Warps and Morphs

Applications of Linear Algebra

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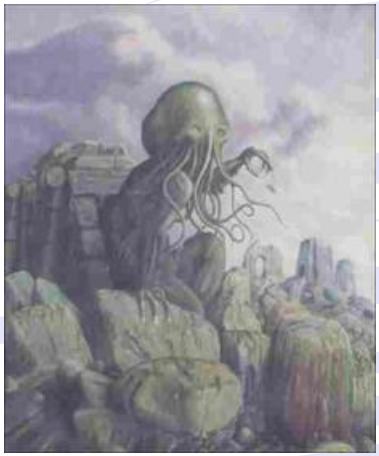
Introduction

- What is Morphing?
- The word morph derives from the word metamorphosis meaning to change shape or form.
- Morphing is achieved by compiling several images that are gradually distorted and faded out while the destination image is faded in.
- In this presentation we will develop a mathematical process that allows for the metamorphosis of one digital image into another.

An Example of Morphs and Warps

- The girl in the picture morphs into a frog.

Objective

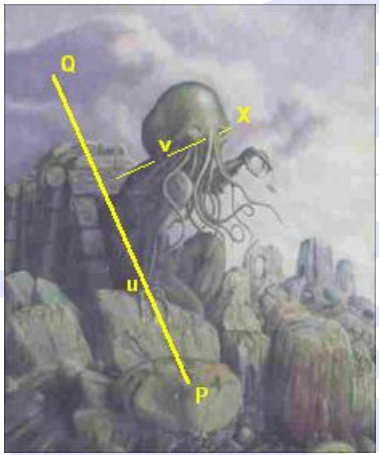


Destination Image

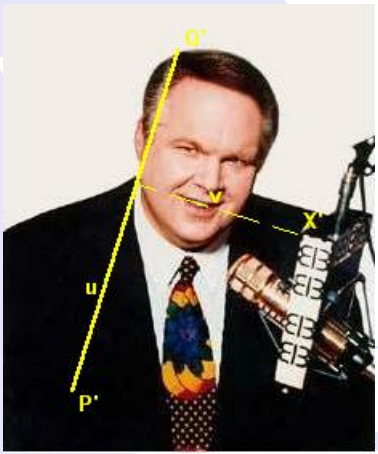
Source Image

Process

- In order to achieve this we will draw a line on the source image and destination image.



Destination Image



Source Image

First Steps

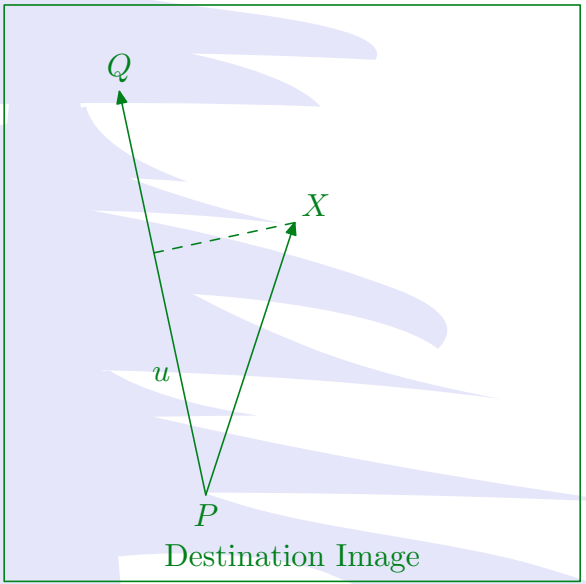
- In order to calculate the color we let each pixel be represented by X and X'.
- We will calculate the color of X' in the source image and pour that color into X in the destination image.
- In order to accomplish this we will need to calculate u and v.
- u is a percentage up the line PQ, and v is a set distance away from the line PQ.

