

# Finite Element Model for Earthquakes

Benjamin Allen

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## Abstract

This document attempts to show how a complex system can be described by using a linear approach. This model will show a relation between the force and the 0th, 1st, and 2nd derivatives of the displacement. Taken into account will be the damping and stiffness of the ground. Two simple examples will be given and evaluated with a brief analysis.

[Introduction](#)

[Description of the...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



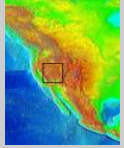
Page 1 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)



*Introduction*

*Description of the . . .*

*The Model*

*Home Page*

*Title Page*



*Page 2 of 26*

*Go Back*

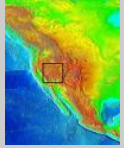
*Full Screen*

*Close*

*Quit*

## 1. Introduction

This paper will take a look at a complex system by a linear finite element approach. In particular this paper will use an example of earthquake modeling used for structural analysis. The approach used will a lumped parameter model where variable layers have different stiffness and damping ratios. In the model layers are evaluated as a sheer beam as if each were a spring and dashpot model.



*Introduction*

*Description of the . . .*

*The Model*

*Home Page*

*Title Page*



*Page 3 of 26*

*Go Back*

*Full Screen*

*Close*

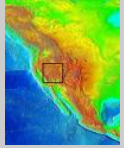
*Quit*

## 2. Description of the Process Used

### 2.1. Introduction of Variables

In this paper there will be a set of variables such that:

- $c_k$  is the damping ratio of the kth node.
- We shall let  $m$  be mass. And  $\mathbf{M}$  will be the matrix of the masses.
- $\rho$  is density
- $\mathbf{K}$  will represent the stiffness matrix that holds the data for the nodal links.
- $\mathbf{C}$  will show the restoring damping of the system in terms of the nodes.
- We will let  $\mathbf{x}$  be displacement vector of the nodes and  $\dot{\mathbf{x}}$  will be the derivative, or velocity. Thus  $\ddot{\mathbf{x}}$  is the acceleration.
- $\kappa$  shall be the bulk modulus of a material,  $G$  shall be the shear modulus of a material.
- $s$  will be displacement
- We will let  $\mathbf{F}$  be the notation for Force
- $x$  shall be the horizontal direction and  $y$  shall be the vertical direction



[Introduction](#)

[Description of the . . .](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



Page 4 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

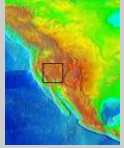
[Quit](#)

## 2.2. Description of Method

To calculate the force from the displacement that is caused we will look at the ground as a series of soil layers having different properties and variables. These soils are then analyzed with an outside force coming from a variable direction and of a variable magnitude. This force will be modeled as a pressure wave or longitudinal force that acts upon each layer.

Each soil layer reacts with its own set of properties but is bound to the other layers as they also react to the force. By placement of nodes at the boundary of each layer and assuming that the layers are homogeneous within themselves and have no vertical edges. This allows us to ignore the effects of their surfaces and boundaries. Assuming this we then go on to assume 2 degrees of freedom. The soil layers can move in a horizontal direction or in a vertical direction. The resistances and compression values of each degree of freedom are generally different so we then will evaluate at the nodes for each degree of freedom separately. This is a 2-dimensional model.

This can best be accomplished by breaking up the problem into different elements and relationships. Then these relationships will tell us what affects each node and the movement of that node.



We shall be using displacements of the form:

$$\mathbf{x}(t) = \begin{bmatrix} s_{x_1}(t) \\ s_{y_1}(t) \\ s_{x_2}(t) \\ s_{y_2}(t) \\ \vdots \\ s_{x_n}(t) \\ s_{y_n}(t) \end{bmatrix}$$

