College of the Redwoods
Mathematics Department
Math 120 — Intermeidate Algebra

Review — Final Examination

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Multiple Choice Questions

Instructions: *For each of the following questions, select the "best" answer and darken the corresponding oval. Good luck!*

1. Solve the following equation for x.

$$3.2x - 1.2 = 0.4x + 2.5$$

(a) 33/11 (b) $-32/13$ (c) 24/17

(d) 28/19 (e) 37/28

2. Solve the following equation for x.

$$x - \frac{x+1}{2} = 3$$

(a) 7 (b) -3 (c) 2

(d) -5 (e) 4

3. Solve the following equation for p.

$$\frac{1}{p} = \frac{1}{q} + \frac{1}{r}$$

(a) $q + r$ (b) $1/(q + r)$ (c) $qr/(q + r)$

(d) $(q + r)/qr$ (e) q/r

4. Solve the following inequality for x.

$$3x - 2(4 - x) < 8$$

(a) $x > 8/3$ (b) $x < 4/5$ (c) $x < 8/5$

(d) $x > 5/16$ (e) $x < 16/5$

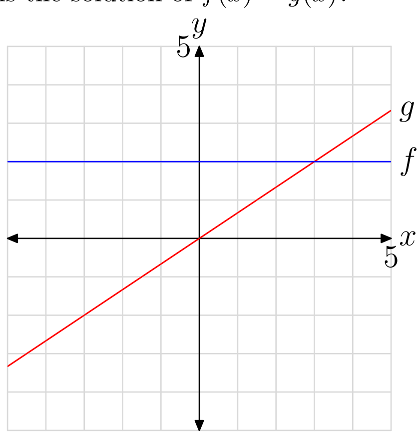
5. Solve the following inequality for x.

$$-x < 4$$

(a) $x > 4$ (b) $x > -4$ (c) $x < 4$

(d) $x < -4$ (e) $x < 5$

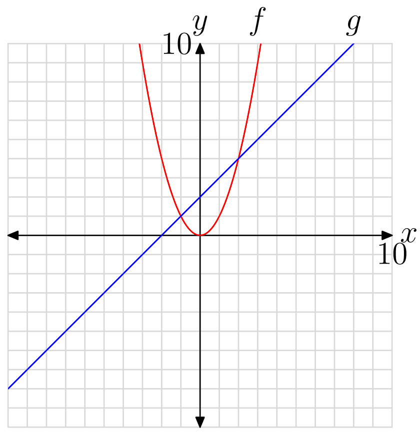
6. Based on the graph below, what is the solution of $f(x) = g(x)$?



(a) $x = 0$ (b) $x = 1$ (c) $x = -3$

(d) $x = 3$ (e) $x = 0$

7. Consider the following plot of the functions f and g.



Which of the following best describes the solution of $f(x) > g(x)$?

(a) $(-1, 2)$ (b) $(-\infty, -1)$ (c) $(2, +\infty)$

(d) $(-\infty, -1) \cup (2, +\infty)$ (e) $(2, -1)$

8. Consider the following inequality.

$$-2 < 3 - 2x < 5$$

Which of the following best describes the solution set?

(a) $(-\infty, 1) \cup (5/2, +\infty)$ (b) $[5/2, -1]$ (c) $(-1, +\infty)$

(d) $(-\infty, 5/2)$ (e) $(-1, 5/2)$

9. Consider the following inequality.

$$2x + 3 < 5 \qquad \text{or} \qquad 3 - x > -5$$

Which of the following best describes the solution set?

(a) $(-\infty, 8)$ (b) $(1, +\infty)$ (c) $(1, 8)$

(d) $(-\infty, 1) \cup (8, +\infty)$ (e) $[8, 1]$

10. Consider the following inequality.

$$|x - a| < \delta$$

Which of the following best describes the solution set?

(a) $(a - \delta, +\infty)$ (b) $(a - \delta, a + \delta)$ (c) $(-\infty, a + \delta)$

(d) $(-\infty, a - \delta, a + \delta, +\infty)$ (e) $[a - \delta, a + \delta]$

11. Consider the following inequality.

$$|3x + 2| > 7$$

Which of the following best describes the solution set?

(a) $(-3, 5/3)$ (b) $(-\infty, 5/3)$ (c) $(-3, +\infty)$

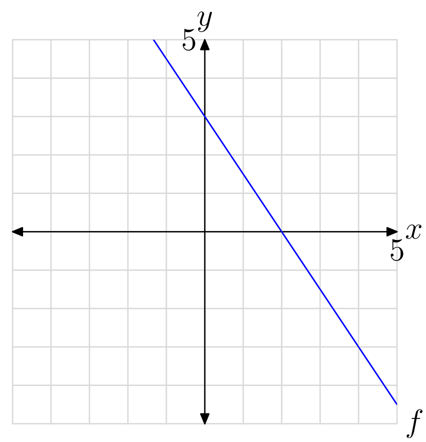
(d) $[5/3, -3]$ (e) $(-\infty, -3) \cup (5/3, +\infty)$

12. What is the slope of the line passing through the points $(2, 3)$ and $(-3, 5)$?

(a) $-5/2$ (b) $-2/5$ (c) $8/5$

(d) -8 (e) $-1/8$

13. Consider the following graph of f.



Which of the following is the equation of the line pictured above?

