College of the Redwoods
Mathematics Department
Math Math 45 — Linear Algebra

Quiz #8—Linear Algebra

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Directions: *Place your solution in the space provided.* **No calculators allowed!**

Exercise 1. Use elimination to compute the following determinant. Show all of your work, connecting consecutive computations with equal signs.

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & -1 & 2 \\ 2 & 2 & -3 \end{vmatrix} =$$

Exercise 2. Suppose that matrix A is a 4×4 matrix. Answer each of the following questions, assuming $|A| = 3$.

(a) $|A^T| =$

(b) $|A^{-1}| =$

(c) $|3A| =$

Exercise 3. Each of the following determinants is easily calculated using one or more of the 10 properties of the determinant proposed by

Strang. In each exercise, compute the determinant without resorting to elimination. Give a handwritten reason for your conclusion; i.e., state which properties you use to arrive at your answer.

(a) $\begin{vmatrix} 0 & 0 & 3 \\ 0 & 2 & -1 \\ 1 & 4 & 5 \end{vmatrix} =$

(b) $\begin{vmatrix} a & c & a+c \\ b & d & b+d \\ c & e & c+e \end{vmatrix} =$

(c) $\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} =$

Solutions to Exercises

Exercise 1. Swap rows 1 and 2

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & -1 & 2 \\ 2 & 2 & -3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \\ 2 & 2 & -3 \end{vmatrix}$$

Subtract 2 times row 1 from row 3.

$$= - \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \\ 0 & 4 & -7 \end{vmatrix}$$

Subtract 4 times row 2 from row 3.

$$\begin{aligned} &= \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -15 \end{vmatrix} \\ &= -(1)(1)(15) \\ &= 15 \end{aligned}$$

Exercise 1

Exercise 2(a) The determinant of the transpose equals the determinant of the original matrix. Therefore,

$$|A^T| = |A| = 3.$$

□

Exercise 2(b) Because $AA^{-1} = I$, we can write

$$|AA^{-1}| = |I| = 1.$$

The determinant of a product is the product of the determinants. Thus,

$$|A||A^{-1}| = 1$$

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{3}.$$

□

Exercise 2(c) Every time we multiply a row by 3, we multiply the value of the determinant by 3. But $3A$ multiplies each of 4 rows of A ($A \quad \text{is} \quad 4 \times 4$) by 3. Thus,

$$|3A| = 3^4 |A| = 81 \cdot 3 = 243$$

$\square$

Exercise 3(a) Swapping rows 1 and 3 negates the determinant

$$\begin{vmatrix} 0 & 0 & 3 \\ 0 & 2 & -1 \\ 1 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 4 & 5 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{vmatrix}$$

The determinant of a triangular matrix is the product of its diagonal elements.

$$\begin{aligned} &= (1)(2)(3) \\ &= 6. \end{aligned}$$

□

Exercise 3(b) A determinant is linear in its columns. Thus,

$$\begin{vmatrix} a & c & a+c \\ b & d & b+d \\ c & e & c+e \end{vmatrix} = \begin{vmatrix} a & c & a \\ b & d & b \\ c & e & c \end{vmatrix} + \begin{vmatrix} a & c & c \\ b & d & d \\ c & e & e \end{vmatrix}$$

If a matrix has two equal columns, its determinant is zero.

$$\begin{aligned} &= 0 + 0 \\ &= 0 \end{aligned}$$

□

Exercise 3(c) Swapping rows 1 and 3 negates the determinant

$$\begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$$

Swapping row 2 and 3 negates again.

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= 1$$

□