College of the Redwoods
Mathematics Department
Math 45 — Linear Algebra

Matlab Exam #1

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Essay Questions

Read Carefully! *This part of the exam is closed book, closed notes, closed activity, etc. The only online materials you may use are the Matlab help files. The exam must be completed at one sitting in the lab. Once you start the exam, you must finish the exam before you leave. You cannot start the exam, leave, then return later to work on the exam. Please follow directions exactly.*

EXERCISE 1. This activity requires that you find two fourth degree interpolating polynomials passing through each of the following points.

$$(1, 2),\ (3, -4),\ (5, 5),\ (6, -1) \tag{1}$$

(a) On a sheet of college ruled paper, use the data points in (1) to set up a system four equations in five unknowns.

(b) On your college ruled paper, set up the augmented matrix for the system crafted in part (a).

(c) Use Matlab to place the augmented matrix in part (b) in reduced row echelon form. Copy the result onto your college ruled paper.

(d) On your college ruled paper, write down the system of equations represented by the reduced form of the augmented matrix in part (c). Solve each equation for the pivot variable in terms of any free variable(s)[1]. Write these solutions on your college ruled paper.

(e) Select a particular value(s) for the free variable(s) in part (d) to find one instance of an interpolating polynomial passing through each data point in (1). Clearly state the equation of this polynomial on your college ruled paper. Use Matlab to craft a single plot the contains each of the following items.

1. The data points in (1) plotted as discrete points.
2. The graph of the interpolating polynomial.
3. Appropriate labels for each axis and a title containing the equation of your interpolating polynomial.

Print the resulting figure window and include the printout with your examination results.

(f) Select a second particular value(s) for the free variable(s) in part (d) to find a second instance of an interpolating polynomial passing through each data point in (1). Clearly state the equation of this polynomial on your college ruled paper. Use Matlab to craft a single plot the contains each of the following items.

1. The data points in (1) plotted as discrete points.
2. The graph of the interpolating polynomial.
3. Appropriate labels for each axis and a title containing the equation of your interpolating polynomial.

Print the resulting figure window and include the printout with your examination results.

[1]When we write variable(s), we mean that it could be either "variable" or "variables." Thus we are not forced to reveal whether there is only one free variable or perhaps there is more than one free variable. You, of course, have to determine this on your own.

Solutions to Exercises

Exercise 1(a) We wish to find a fourth degree polynomial

$$y = ax^4 + bx^3 + cx^2 + dx + e \tag{2}$$

that passes through each of the data points $(1, 2)$, $(3, -4)$, $(5, 5)$, and $(6, -1)$. Substituting each of these points in (2) leads to the following system of equations.

$$\begin{aligned} 2 &= a(1)^4 + b(1)^3 + c(1)^2 + d(1) + e \\ -4 &= a(3)^4 + b(3)^3 + c(3)^2 + d(3) + e \\ 5 &= a(5)^4 + b(5)^3 + c(5)^2 + d(5) + e \\ -1 &= a(6)^4 + b(6)^3 + c(6)^2 + d(6) + e \end{aligned} \tag{3}$$

□

Exercise 1(b) The augmented matrix for system (3) is

$$\begin{pmatrix} 1^4 & 1^3 & 1^2 & 1 & 1 & 2 \\ 3^4 & 3^3 & 3^2 & 3 & 1 & -4 \\ 5^4 & 5^3 & 5^2 & 5 & 1 & 5 \\ 6^4 & 6^3 & 6^2 & 6 & 1 & -1 \end{pmatrix}. \tag{4}$$

□

Exercise 1(c) The augmented matrix (4) is easily entered in Matlab. Begin by entering the data points in vectors `x` and `y`.

```
>> x=[1;3;5;6]
x =
     1
     3
     5
     6

>> y=[2;-4;5;-1]
y =
     2
    -4
     5
    -1
```

Now, enter the augmented matrix as follows.

```
>> M=[x.^4,x.^3,x.^2,x,ones(size(x)),y]
M =
           1           1           1           1           1           2
          81          27           9           3           1          -4
         625         125          25           5           1           5
        1296         216          36           6           1          -1
```

Reduce.

```
>> R=rref(M)
R =
      1           0           0           0         -1/90      -107/360
      0           1           0           0          1/6        203/60
      0           0           1           0        -77/90     -1349/119
      0           0           0           1         17/10        41/4
```

□

Exercise 1(d) The augmented matrix from part (c) represents the following system of equations.

$$\begin{aligned} a - \frac{1}{90}e &= -\frac{107}{360} \\ b + \frac{1}{6}e &= \frac{203}{60} \\ c - \frac{77}{90}e &= -\frac{1349}{119} \\ d + \frac{17}{10}e &= \frac{41}{4} \\ e &= \text{free} \end{aligned} \tag{5}$$