(a) $f(x) = \frac{2}{3}x - 3$ (b) $f(x) = \frac{2}{3}x + 3$ (c) $f(x) = \frac{3}{2}x + 3$

(d) $f(x) = -\frac{2}{3}x + 3$ (e) $f(x) = -\frac{3}{2}x + 3$

14. What is the slope of the line represented by the equation $3x + 4y = 12$?

(a) $-3/4$ (b) $3/4$ (c) $-4/3$

(d) $4/3$ (e) 4

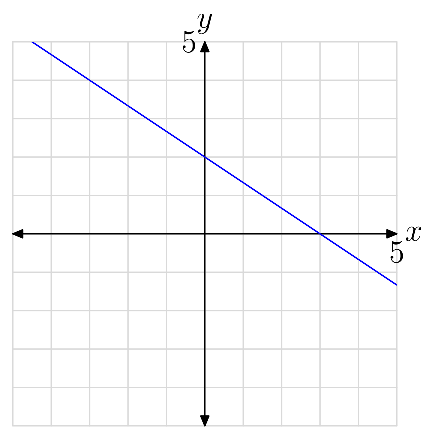
15. What is the equation of the line passing through the points $(2, 3)$ and $(5, 1)$?

(a) $y - 3 = -\frac{2}{3}(x - 5)$ (b) $y - 3 = \frac{3}{2}(x - 2)$

(c) $y - 3 = \frac{4}{7}(x - 2)$ (d) $y - 2 = -\frac{2}{3}(x - 3)$

(e) $y - 3 = -\frac{2}{3}(x - 2)$

16. Consider the line pictured below.

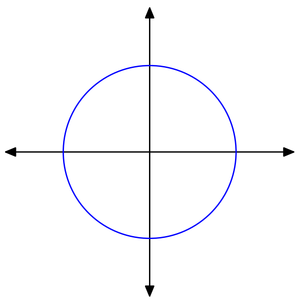


What is the slope of *any* line perpendicular to the line drawn above?

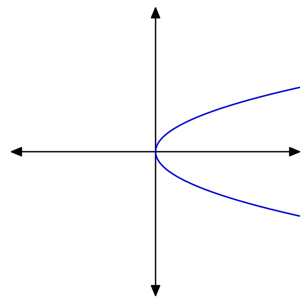
(a) $2/3$ (b) $-2/3$ (c) $-3/2$

(d) $3/2$ (e) -1

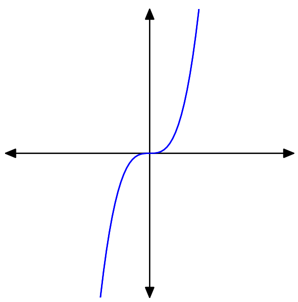
17. Which of the following depicts the graph of a function?



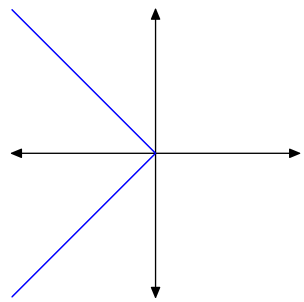
(a)



(b)

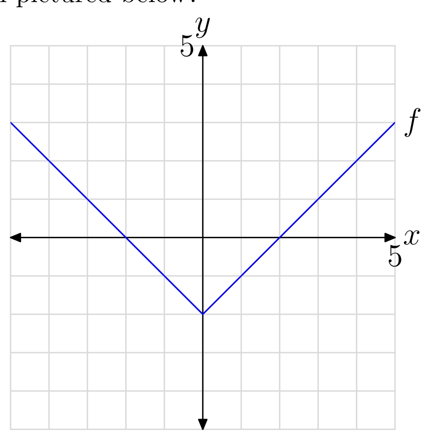


(c)



(d)

18. What is the range of the function pictured below?



(a) $[0, +\infty)$ (b) $(-\infty, +\infty)$ (c) $[-2, +\infty)$

(d) $(-\infty, -2]$ (e) $(-\infty, 0]$

19. What is the domain of the function defined by the following equation?

$$f(x) = \sqrt{x - 2}$$

(a) $[1, +\infty)$ (b) $[2, +\infty)$ (c) $(-\infty, 2)$

(d) $[-2, +\infty)$ (e) $(-2, 2)$

20. Given $f(x) = x^2 + 2x - 3$, evaluate $f(-3)$.

(a) -2 (b) -1 (c) 0

(d) 1 (e) 3

21. Given $f(x) = x^2 + x$, evaluate $f(a + 3)$.

(a) $(a + 3)(x^2 + x)$ (b) $a^2 + 6a + 9$ (c) $2a + 6$

(d) $a^2 + a + 12$ (e) $a^2 + 7a + 12$

22. Given $f(x) = x^2$, simplify

$$\frac{f(x+h) - f(x)}{h}.$$

(a) h

(b) $\dfrac{x^2 + h^2}{h}$

(c) $2x + h$

(d) $x + 2h$

(e) $1 + \dfrac{x}{h}$

23. Given $f(x) = x^2 - 3x$, simplify

$$\frac{f(x) - f(5)}{x - 5}.$$

(a) $x^2 - 3x$

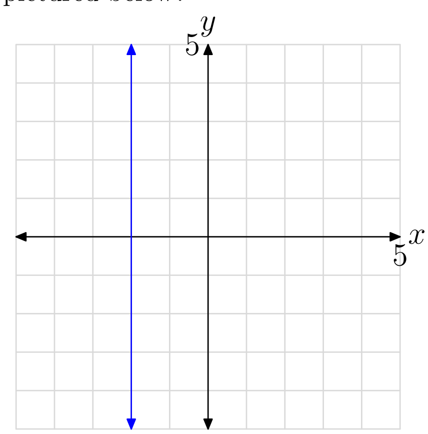
(b) $x - 5$

(c) $x - 2$

(d) $\dfrac{1}{x - 5}$

(e) $x + 2$

24. What is the equation of the line pictured below?



(a) $y = -2$

(b) $x = -2$

(c) $y = -1$

(d) $x = -1$

(e) $x + y = -2$

25. Use the regression capability of your graphing calculator to determine the equation of the *Line of Best Fit* for the data in the following table.