### 2.2.1. Finding the Mass Matrix

Mass opposes acceleration as can be gotten from Newton's second law  $m\mathbf{a} = \mathbf{F}$  where  $m$  is the mass and such a scalar quantity. The vectors  $\mathbf{a}$ ,  $\mathbf{F}$  are the acceleration and force respectively. Both of these are vector quantities as they have both direction and magnitude. The mass matrix will take the general form

$$\mathbf{M} = \rho \int \int \mathbf{N}\mathbf{N}^T dx, dy \quad (1)$$

where  $\mathbf{N}$  is the shape function so you can integrate over the density distribution. For our case this is a simple shape. So now we evaluate a matrix so that it corresponds with the masses acting at the nodes. For each node and direction one must evaluate the matrix for the mass to resist movement. As you look at what moves you will discover that the masses acting on the nodes increase from the top down. This matrix will therefore resemble a lower triangular as

[Introduction](#)

[Description of the ...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



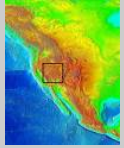
Page 5 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)



[Introduction](#)

[Description of the . . .](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



Page 6 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

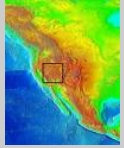
[Quit](#)

the reacting elements descend in layers with  $x$  and  $y$  separated. By inspection:

$$\mathbf{M} = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ m_1 & 0 & m_2 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & m_1 & 0 & m_2 & 0 & 0 & \dots & 0 & 0 \\ m_1 & 0 & m_2 & 0 & m_3 & 0 & \dots & 0 & 0 \\ 0 & m_1 & 0 & m_2 & 0 & m_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ m_1 & 0 & m_2 & 0 & m_3 & 0 & \dots & m_n & 0 \\ 0 & m_1 & 0 & m_2 & 0 & m_3 & \dots & 0 & m_n \end{bmatrix} \quad (2)$$

### 2.2.2. Finding the Stiffness Matrix

The stiffness matrix of the system will analyze the stiffness at each degree of freedom with respect to the other elements. This analyzes each layer as if it were a spring and will create a symmetric matrix for the same reasons stated



Introduction

Description of the ...

The Model

Home Page

Title Page



Page 7 of 26

Go Back

Full Screen

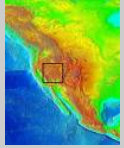
Close

Quit

before in the creation of the mass matrix.

$$\mathbf{K} = \begin{bmatrix} k_{1x} & k_{1y} & k_{2x} & k_{2y} & k_{3x} & k_{3y} & \dots & k_{(n-1)x} & k_{ny} \\ k_{1x} & k_{1y} & k_{2x} & k_{2y} & k_{3x} & k_{3y} & \dots & k_{(n-1)x} & k_{ny} \\ k_{1x} & k_{1y} & k_{2x} & k_{2y} & k_{3x} & k_{3y} & \dots & k_{(n-1)x} & k_{ny} \\ k_{1x} & k_{1y} & k_{2x} & k_{2y} & k_{3x} & k_{3y} & \dots & k_{(n-1)x} & k_{ny} \\ k_{1x} & k_{1y} & k_{2x} & k_{2y} & k_{3x} & k_{3y} & \dots & k_{(n-1)x} & k_{ny} \\ k_{1x} & k_{1y} & k_{2x} & k_{2y} & k_{3x} & k_{3y} & \dots & k_{(n-1)x} & k_{ny} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ k_{1x} & k_{1y} & k_{2x} & k_{2y} & k_{3x} & k_{3y} & \dots & k_{(n-1)x} & k_{ny} \\ k_{1x} & k_{1y} & k_{2x} & k_{2y} & k_{3x} & k_{3y} & \dots & k_{(n-1)x} & k_{ny} \end{bmatrix}$$

The stiffness is the analog of the spring constant for the different layers. As our  $k$  is the elasticity, the stiffness is the inverse and in the  $y$  direction they will add inversely where in the  $x$  direction they will add normally. There is no interaction between layers 1 and 3, 1 and 4 down to 1 and  $n$  and then 2 and 4, etc.