Finding u

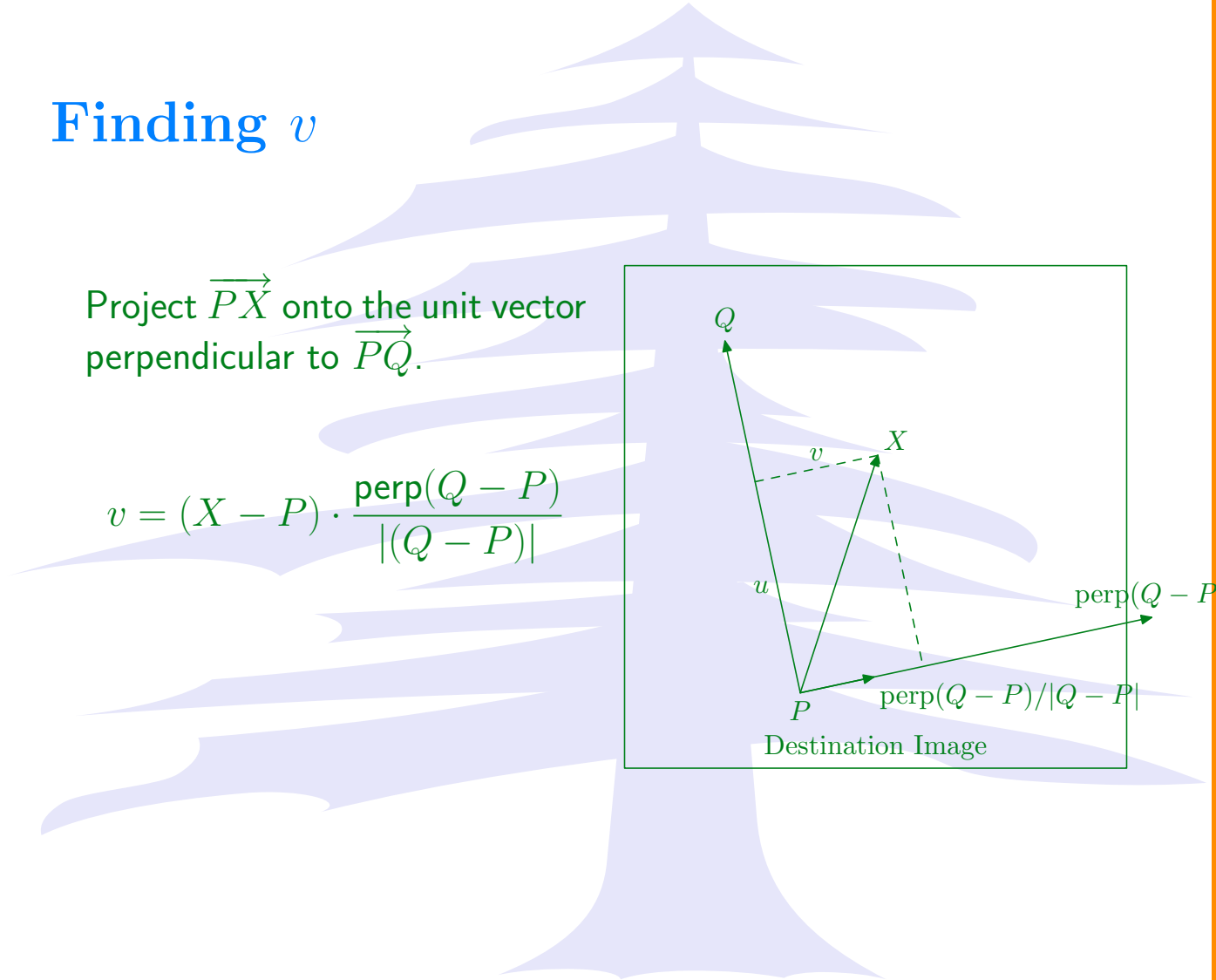
Start with X in the destination image.

Project $\overrightarrow{PX}$ onto $\overrightarrow{PQ}$ to determine u.

$$u = \frac{(X - P) \cdot (Q - P)}{(Q - P) \cdot (Q - P)}$$



Finding v



Project $\overrightarrow{PX}$ onto the unit vector perpendicular to $\overrightarrow{PQ}$.

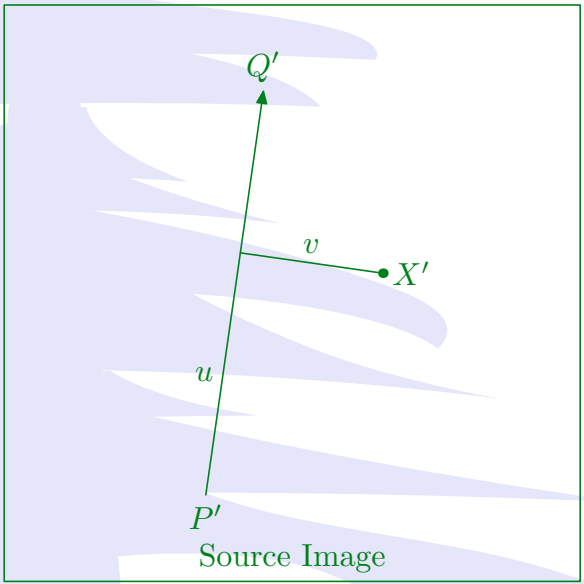
$$v = (X - P) \cdot \frac{\text{perp}(Q - P)}{|(Q - P)|}$$

Finding X' in Source Image

Start at P'.