x	-2	-1	0	1	2
y	4	3	1	-1	-2

(a) $y = 2.8x + 3$

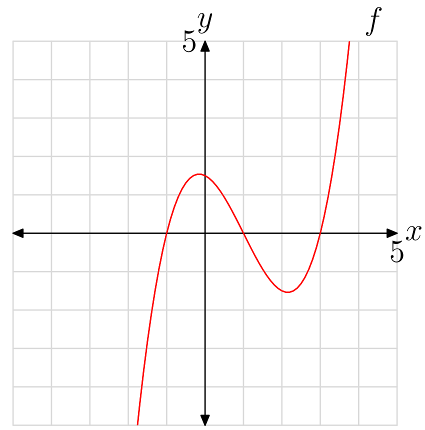
(b) $y = 1.6x + 2$

(c) $y = -1.1x + 3$

(d) $y = -1.6x + 1$

(e) $y = -2.2x + 3$

26. One of the zeros of the polynomial



shown above is

(a) -4 (b) -2 (c) -1

(d) 0 (e) 2

27. If -3 is a zero of the polynomial p, then which of the following must be a factor of $p(x)$?

(a) $x-3$ (b) $x-1$ (c) x

(d) $x+2$ (e) $x+3$

28. One of the zeros of

$$p(x) = x^2 + ax + bx + ab$$

is

(a) a (b) b (c) $-a$

(d) b/a (e) a/b

29. Reduce the rational expression

$$\frac{3t - t^2}{t^2 - 9}.$$

(a) $\frac{t}{3-t}$ (b) $\frac{t}{t-3}$ (c) $\frac{t}{t+3}$

(d) $-\frac{t}{t+3}$ (e) $\frac{3-t}{t+3}$

30. Consider the rational expression

$$f(x) = \frac{x+3}{x-5}.$$

The graph of f has a vertical asymptote

(a) $x = 5$ (b) $x = -3$ (c) $y = -3$

(d) $y = 5$ (e) $x = -3/5$

31. Consider the rational expression

$$f(x) = \frac{x^2 + 6x - 16}{x+4}.$$

One of the x-intercepts of the graph of f is

(a) -2 (b) -8 (c) 8

(d) 4 (e) -16

32. $\frac{a}{b} + \frac{c}{d}$ equals

(a) $\dfrac{a+c}{b+d}$ (b) $\dfrac{a+c}{bd}$ (c) $\dfrac{ad-bc}{bd}$

(d) $\dfrac{ad+bc}{bd}$ (e) $\dfrac{ac}{bd}$

33. Simplify

$$\frac{y^2-2y-15}{2y^2+6y}\cdot\frac{y^2-5y}{y^2-10y+25}.$$

(a) $\dfrac{y}{2y+6}$ (b) $\dfrac{y-5}{2y}$ (c) $\dfrac{1}{2}$

(d) $\dfrac{y-5}{2y(y+3)}$ (e) $\dfrac{y(y-5)}{y+3}$

34. Given that $f(x)=1/x$, simplify

$$\frac{f(x)-f(5)}{x-5}.$$

(a) $x-5$ (b) $\dfrac{1}{x}-\dfrac{1}{5}$ (c) $-\dfrac{1}{5x}$

(d) $\dfrac{1}{x+5}$ (e) $\dfrac{5x}{x-5}$

35. If

$$f(x)=\frac{x+1}{x} \qquad \text{and} \qquad g(x)=\frac{x}{x+1},$$

then $f(x)+g(x)$ equals

(a) 1 (b) $\dfrac{x^2}{(x+1)^2}$ (c) $\dfrac{x^2+2x+1}{x^2}$

(d) $\dfrac{2x^2+2x+1}{x(x+1)}$ (e) $\dfrac{x^2+2x+2}{x(x+1)}$

36. Given

$$f(x)=x-\frac{3}{x},$$

find one solution of $f(x)=2$.

(a) -1 (b) 1 (c) -3

(d) 2 (e) -4

37. One solution of

$$\frac{x}{x+a}=\frac{x+b}{x}$$

is

(a) ab (b) $\dfrac{a+b}{ab}$ (c) $ab(a+b)$

(d) a^2+b^2 (e) $\dfrac{-ab}{a+b}$

38. Working alone, it takes Jane 4 hours to paint a room. If it takes Jane and Liz 2 hours to paint the room working together, how long would it take Liz working alone?

(a) 1 hr (b) 2 hr (c) 3 hr

(d) 4.5 hr (e) 4 hr

39. Mary's boat travels at 5 miles per hour in still water. Mary travels upstream 24 miles, then returns to her starting point. If the total time of the trip is 10 hours, what is the speed of the current of the river?

