{x}C$.

```
>> C=[r1;r2;r3]

C =

     1     2     3
     4     5     6
     7     8     9

>> x*C

ans =

     9    12    15
```

What's the point? You may compute the matrix product $\mathbf{x}C$ by forming a linear combination of the rows of C using the elements of $\mathbf{x}$ as weights.

Theorem 4 *If* $\mathbf{x} = [x_1\ x_2\ \cdots\ x_n]$ *and* $C = \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_n \end{bmatrix}$, *then* $\mathbf{x}C = x_1\mathbf{r}_1 + x_2\mathbf{r}_2 + \cdots + x_n\mathbf{r}_n$.

4.1 Homework

From time to time during the remainder of the semester, take out this sheet and practice these various forms of multiplication. They will greatly aid your study of linear algebra.