Move along $\overrightarrow{P'Q'}$ the same percentage u that we moved along $\overrightarrow{PQ}$ in the destination image.

Move perpendicular to $\overrightarrow{P'Q'}$ a distance v, the same v computed in the destination image.



Calculating X'

$$X' = P' + u \cdot (Q' - P') + v \cdot \frac{\mathsf{perp}(Q' - P')}{|(Q' - P')|}$$

One Line

Destination Image

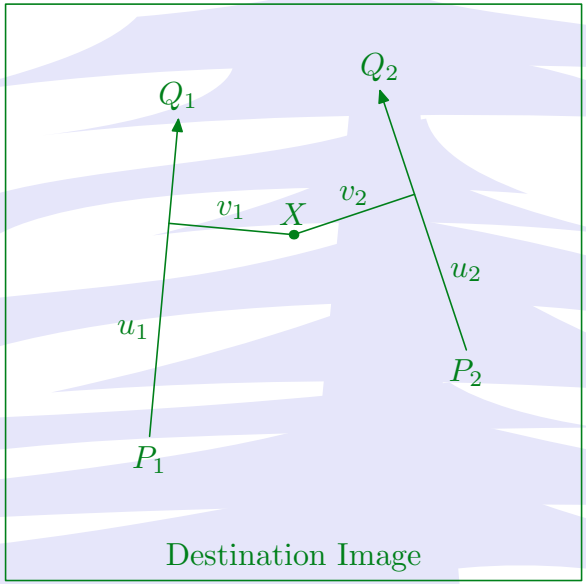
Source Image

Warp with One Line

Destination Image

Source Image

Two Lines



Calculating u_1, u_2, v_1, and v_2

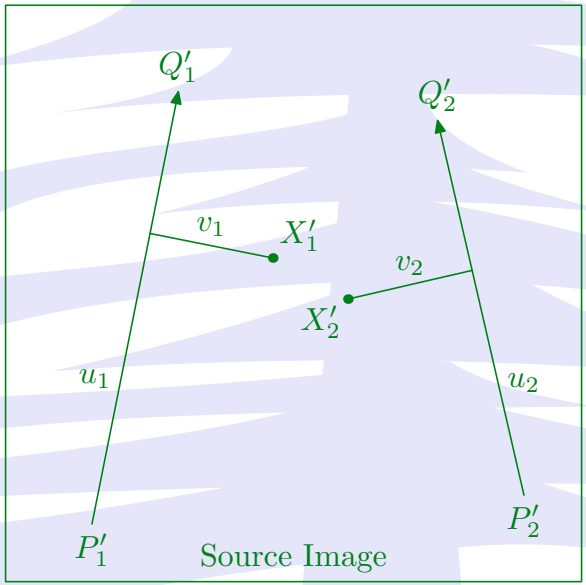
$$u_1 = \frac{(X - P_1) \cdot (Q_1 - P_1)}{(Q_1 - P_1) \cdot (Q_1 - P_1)}$$