(a) 1 mi/hr (b) 2 mi/hr (c) 2.5 mi/hr

(d) 1.5 mi/hr (e) 3 mi/hr

40. Consider the linear system

$$x + 2y = a$$
$$2x - y = b.$$

What is the x component of the solution?

(a) $\frac{2a-b}{5}$ (b) $\frac{a+2b}{5}$ (c) $\frac{2a+b}{5}$
(d) $\frac{a+b}{5}$ (e) $\frac{a-b}{5}$

41. $\sqrt{x^2}$ equals

(a) $\pm x$ (b) x (c) $-x$
(d) $|x|$ (e) x^2

42. Consider the function defined by

$$f(x) = \sqrt{2x+3}.$$

Which of the following best describes the domain of f?

(a) $[0, +\infty)$ (b) $(-2/3, 2/3)$ (c) $(-3/2, +\infty)$
(d) $[-3/2, +\infty)$ (e) $(-\infty, -3/2]$

43. Assuming $x \geq 0$, then $\sqrt{25x^3}$ equals

(a) $x\sqrt{5x}$ (b) $x^2\sqrt{5x}$ (c) $5x\sqrt{x}$
(d) $-5x\sqrt{x}$ (e) $-x\sqrt{5x}$

44. $\frac{1}{\sqrt[3]{3a^2}}$ equals

(a) $\frac{1}{3a^2}$ (b) $\frac{\sqrt[3]{3a^2}}{3a^2}$ (c) $\frac{\sqrt[3]{3a}}{9a}$
(d) $\frac{\sqrt[3]{9a}}{3a}$ (e) $\frac{\sqrt[3]{3a}}{3a^2}$

45. $\frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}-\sqrt{a}}$ equals

(a) $\frac{x+a}{x-a}$ (b) $\frac{\sqrt{x}-\sqrt{a}}{x-a}$ (c) $\frac{x^2+2ax+a^2}{x^2-2ax+a^2}$
(d) $\frac{x-a}{x+a}$ (e) $\frac{x+2\sqrt{xa}+a}{x-a}$

46. $(a+1)^{-2/3}$ equals

(a) $\frac{1}{\sqrt{(a+1)^3}}$ (b) $-\sqrt[3]{(a+1)^2}$ (c) $-\sqrt{(a+1)^3}$
(d) $\frac{1}{\sqrt[3]{(a+1)^2}}$ (e) $\frac{-1}{\sqrt[3]{(a+1)^2}}$

47. Given

$$f(x) = a - \sqrt{x^2 + a^2},$$

find a solution of $f(x) = -1$.

(a) $x = 2a+1$ (b) $x = \sqrt{1+a^2}$ (c) $x = \sqrt{2a+1}$
(d) $x = a+2$ (e) $x = 2+\sqrt{a}$

48. The parabola

$$y = x^2 - 2x - 4$$

has x-intercept

(a) $1-2\sqrt{5}$ (b) $2+2\sqrt{5}$ (c) $1-\sqrt{5}$

(d) $2+5\sqrt{2}$ (e) $2-4\sqrt{2}$

49. The parabola

$$y=-3x^2+9x+12$$

has axis of symmetry

(a) $x=-3/2$ (b) $x=3/2$ (c) $x=2/3$

(d) $x=-3$ (e) $x=3$

50. The parabola

$$y=x^2-4x-6$$

has a minimum value of

(a) $x=-4$ (b) $x=-6$ (c) $x=-8$

(d) -10 (e) -12

51. The total surface area of a right-circular cylinder with base radius r and height h is given by the formula

$$S=2\pi r^2+2\pi rh.$$

Solve this formula for r.

(a) $\dfrac{S}{2\pi r+2\pi h}$ (b) $\dfrac{S}{\pi(r+h)}$ (c) $\dfrac{\sqrt{8\pi S}}{4\pi}$

(d) $\dfrac{-\pi h+\sqrt{\pi^2h^2+2\pi S}}{\pi}$ (e) $\dfrac{-\pi h+\sqrt{\pi^2h^2+2\pi S}}{2\pi}$

52. Two numbers add to 10 so that the sum of the squares of the two numbers is a minimum. One of the numbers is

(a) 8 (b) 7 (c) 6

(d) 5 (e) 9

53. The city of Fortuna is growing at a rate of 4.2% per year. If the present population is 12,000, what will be the population in 5 years?

(a) $13,800$ (b) $14,740$ (c) $14,950$

(d) $15,120$ (e) $15,325$

54. What is the range of the function

$$f(x)=2^x-3$$

(a) $(-3,+\infty)$ (b) $[-3,+\infty)$ (c) $(-\infty,-3)$

(d) $(-2,+\infty)$ (e) $(-\infty,-3]$

55. If \$10,000 is invested in an account paying 6% compunded quarterly, how much will be in the account at the end of 10 years. Assume no additional deposits or withdrawals.

(a) \$16,943,82 (b) \$17,250.52 (c) \$18,140.18

(d) \$18,960.43 (e) \$19,470.44

56. How much should be invested now in an account paying 6% compounded continuously so that \$10,000 will be in the account at the end of 5 years? Assume no additional deposits of withdrawals.