So these entries will become zero bringing this matrix down to:

$$\rightarrow \mathbf{K} = \begin{bmatrix} k_{1x} & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{k_{1y}} & 0 & \frac{-1}{k_{2y}} & 0 & 0 & \dots & 0 & 0 \\ k_{1x} & 0 & k_{2x} & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{k_{1y}} & 0 & \frac{1}{k_{2y}} & 0 & \frac{-1}{k_{3y}} & \dots & 0 & 0 \\ k_{1x} & 0 & k_{2x} & 0 & k_{3x} & 0 & \dots & 0 & 0 \\ 0 & \frac{1}{k_{1y}} & 0 & \frac{1}{k_{2y}} & 0 & \frac{1}{k_{3y}} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ k_{1x} & 0 & k_{2x} & 0 & k_{3x} & 0 & \dots & k_{nx} & 0 \\ 0 & \frac{1}{k_{1y}} & 0 & \frac{1}{k_{2y}} & 0 & \frac{1}{k_{3y}} & \dots & 0 & \frac{1}{k_{ny}} \end{bmatrix} \quad (3)$$

As you see this will create a tridiagonal matrix.

### 2.2.3. Finding the Damping Matrix

The damping matrix is the collection of the resistance against displacement. This is reacts against the displacements of the nodes and will be assumed to approximate a linear reaction while we have reasonably small displacements.

[Introduction](#)

[Description of the ...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



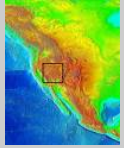
Page 8 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)



Introduction

Description of the ...

The Model

Home Page

Title Page



Page 9 of 26

Go Back

Full Screen

Close

Quit

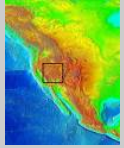
Once again this is asymmetric matrix.

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & \dots & c_{1n} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & \dots & c_{2n} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} & \dots & c_{3n} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} & \dots & c_{4n} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} & \dots & c_{5n} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} & \dots & c_{6n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{(n-1)1} & c_{(n-1)2} & c_{(n-1)3} & c_{(n-1)4} & c_{(n-1)5} & c_{(n-1)6} & \dots & c_{(n-1)n} \\ c_{n1} & c_{n2} & c_{n3} & c_{n4} & c_{n5} & c_{n6} & \dots & c_{nn} \end{bmatrix}$$

This has the same interactions as the stiffness matrix except that the  $y$  only reacts to the level above it. so will reduce to:

$$\mathbf{C} = \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & c_{22} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & c_{12} & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{33} & 0 & \dots & 0 & 0 & 0 \\ 0 & c_{12} & 0 & c_{23} & 0 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & c_{nn} & 0 \\ 0 & c_{12} & 0 & c_{23} & 0 & c_{34} & \dots & c_{(n-1)n} & 0 & 0 \end{bmatrix}$$

The damping values are also proportional to the energy that the layer will absorb as the force acts upon it. This is very important as this will allow one to



Introduction

Description of the ...

The Model

Home Page

Title Page

◀ ▶

◀ ▶

Page 10 of 26

Go Back

Full Screen

Close

Quit

determine the failure modes of the layers. Because if they absorb more energy than the layer material can ‘hold’<sup>1</sup> there will be a failure at that layer of the type of a fissure or the layer will shear apart depending on the direction incorporating the energy.

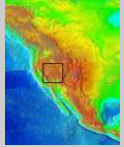
$$\rightarrow \mathbf{C} = \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & c_{22} & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & c_{12} & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{33} & 0 & \dots & 0 & 0 & 0 \\ 0 & c_{12} & 0 & c_{23} & 0 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & c_{nn} & 0 \\ 0 & c_{12} & 0 & c_{23} & 0 & c_{34} & \dots & c_{(n-1)n} & 0 & 0 \end{bmatrix} \quad (4)$$

### 2.3. Creating the Model

To set up the problem we first must look at the components of the model. The problem that we will look at has layers of soil with varying properties.<sup>2</sup> We shall assume these soils resemble elastic substances and therefore deform elastically. We shall also assume the elasticity is linear.

<sup>1</sup>Within the degrees of freedom only so much energy can be incorporated before a crack is formed as soil layers have mostly ceramic properties.