Solve each equation in (5) for the pivot variable.

$$\begin{aligned} a &= -\frac{107}{360} + \frac{1}{90}e \\ b &= \frac{203}{60} - \frac{1}{6}e \\ c &= -\frac{1349}{119} + \frac{77}{90}e \\ d &= \frac{41}{4} - \frac{17}{10}e \\ e &= \text{free} \end{aligned} \tag{6}$$

□

Exercise 1(e) Substitute $e = 0$ in the system (6) to obtain

$$\begin{aligned} a &= -\frac{107}{360} \\ b &= \frac{203}{60} \\ c &= -\frac{1349}{119} \\ d &= \frac{41}{4} \\ e &= 0. \end{aligned} \tag{7}$$

This gives an interpolating polynomial

$$y = -\frac{107}{360}x^4 + \frac{203}{60}x^3 - \frac{1349}{119}x^2 + \frac{41}{4}x. \tag{8}$$

Note that the plot of this polynomial passes through each of the given data points in Figure 1.

□

Exercise 1(f) Substitute $e = 10$ in system (6) to obtain

$$\begin{aligned} a &= -\frac{67}{360} \\ b &= \frac{103}{60} \\ c &= -\frac{2978}{1071} \\ d &= -\frac{27}{4} \\ e &= 10. \end{aligned} \tag{9}$$

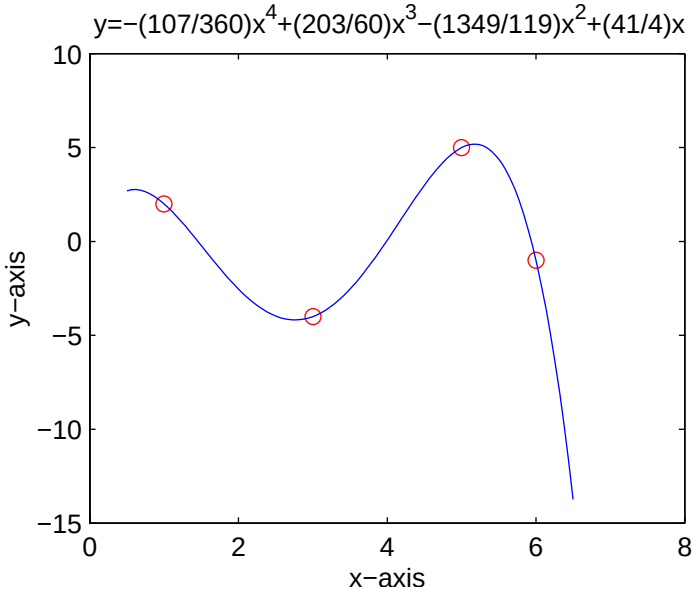

Figure 1: An interpolating polynomial must pass through each data point.

This leads to the interpolating polynomial

$$y = -\frac{67}{360}x^4 + \frac{103}{60}x^3 - \frac{2978}{1071}x^2 - \frac{27}{4}x + 10. \tag{10}$$

Note that the plot of this polynomial passes through each of the given data points in Figure 2.

□

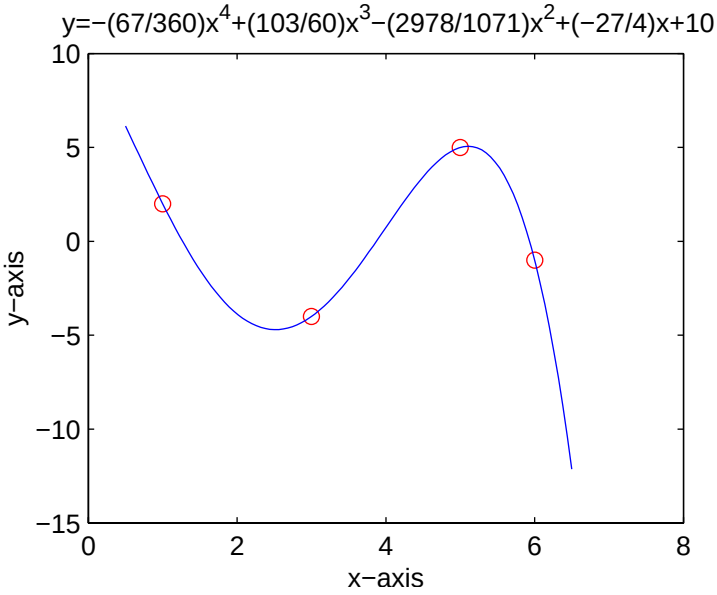

Figure 2: An interpolating polynomial must pass through each data point.