(a) \$6,153.22 (b) \$6,722.34 (c) \$6,935.22

(d) \$7,120.52 (e) \$7,408.18

57. Given

$$f(x)=2x+3 \qquad \text{and} \qquad g(x)=x^2,$$

find $g(f(x))$.

(a) $2x^2 + 3$ (b) $4x^2 + 12x + 9$ (c) $4x^2 + 9$

(d) $4x^2 + 6x + 9$ (e) $2x^2 + 2x + 3$

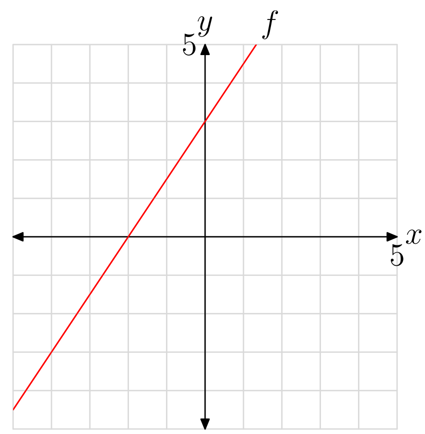
58. Which of the following is the inverse of the function defined by

$$f(x) = 3x + 4.$$

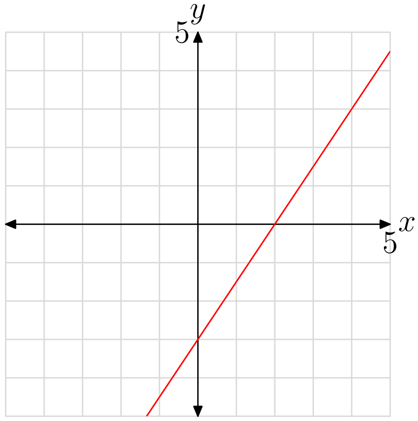
(a) $g(x) = 3x - 4$ (b) $g(x) = \frac{x}{3} - 4$ (c) $g(x) = \frac{x-4}{3}$

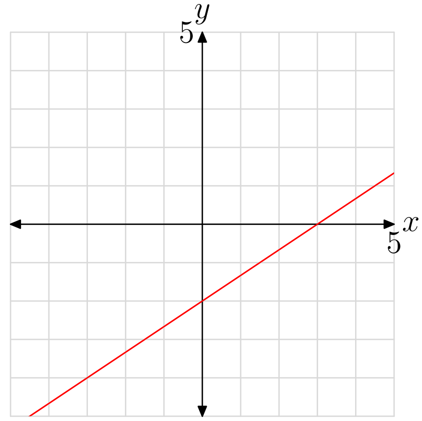
(d) $g(x) = \frac{x+4}{3}$ (e) $g(x) = x + \frac{4}{3}$

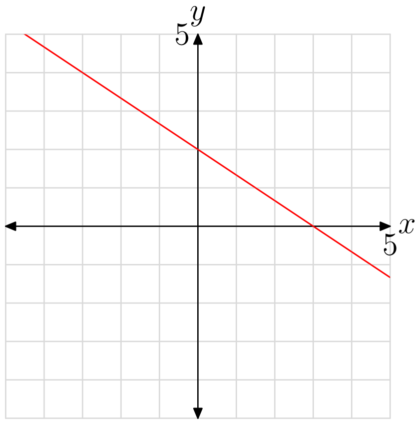
59. Consider the following graph of f.

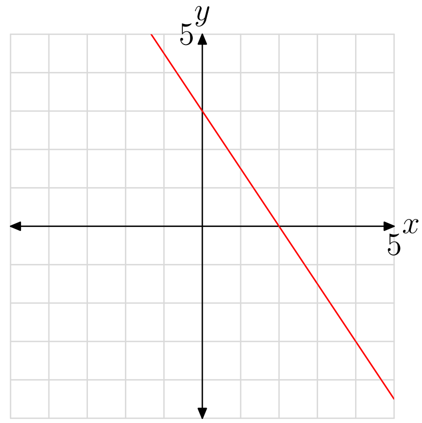


Which of the following is the graph of the inverse of f?

(a) 

(b) 

(c) 

(d) 

60. What is the domain of the function defined by

$$f(x) = \ln(x + 4).$$

(a) $(0, +\infty)$ (b) $(1, +\infty)$ (c) $(4, +\infty)$
(d) $(-4, +\infty)$ (e) $(-3, +\infty)$

61. Solve for x:

$$\log_x 9 = -2$$

(a) 81 (b) $\frac{1}{81}$ (c) $\frac{1}{3}$
(d) 3 (e) 9

62. Solve for x:

$$\log_9 27 = x$$

(a) 2/3 (b) -3 (c) -2
(d) 3/2 (e) $-1/3$

Solutions to Multiple Choice Questions

Solution to Question 1: Multiply both sides of the equation by 10.

$$10(3.2x - 1.2) = 10(0.4x + 2.5)$$
$$32x - 12 = 4x + 25$$

Isolate terms with x on one side of the equation.

$$32x - 4x = 25 + 12$$
$$28x = 37$$

Divide both sides by 28.

$$x = \frac{37}{28}$$

□

Solution to Question 2: Multiply both sides of the equation by 2.

$$2\left(x - \frac{x+1}{2}\right) = 2(3)$$
$$2x - 2\left(\frac{x+1}{2}\right) = 6$$
$$2x - (x+1) = 6$$

Distribute the minus sign and simplify.