<sup>2</sup>See Figure 1 in the Appendix



[Introduction](#)

[Description of the . . .](#)

[The Model](#)

[Home Page](#)

[Title Page](#)

[◀](#) [▶](#)

[◀](#) [▶](#)

Page 11 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

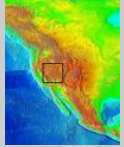
So looking at this we have  $n$  nodes corresponding to the different soil layers. During the earthquake a force that varies with time will come from some arbitrary direction. This force can then be broken up into two components, a horizontal component and a vertical component. This force will be in the form:

$$\mathbf{F}(t) = \begin{bmatrix} f_{x_1}(t) \\ f_{y_1}(t) \\ f_{x_2}(t) \\ f_{y_2}(t) \\ \vdots \\ f_{x_n}(t) \\ f_{y_n}(t) \end{bmatrix}$$

In the horizontal direction,  $x$ , the force will propagate as a pressure wave at  $v_p = \sqrt{(\kappa + 4G_n)/(3\rho)}$  where  $v_p$  is the velocity modifying the previous force. The force will cause a displacement which we will put into the form as

$$\mathbf{x}(t) = \begin{bmatrix} s_{x_1}(t) \\ s_{y_1}(t) \\ s_{x_2}(t) \\ s_{y_2}(t) \\ \vdots \\ s_{x_n}(t) \\ s_{y_n}(t) \end{bmatrix}$$

but all the vertical displacements,  $y$ , will be the same. Because otherwise the rise of the layers pushes on each other and either one layer goes through the above



*Introduction*

*Description of the . . .*

*The Model*

*Home Page*

*Title Page*



*Page 12 of 26*

*Go Back*

*Full Screen*

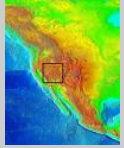
*Close*

*Quit*

layer or there is a vacuum created between the layers, neither shall be allowed within this model. From force and displacement we have a tree of reactions when a force is applied. Mass opposes acceleration, stiffness opposes velocity and damping opposes displacement.

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\dot{\mathbf{x}}(t) + \mathbf{C}\mathbf{x}(t) = \mathbf{F}(t)$$

From this we can put in the mass, stiffness and damping matrices that were discussed earlier. We will start knowing the initial displacement functions and from this determine the resultant force in each direction of each node.



[Introduction](#)

[Description of the...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)

◀▶

◀▶

Page 13 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

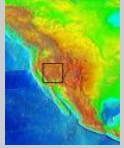
## 3. The Model

### 3.1. Initial Values

For this model because of scarcity of data we shall take assumed values for the damping. Sand is a combination of small crystalline structures and does not damp movement so much as excite and amplify in the proper frequencies so it will have a low value. For this paper we will pull initial soil properties as with the damping approximating a ratio:

Properties	Sand	Silt Loam	Clay	Aggregated Clay
Density	1.55 g/cm <sup>3</sup>	1.15 g/cm <sup>3</sup>	1.05 g/cm <sup>3</sup>	1.00 g/cm <sup>3</sup>
Horizontal Damping	.015	.18	.08	.11
Vertical Damping	.015	.18	.08	.11
Young's Modulus	20-60MPa	10-50MPa	3-10 MPa	.1-.6 MPa
Shear Modulus	27-81MPa	8.4-17MPa	4-6GPa	42-128MPa
Bulk Modulus	52-130MPa	2.0-170MPa	6-12GPa	21-55GPa

Also initially the displacement  $\mathbf{x}(0) = 0$ , and the velocity  $\dot{\mathbf{x}}(0) = 0$  as there is nothing started.



[Introduction](#)

[Description of the...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



Page 14 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

## 3.2. Evaluation

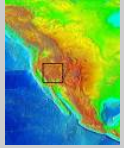
### 3.2.1. Mass Matrix

To evaluate the mass matrix we will take an unit wide and thick slice of the layer above the node so we get the mass directly over the node. So then  $m = \rho V$  where  $V$  is the volume of the piece that we are examining. So  $m = \rho_n h$  where  $h$  is the depth of the layer. We shall assume  $h$  to be a unit for our analysis. Using the previous mass matrix formation this will give us before substitution:

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & m_1 & 0 & 0 & 0 & 0 & \dots & 0 & 0 \\ m_1 & 0 & m_2 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & m_1 & 0 & m_2 & 0 & 0 & \dots & 0 & 0 \\ m_1 & 0 & m_2 & 0 & m_3 & 0 & \dots & 0 & 0 \\ 0 & m_1 & 0 & m_2 & 0 & m_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ m_1 & 0 & m_2 & 0 & m_3 & 0 & \dots & m_n & 0 \\ 0 & m_1 & 0 & m_2 & 0 & m_3 & \dots & 0 & m_n \end{bmatrix}$$

### 3.2.2. Stiffness Matrix

Inputting the Young's Modulus of our layers into the previously evaluated matrix we get the stiffness matrices for the difference estimated moduli.



[Introduction](#)

[Description of the...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)

◀▶

◀▶

Page 15 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

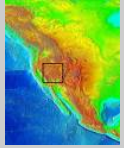
### 3.2.3. Damping Matrix

The Damping matrix as previously found will be updated with numbers.

### 3.2.4. Force Vector

The force vector shall be our evaluation and the output of our data set. So it shall stay as previously mentioned.

$$\mathbf{F}(t) = \begin{bmatrix} f_{x_1}(t) \\ f_{y_1}(t) \\ f_{x_2}(t) \\ f_{y_2}(t) \\ \vdots \\ f_{x_n}(t) \\ f_{y_n}(t) \end{bmatrix}$$



Introduction

Description of the...

The Model

Home Page

Title Page



Page 16 of 26

Go Back

Full Screen

Close

Quit

### 3.2.5. Displacement Vector

We know that the vertical displacement will be the same for every layer and therefore be:

$$\mathbf{x}(t) = \begin{bmatrix} s_{x_1}(t) \\ s_{y_1}(t) \\ s_{x_2}(t) \\ s_{y_1}(t) \\ \vdots \\ s_{x_n}(t) \\ s_{y_1}(t) \end{bmatrix}$$

### 3.2.6. Computation

As was outlined at the beginning of this paper we will be using the equation:

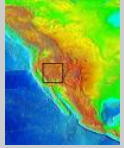
$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{K}\dot{\mathbf{x}}(t) + \mathbf{C}\mathbf{x}(t) = \mathbf{F}(t)$$

so putting in our components and displacement vectors we get a nasty bit of work, just what a computer is for.

### 3.2.7. Assuming Displacement to find Force

For this I have written a Matlab code which can be accessed at:

<http://online.redwoods.edu/instruct/darnold/laproj/fall2006/ben/evalearth.m>



Introduction

Description of the...

The Model

Home Page

Title Page



Page 17 of 26

Go Back

Full Screen

Close

Quit

## Example 1.

Putting in that a simple function that one might link with an earthquake namely that the displacement varies with time by  $s_x = .02(1 + 5n^t) \ln(t)$  and  $s_y = .006t^2$  in meters, where  $n$  is the node from top to bottom and the stiffness is broken into ten equal intervals we will look at the sixth:

$$f_{x1} = - .186/t^2 + .387E7/t + .216E - 1 * \ln(t)$$

$$f_{y1} = 162 * .600E - 2(t^2)t^2 - 15.9 * .600E - 2(t^2) - .753E - 7 * .600E - 2(t^2)t$$

$$f_{x2} = - .186/t^2 - .230E - 1/t^2(1. + 5.2^t) + .159/t2^t + .292E7\ln(t)2^t + .387E7/t + .844E6/t(1. + 5.2^t)$$