$$u_2 = \frac{(X - P_2) \cdot (Q_2 - P_2)}{(Q_2 - P_2) \cdot (Q_2 - P_2)}$$

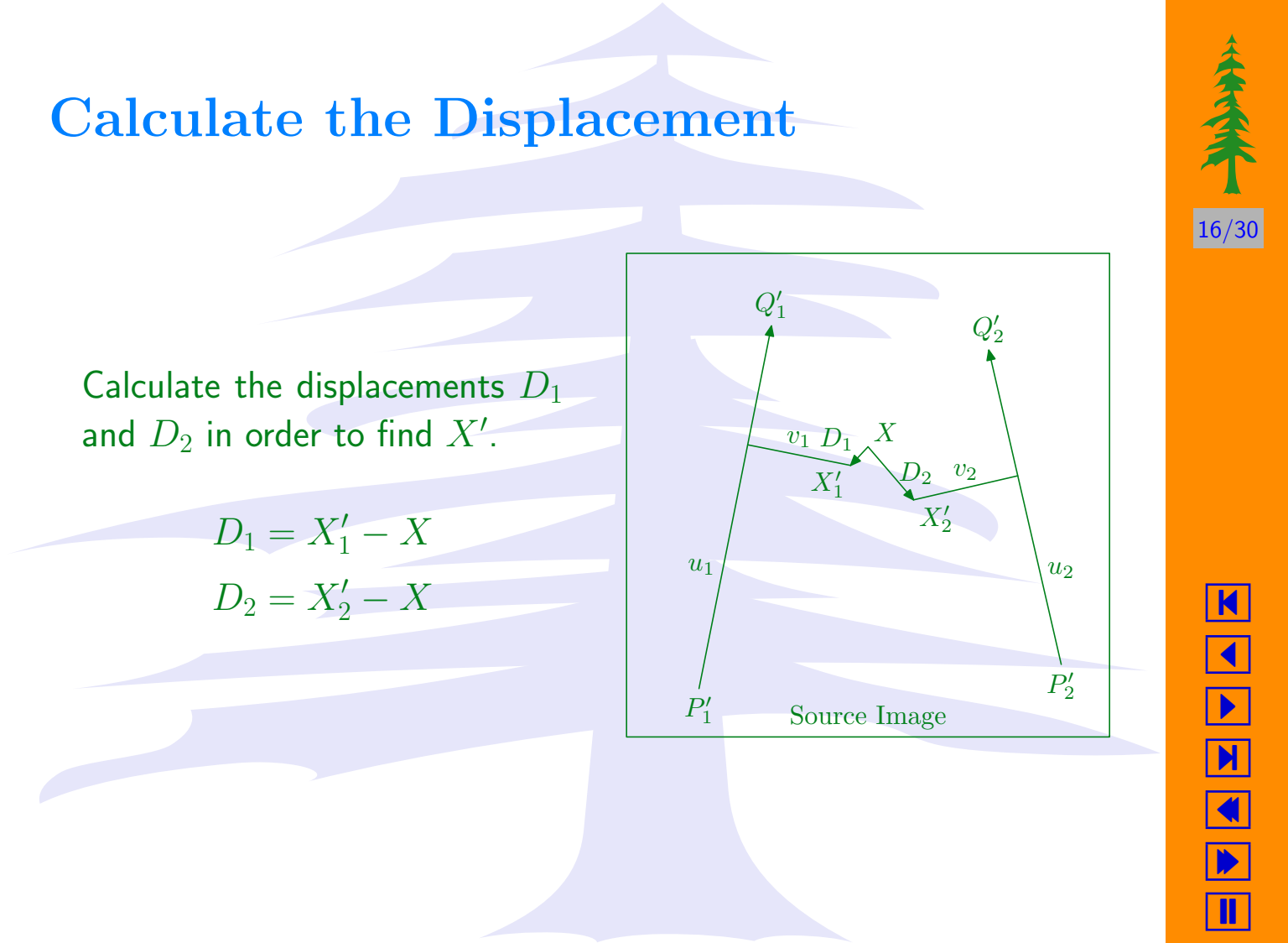
$$v_1 = (X - P_1) \cdot \frac{\mathsf{perp}(Q_1 - P_1)}{|(Q_1 - P_1)|}$$

$$v_2 = (X - P_2) \cdot \frac{\mathsf{perp}(Q_2 - P_2)}{|(Q_2 - P_2)|}$$

Calculate X_1' and X_2'



Calculate the Displacement



Calculate the displacements D_1 and D_2 in order to find X'.

$$D_1 = X_1' - X$$

$$D_2 = X_2' - X$$

Calculate the Weight for Each Displacement.

We now want to compute a weighted average of our displacements. We use the following formula for the weights.

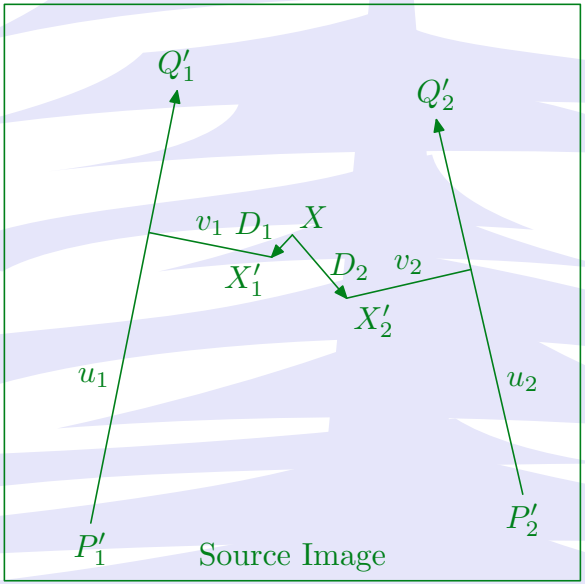
$$\text{Weight} = \left(\frac{\text{length}^p}{a + \text{dist}} \right)^b$$

Length is the length of the line $P_i Q_i$

Dist is the distance from the pixel to the line.