$$2x - x - 1 = 6$$
$$x - 1 = 6$$

Add 1 to both sides of the equation.

$$x = 7$$

□

Solution to Question 3: Multiply both sides of the equation by pqr.

$$pqr\left(\frac{1}{p}\right) = pqr\left(\frac{1}{q} + \frac{1}{r}\right)$$
$$pqr\left(\frac{1}{p}\right) = pqr\left(\frac{1}{q}\right) + pqr\left(\frac{1}{r}\right)$$
$$qr = pr + pq$$

Factor out a p on the right-hand side.

$$qr = p(r+q)$$

Divide both sides by $r + q$.

$$\frac{qr}{r+q} = \frac{p(r+q)}{r+q}$$
$$\frac{qr}{r+q} = p$$

□

Solution to Question 4: Distribute -2, then simplify.

$$\begin{aligned} 3x - 2(4 - x) &< 8 \\ 3x - 8 + 2x &< 8 \\ 5x - 8 &< 8 \end{aligned}$$

Add 8 to both sides.

$$5x < 16$$

Divide both sides by 5.

$$x < \frac{16}{5}$$

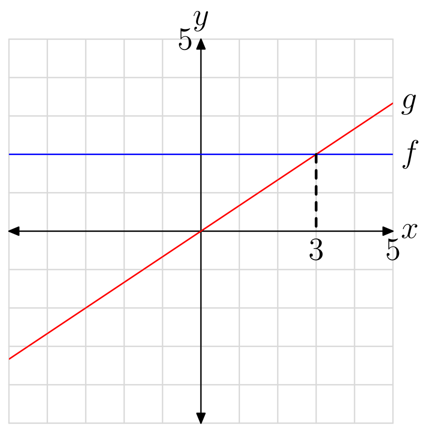
□

Solution to Question 5: Multiply both sides of the inequality by -1, reversing the inequality symbol.

$$\begin{aligned} -x &< 4 \\ -1(-x) &> -1(4) \\ x &> -4 \end{aligned}$$

□

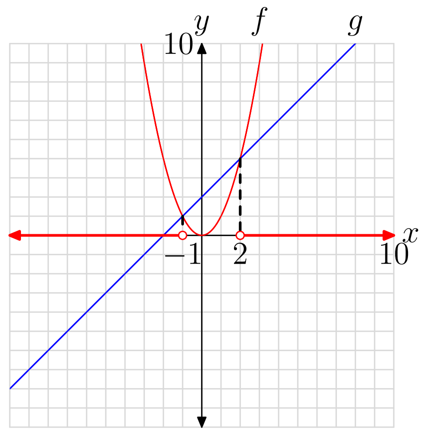
Solution to Question 6: Note that the graphs intersect at $x = 3$.



Thus, $x = 3$ is the solution of $f(x) = g(x)$.

□

Solution to Question 7: To find the solution of $f(x) > g(x)$, take note of where the graph of f lies *above* the graph of g.



The graph of f is above the graph of g when x is to the left of -1 or to the right of 2. Thus, the solution of $f(x) > g(x)$ is best described by

$$(-\infty, -1) \cup (2, +\infty) = \{x : x < -1 \text{ or } x > 2\}.$$

□

Solution to Question 8: Add -3 to all three members of the inequality.

$$-2 < 3 - 2x < 5 \tag{1}$$
$$-3 - 2L - 3 + 3 - 2x < -3 + 5 \tag{2}$$
$$-5 < -2x < 2 \tag{3}$$

Divide all three members by -2, reversing the inequality symbols.

$$\frac{-5}{-2} > \frac{-2x}{-2} > \frac{2}{-2} \tag{4}$$
$$\frac{5}{2} > x > -1 \tag{5}$$

It is instructive to read this from right to left and rearrange as follows.

$$-1 < x < \frac{5}{2}$$

Thus, the solution set is

$$(-1, 5/2) = \{x : -1 < x < 5/2\}.$$

□

Solution to Question 9: Solve each piece of

$$2x + 3 < 5 \qquad \text{or} \qquad 3 - x > -5$$

separately. Thus,

$$\begin{aligned} 2x + 3 &< 5 \\ 2x &< 2 \\ x &< 1. \end{aligned}$$

Similary,

$$\begin{aligned} 3 - x &> -5 \\ -x &> -8 \\ x &< 8. \end{aligned}$$

Note how we reversed the inequality in the last step because we multiplied both sides by -1. Thus, the original inequality is equivalent to $x < 1 \quad \text{or} \quad x < 8$. Now, the numbers that are either less than 1 or less than 8 are

$$(-\infty, 8) = \{x : x < 8\}.$$

□

Solution to Question 10: Recall that $|x| < a$, $a > 0$, if and only if $-a < x < a$. Thus,

$$|x - a| < \delta$$

becomes

$$-\delta < x - a < \delta.$$

Now add a to all three members of the inequality.