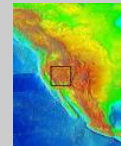
$$f_{y2} = 283. * .600E - 2(t^2)t^2 - 27.4 * .600E - 2(t^2) - .318E - 6.600E - 2(t^2)t + .300E - 2\ln(t)(1. + 5.2^t)$$

$$f_{x3} = - .186/t^2 - .230E - 1/t^2(1. + 5.2^t) + .159/t2^t + .292E7\ln(t)2^t - .210E - 1/t^2(1. + 5.3^t) + .231/t3^t + .758E6\ln(t)3^t + .387E7/t + .844E6/t(1. + 5.2^t) + .138E6/t(1. + 5.3^t) + .160E - 2\ln(t)(1. + 5.3^t)$$

$$f_{y3} = 393. * .600E - 2(t^2)t^2 - 38.1 * .600E - 2(t^2) - .558E - 6.600E - 2(t^2)t$$

$$f_{x4} = - .186/t^2 - .230E - 1/t^2(1. + 5.2^t) + .159/t2^t + .292E7 * \ln(t)2^t - .210E - 1/t^2 * (1. + 5.3^t) + .231/t * 3^t + .758E6 * \ln(t)3^t - .200E - 1/t^2(1. + 5.4^t) + .278/t4^t + .626E7 * \ln(t)4^t + .387E7/t + .844E6/t(1. + 5.2^t) + .138E6/t(1. + 5.3^t) + .901E6/t(1. + 5.4^t) + .220E - 2 * \ln(t)(1. + 5.4^t)$$

$$f_{y4} = 498. * .600E - 2(t^2)t^2 - 48.2 * .600E - 2(t^2) - .104E - 5 * .600E - 2(t^2)t$$



Now we will evaluate for  $t = 1$  seconds. Putting this values for t in we get<sup>3</sup>:

Time	Force	Angle
1s	3.43E7	7.87E-8
3s	6.04E8	7.40E-26
7s	2.14E11	1.53E-116
12s	2.68E14	0

### Example 2.

Here we will take the displacement equations to be:

$$s_x = .008 \frac{n}{t} * \sin(t) \quad \text{and} \quad s_y = (.03 * t - .012)^t$$

Once again we will look at the sixth interval for the evaluation. These two equations then produce this set of equations:

---

<sup>3</sup>The full calculations are in [Appendix B: Matlab Calculations](#)

Introduction

Description of the...

The Model

Home Page

Title Page



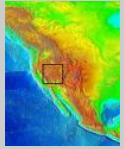
Page 18 of 26

Go Back

Full Screen

Close

Quit



[Introduction](#)

[Description of the ...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



Page 19 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

$$\mathbf{f}_{x1} = 1.55 * (.03 * t - .012)^t * (\ln(.03 * t - .012) + .03 * t / (.03 * t - .012))^2 + 1.55 * (.03 * t - .012)^t * (.600E - 1 / (.03 * t - .012) - .900E - 3 * t / (.03 * t - .012)^2) + .735E - 8 * (.03 * t - .012)^t * (\ln(.03 * t - .012) + .03 * t / (.03 * t - .012))$$

$$\mathbf{f}_{y1} = - .308E - 1 * \sin(t) / t - .616E - 1 * \cos(t) / t^2 + .616E - 1 * \sin(t) / t^3 + .933E6 * \cos(t) / t - .933E6 * \sin(t) / t^2$$

$$\mathbf{f}_{x2} = 2.70 * (.03 * t - .012)^t * (\ln(.03 * t - .012) + .03 * t / (.03 * t - .012))^2 + 2.70 * (.03 * t - .012)^t * (.600E - 1 / (.03 * t - .012) - .900E - 3 * t / (.03 * t - .012)^2) + .310E - 7 * (.03 * t - .012)^t * (\ln(.03 * t - .012) + .03 * t / (.03 * t - .012)) + .180 * (.03 * t - .012)^t + .240E - 2 * \sin(t) / t$$

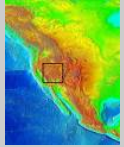
$$\mathbf{f}_{y2} = - .541E - 1 * \sin(t) / t - .112 * \cos(t) / t^2 + .112 * \sin(t) / t^3 + .110E7 * \cos(t) / t - .110E7 * \sin(t) / t^2$$

$$\mathbf{f}_{x3} = 3.75 * (.03 * t - .012)^t * (\ln(.03 * t - .012) + .03 * t / (.03 * t - .012))^2 + 3.75 * (.03 * t - .012)^t * (.600E - 1 / (.03 * t - .012) - .900E - 3 * t / (.03 * t - .012)^2) + .547E - 7 * (.03 * t - .012)^t * (\ln(.03 * t - .012) + .03 * t / (.03 * t - .012)) + .330 * (.03 * t - .012)^t$$