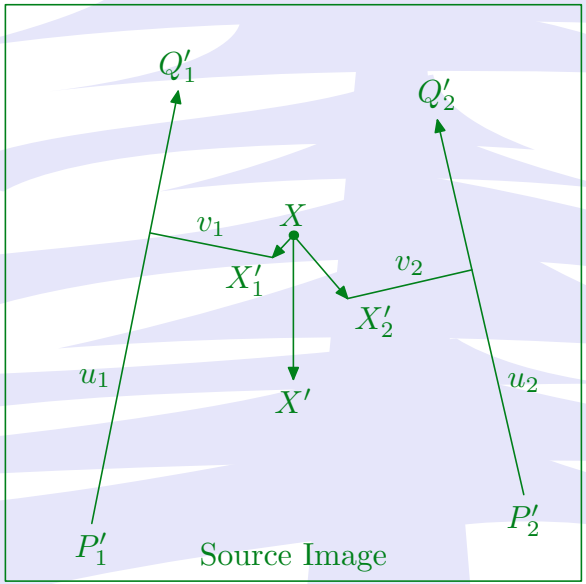
Parameters a, b, and p are constants that can be used to change the relative effect of the lines.

Weighted Average



$$\frac{W_1 D_1 + W_2 D_2}{W_1 + W_2}$$

Calculating X' by Adding the Weighted Average



$$X' = X + \frac{W_1 D_1 + W_2 D_2}{W_1 + W_2}$$







Warp with Two Lines

Destination Image

Source Image

Warp with Two Lines

Destination Image

Source Image

Back to Our Objective

Destination Image

Source Image

Rush Lines to Squid man Lines

Rush Destination Image

Rush Source Image

Rush Sequence Lines

Warp Rush Sequence

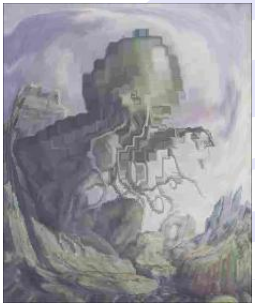
Rush to Squid Man Lines

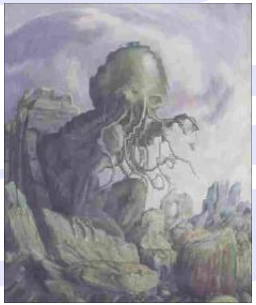


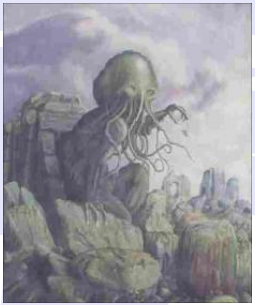




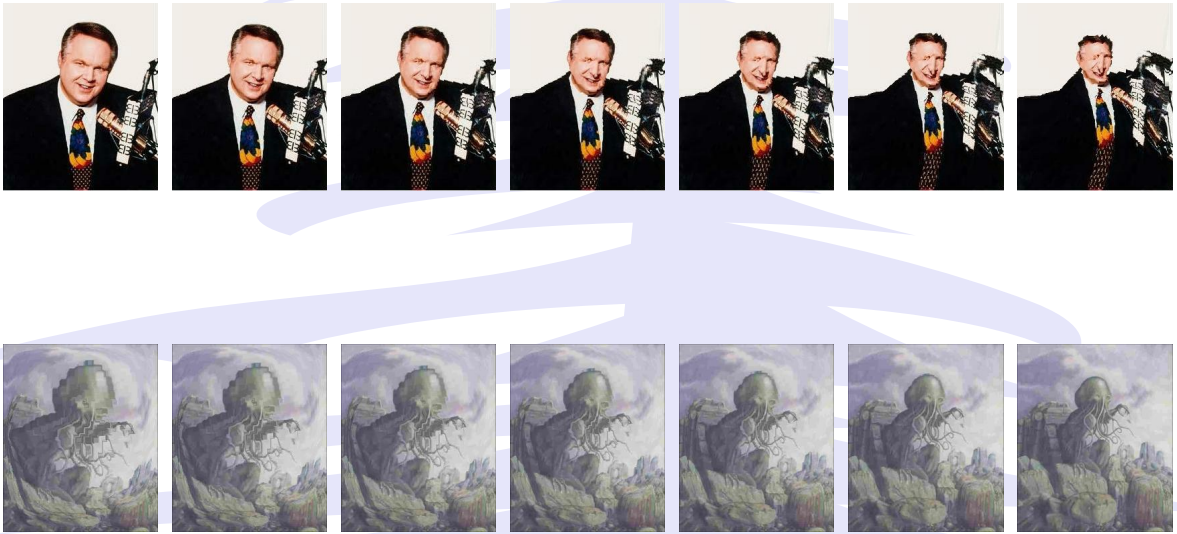
Squid Man to Rush Lines






Forwards and Backwards



Blending the Morph