$$a - \delta < x < a + \delta$$

Hence, the solution set is

$$(a - \delta, a + \delta) = \{x : a - \delta < x < a + \delta\}.$$

□

Solution to Question 11: Recall that $|x| > a$, $a > 0$, if and only if $x < -a$ or $x > a$. Thus,

$$|3x + 2| > 7$$

becomes

$$3x + 2 < -7 \qquad \text{or} \qquad 3x + 2 > 7.$$

Solve each side separately. Thus,

$$\begin{aligned} 3x + 2 &< -7 \\ 3x &< -9 \\ x &< -3. \end{aligned}$$

Also,

$$\begin{aligned} 3x + 2 &> 7 \\ 3x &> 5 \\ x &> \frac{5}{3}. \end{aligned}$$

Thus, the original inequality is equivalent to

$$x < -3 \qquad \text{or} \qquad x > \frac{5}{3},$$

and the solution set is

$$(-\infty, -3) \cup (5/3, +\infty) = \{x : x < -3 \text{ or } x > 5/3\}.$$

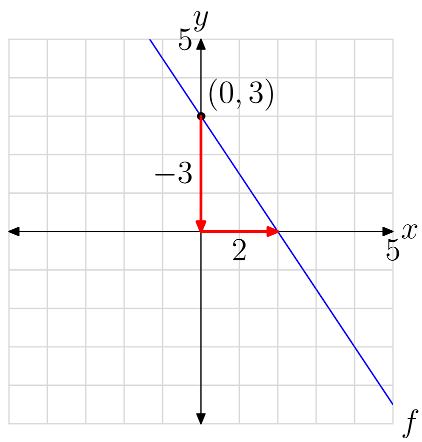
□

Solution to Question 12: The slope is defined as

$$\begin{aligned} \text{slope} &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{5 - 3}{-3 - 2} \\ &= \frac{2}{-5} \\ &= -\frac{2}{5}. \end{aligned}$$

□

Solution to Question 13: First, note that the y-intercept of the line is $b = 3$. Secondly, use the graph to determine the slope of the line.



Thus, if m represents the slope of the line, then

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{-3}{2} \\ &= -\frac{3}{2}. \end{aligned}$$

Hence, the equation of the line is

$$\begin{aligned} f(x) &= mx + b \\ f(x) &= -\frac{3}{2}x + 3. \end{aligned}$$

□

Solution to Question 14: Solve the equation for y.

$$\begin{aligned} 3x + 4y &= 12 \\ 4y &= -3x + 12 \\ y &= \frac{-3x + 12}{4} \\ y &= \frac{-3x}{4} + \frac{12}{4} \\ y &= -\frac{3}{4}x + 3 \end{aligned}$$

Comparing this with $y = mx + b$, we see that the slope is $m = -3/4$.

□

Solution to Question 15: First, find the slope of the line passing through $(2, 3)$ and $(5, 1)$.

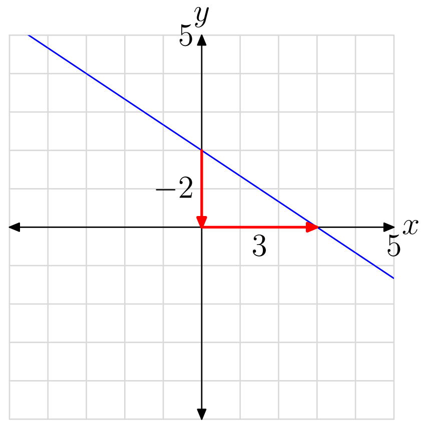
$$m = \frac{1 - 3}{5 - 2} = \frac{-2}{3}.$$

Now, sub $m = -2/3$ and $(x_0, y_0) = (2, 3)$ into the point-slope form of the line.

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 3 &= -\frac{2}{3}(x - 2) \end{aligned}$$

□

Solution to Question 16: Measure rise and run.



Hence, the slope of the given line is

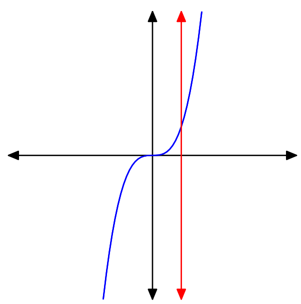
$$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{3}.$$

Perpendicular lines have slopes that are negative reciprocals of one another. Hence, the slope of *any* line perpendicular to the given line is

$$m = \frac{-1}{-\frac{2}{3}} = \frac{3}{2}.$$

□

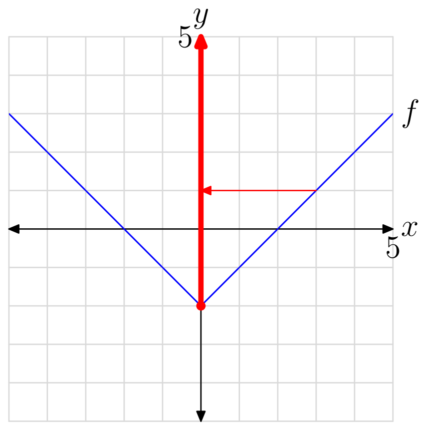
Solution to Question 17: Only one of the graphs passes the *Vertical Line Test*.



No vertical line will cut this graph more than once. Thus, this graph represents a function.

□

Solution to Question 18: To find the range of the given function, project all points on the graph onto the vertical axis to determine the y-value of each point (x, y) on the graph of f.