$$\mathbf{f}_{y3} = - .845E - 1 * \sin(t) / t - .176 * \cos(t) / t^2 + .176 * \sin(t) / t^3 + .254E7 * \cos(t) / t - .254E7 * \sin(t) / t^2$$

$$\mathbf{f}_{x4} = 4.75 * (.03 * t - .012)^t * (\ln(.03 * t - .012) + .03 * t / (.03 * t - .012))^2 + 4.75 * (.03 * t - .012)^t * (.600E - 1 / (.03 * t - .012) - .900E - 3 * t / (.03 * t - .012)^2) + .102E - 6 * (.03 * t - .012)^t * (\ln(.03 * t - .012) + .03 * t / (.03 * t - .012)) + .410 * (.03 * t - .012)^t$$

$$\mathbf{f}_{y4} = 498. * .600e - 2(t^2) * t^2 - 48.2 * .600e - 2(t^2) - .104e - 5 * .600e - 2(t^2) * t$$



Now we will evaluate for  $t = \begin{bmatrix} 1 \text{ seconds} \\ 3 \text{ seconds} \\ 7 \text{ seconds} \\ 12 \text{ seconds} \end{bmatrix}$ . This gives<sup>4</sup>:

$$F = \begin{bmatrix} -7.77E4 & -8.92E4 & 2.43E4 & 1.91E4 \\ 0.170 & 0.00168 & 8.41E-006 & 4.09E-007 \\ -2.81E+005 & -3.23E+005 & 8.80E4 & 6.91E4 \\ 0.301 & 0.00312 & 0.000242 & -0.000106 \\ -3.31E+005 & -3.80E+005 & 1.03E+005 & 8.15E4 \\ 0.416 & 0.00421 & 2.43E-005 & 2.03E-006 \\ -7.65E+005 & -8.78E+005 & 2.40E+005 & 1.88E+005 \\ 0.527 & 0.00533 & 3.07E-005 & 2.55E-006 \end{bmatrix}$$

Now at the lowest node:

Time	Newtons	Radians
1s	7.65E+005	-6.89E-007
3s	8.78E+005	-6.07E-009
7s	2.40E+005	1.28E-010
12s	1.88E+005	-1.35e-011

<sup>4</sup>These are simplified calculations. Full calculations appended in [Appendix B: Matlab Calculations](#)

[Introduction](#)

[Description of the...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



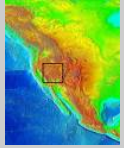
Page 20 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)



[Introduction](#)

[Description of the...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)

◀▶

◀▶

Page 21 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

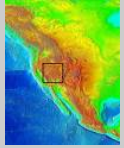
[Quit](#)

### 3.3. Analysis

Now to look at our data let us look at the analysis for the Richter scale  $M = \log_{10}(I/S)$  where  $M$  is the magnitude and  $S = 10$  micrometers or  $10^{-6}$  meters as a arbitrary standard earthquake where this is a very small earthquake. The examples used in this paper are of a ground movement of several centimeters thus correlating to around 5 to 6 on the Richter scale depending on the distance to the epicenter. It should be noted that the force impacts at a very, very slight angle to the horizontal thus indicating a more probable vertical movement. The force appears to be of the right magnitude perhaps a little on the low side but it must be remembered that we estimated numbers for the damping of the layers.

The examples are around the numbers expected by an order of magnitude or so. To get a better model many of the assumptions that were stated could be addressed. For instance one could take a discrete portion of the sample and array this across in three dimensions to create a 3 dimensional cube. Then you can calculate at all portions the force from a fixed or possibly moving source. This would also add in yet another degree of freedom to analyze. Also for accuracy one should add in the degree of freedom for the rotations. With the sand layer there should also be another degree of freedom for the vibration as sand can liquify under excitation.

