College of the Redwoods
Mathematics Department
Math 45 – Linear Algebra

Exam #1—Matlab Component

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Directions: *This examination is to be completed at one contiguous sitting in PS116. There is a maximum time limit of two hours. You are honor bound to complete all of this examination without any aid: fellow students, texts, manuals, notes, activities from the web, etc. The only aid you may use is the Matlab help files, available by typing* `help` *at the Matlab prompt. Furthermore, you are not to discuss the contents of this examination with anyone until the examination is scored and returned in class. Good luck!*

Exercise 1. **The Interpolating Polynomial**. The goal of this examination question is to find an interpolating polynomial of degree four that passes through each of the points $(-3, 12)$, $(-2, 4)$, $(0, 0)$, $(1, -6)$, and $(4, 12)$.

(a) Substitute each point into the general form for a fourth degree polynomial,

$$p(x) = ax^4 + bx^3 + cx^2 + dx + e,$$

generating five equations in five unknowns. Record these equations on your examination paper.

(b) Set up the augmented matrix for the system in part (a). Record this augmented matrix on your examination paper.

(c) Use Matlab to place the augmented matrix from part (b) in reduced row echelon form.

(d) Use the results of part (c) to record the interpolating polynomial on your examination paper.

(e) Use Matlab to prepare a sketch of the data and the interpolating polynomial. Plot the data points as discrete points. Plot the interpolating polynomial as a smooth curve traveling through each of the data points.

When you have completed the examination, arrange your papers in order, then staple this examination on top as a cover page. Place your name on this examination and record your starting and stopping *times. Finally, if you used script files to produce any work in Matlab, include a copy of your script file with your work.*

Solutions to Exercises

Exercise 1(a) Substitute each of the given points into the general form

$$p(x) = ax^4 + bx^3 + cx^2 + dx + e.$$

This gives five equations in five unknowns.

$$\begin{aligned}
12 &= a(-3)^4 + b(-3)^3 + c(-3)^2 + d(-3) + e \\
4 &= a(-2)^4 + b(-2)^3 + c(-2)^2 + d(-2) + e \\
0 &= a(0)^4 + b(0)^3 + c(0)^2 + d(0) + e \\
-6 &= a(1)^4 + b(1)^3 + c(1)^2 + d(1) + e \\
12 &= a(4)^4 + b(4)^3 + c(4)^2 + d(4) + e
\end{aligned}$$

□

Exercise 1(b) Set up the augmented matrix.

$$\begin{pmatrix} (-3)^4 & (-3)^3 & (-3)^2 & (-3)^1 & 1 & 12 \\ (-2)^4 & (-2)^3 & (-2)^2 & (-2)^1 & 1 & 4 \\ (0)^4 & (0)^3 & (0)^2 & (0)^1 & 1 & 0 \\ (1)^4 & (1)^3 & (1)^2 & (1)^1 & 1 & -6 \\ (4)^4 & (4)^3 & (4)^2 & (4)^1 & 1 & 12 \end{pmatrix}$$

□

Exercise 1(c) First, enter the vectors of data points.

```
>> x=[-3 -2 0 1 4]'
x =
    -3
    -2
     0
     1
     4
>> y=[12 4 0 -6 12]'
y =
    12
     4
     0
    -6
    12
```

Use element-wise operations to create the augmented matrix.

```
>> M=[x.^4,x.^3,x.^2,x.^1,x.^0,y]
M =
    81   -27     9    -3     1    12
    16    -8     4    -2     1     4
     0     0     0     0     1     0
     1     1     1     1     1    -6
   256    64    16     4     1    12
```

Place the augmented matrix in reduced row echelon form.

```
>> M=[x.^4,x.^3,x.^2,x.^1,x.^0,y]
>> R=rref(M)
R =
     1         0         0         0         0    0.2222
     0         1         0         0         0    0.0556
     0         0         1         0         0   -1.9444
     0         0         0         1         0   -4.3333
     0         0         0         0         1         0
```

Switching to rational format (`format rat`), one can easily see that this matrix equals,

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 2/9 \\ 0 & 1 & 0 & 0 & 0 & 1/18 \\ 0 & 0 & 1 & 0 & 0 & -35/18 \\ 0 & 0 & 0 & 1 & 0 & -13/3 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

□

Exercise 1(d) Thus, $a = 2/9$, $b = 1/18$, $c = -35/18$, $d = -13/3$, $e = 0$, and the interpolating polynomial is

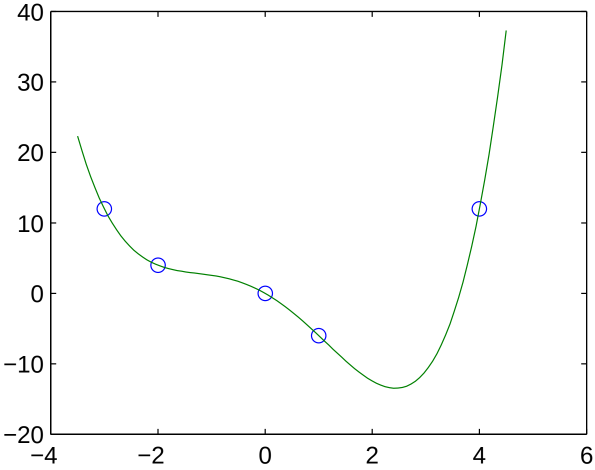
$$p(x) = \frac{2}{9}x^4 + \frac{1}{18}x^3 - \frac{35}{18}x^2 - \frac{13}{3}x.$$

□

Exercise 1(e) The following script file will plot the data points and the interpolating polynomial.

```
>> p=[2/9,1/18,-35/18,-13/3,0];
>> xx=linspace(-3.5,4.5);
>> yy=polyval(p,xx);
>> plot(x,y,'o',xx,yy)
```

This set of commands will produce the image shown in the following figure.


40
30
20
10
0
−10
−20
−4
−2
0
2
4
6

□