For doing these tensors would seem to be the best and easier way to accomplish this task. Also possible with this model would be to take the force and find the displacement, first and second derivatives to analyze the ground movement under a known force. This involves separating an equation to partial integrals and differentiation. This was not solved or attempted as differential equations



*Introduction*

*Description of the...*

*The Model*

*Home Page*

*Title Page*



*Page 22 of 26*

*Go Back*

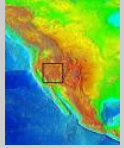
*Full Screen*

*Close*

*Quit*

was not part of the course.

Finding the displacement from the force can be used for various engineering purposes such as building and engineering structures to resist the movement or to analyze slope stabilities. Also mentioned briefly within the paper was the stress modes. This allows a prediction of the failure of the layers. So once you have your calculations one can then separately calculate  $Cx$  and for each layer look at the energies to decide if the layer has passed its failure mode and no longer follows our model.



Introduction

Description of the ...

The Model

Home Page

Title Page



Page 23 of 26

Go Back

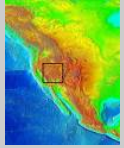
Full Screen

Close

Quit

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[Introduction](#)

[Description of the . . .](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



Page 24 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

## A. Schematic of Ground Layers

This figure shows you the ground layers with the dots representing the nodes and the layers having the properties that are shown in the [table of properties](#).

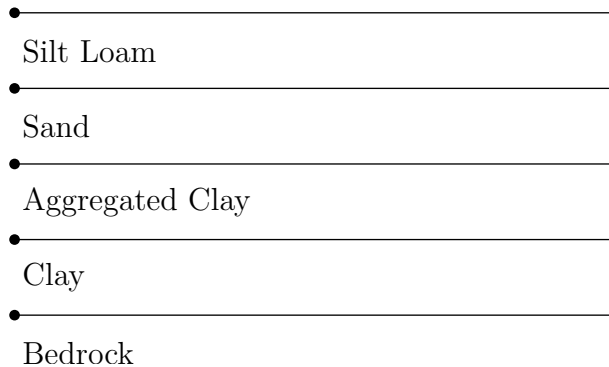
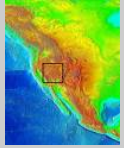


Figure 1: Different Layers of the model and the nodes



[Introduction](#)

[Description of the...](#)

[The Model](#)

[Home Page](#)

[Title Page](#)



Page 25 of 26

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

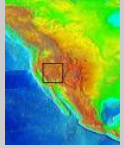
## B. Matlab Calculations

### Example 1

Matlab results for  $s_{x_n} = .02(1 + 5n^t) \log(t)$  and  $s_{y_n} = .006t^2$ .

F(:, :, 6) =

2.69e+008	8.9667e+007	3.8429e+007	2.2417e+007
0.8766	1.4533e-017	1.0672e-105	2.6387e-316
4.6082e+008	6.8654e+007	3.4474e+007	2.2248e+007
1.5336	0.20097	0.022768	0.00090996
4.8235e+008	6.6459e+007	3.4411e+007	2.2248e+007
2.1294	3.5261e-017	2.589e-105	6.4014e-316
5.8949e+008	5.8117e+007	3.434e+007	2.2248e+007
2.6988	4.4682e-017	3.2808e-105	8.1117e-316



Introduction

Description of the...

The Model

Home Page

Title Page



Page 26 of 26

Go Back

Full Screen

Close

Quit

## Example 2

For  $s_x = .008 \frac{n}{t} * \sin(t)$  and  $s_y = (.03 * t - .012)^t$ .

F(:, :, 6) =

-7.7702e+006	-8.9185e+006	2.4327e+006	1.9104e+006
15.87	0.21563	0.012547	0.0022619
-2.8099e+007	-3.2252e+007	8.7975e+006	6.9086e+006
26.212	-1.5136	-1.905	-1.9695
-3.3129e+007	-3.8025e+007	1.0372e+007	8.1452e+006
35.4	-2.9625	-3.5436	-3.5929
-7.6497e+007	-8.7802e+007	2.395e+007	1.8808e+007
44.76	-3.6708	-4.4021	-4.4638