The shadow on the vertical axis is the set containing the y-values of each ordered pair (x, y) on the graph of f. This set is the range and is best described with

$$[-2, +\infty) = \{y : \ y \geq -2\}.$$

□

Solution to Question 19: You cannot take the square root of a negative number. Hence, the expression under the radicand of

$$f(x) = \sqrt{x - 2}$$

must be nonnegative. That is,

$$\begin{aligned} x - 2 &\geq 0 \\ x &\geq 2. \end{aligned}$$

Hence, the domain of f is

$$[2, +\infty) = \{x : \ x \geq 2\}.$$

□

Solution to Question 20: If $f(x) = x^2 + 2x - 3$, then

$$\begin{aligned} f(-3) &= (-3)^2 + 2(-3) - 3 \\ &= 9 - 6 - 3 \\ &= 0. \end{aligned}$$

□

Solution to Question 21: If $f(x) = x^2 + x$, then

$$\begin{aligned} f(a + 3) &= (a + 3)^2 + (a + 3) \\ &= a^2 + 6a + 9 + a + 3 \\ &= a^2 + 7a + 12. \end{aligned}$$

□

Solution to Question 22: Because $f(x + h) = (x + h)^2$, we write

$$\frac{f(x + h) - f(x)}{h} = \frac{(x + h)^2 - x^2}{h}$$

Now, $(x+h)^2 = x^2 + 2xh + h^2$, so

$$= \frac{x^2 + 2xh + h^2 - x^2}{h}$$
$$= \frac{2xh + h^2}{h}.$$

Factor and cancel.

$$= \frac{h(2x+h)}{h}$$
$$= 2x + h$$

Provided, of course, that $h \neq 0$.

□

Solution to Question 23: If $f(x) = x^2 - 3x$, note that $f(5) = (5)^2 - 3(5) = 10$, so

$$\frac{f(x) - f(5)}{x - 5} = \frac{x^2 - 3x - 10}{x - 5}.$$

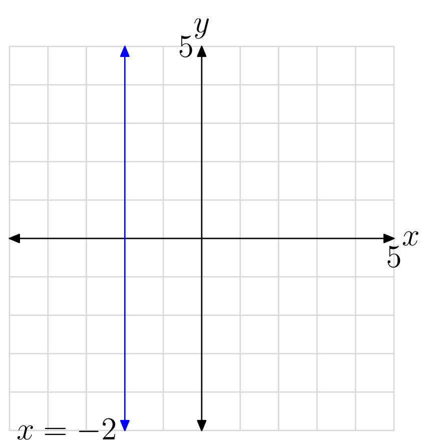
Factor and cancel.

$$= \frac{(x-5)(x+2)}{x-5}$$
$$= x + 2$$

Provided, of course, that $x \neq 5$

□

Solution to Question 24: Every point on the line pictured



has an x-value equal to -2. Hence, the equation of the line is $x = -2$.

□

Solution to Question 25: Push the STAT button on your calculator and select 1:Edit. Load the x-data in column L_1 and the y-data in column L_2. Push the STAT button again, then right cursor to the CALC menu and select 4:LinReg(ax+b). Press Enter and your calculator responds with:

LinReg

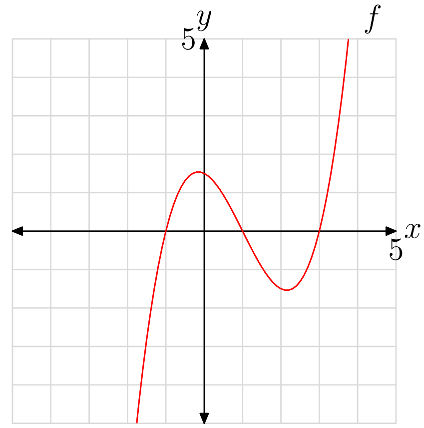
$$y = ax + b$$
$$a = -1.6$$
$$b = 1$$

Hence, the equation of the line of best fit is

$$y = -1.6x + 1.$$

$\square$

Solution to Question 26: Recall that x is a zero of a polynomial p if and only if $p(x) = 0$. The zeros are found by noting where the graph of p crosses the x-axis.



The graph of p crosses the x-axis at -1, 1, and 3. These are the zeros of the polynomial p.

$\square$

Solution to Question 27: If -3 is a zero of p, then

$$p(x) = (x + 3)(\text{something}).$$

Note that in searching for x so that $p(x) = 0$,

$$0 = (x + 3)(\text{something}),$$

which forces

$$\begin{aligned} x + 3 &= 0 \\ x &= -3, \end{aligned}$$

making -3 a zero of the polynomial.

$\square$

Solution to Question 28: Factor $p(x)$ by grouping.

$$\begin{aligned} p(x) &= x^2 + ax + bx + ab \\ p(x) &= x(x + a) = b(x + a) \\ p(x) &= (x + b)(x + a) \end{aligned}$$

Now, to find x such that $p(x) = 0$,

$$0 = (x + b)(x + a).$$

So, either

$$x + b = 0 \qquad \text{or} \qquad x + a = 0.$$

Thus, p has zeros $x = -b$ or $x = -a$.

$\square$

Solution to Question 29: We begin by negating the numerator and the fraction bar.

$$\frac{3t - t^2}{t^2 - 9} = -\frac{t^2 - 3t}{t^2 - 9}$$

This is two negations, so this provides an equivalent rational expression. Now, factor and cancel.

$$= -\frac{t(t-3)}{(t+3)(t-3)}$$
$$= -\frac{t}{t+3}$$

Provided, of course, that $t \neq 3$ and $t \neq -3$.

□

Solution to Question 30: Whatever value of x that makes the denominator zero will place a vertical asymptote in the graph of the function. Note that the denominator of

$$f(x) = \frac{x+3}{x-5}$$

is zero when

$$x - 5 = 0$$
$$x = 5.$$

Hence, the graph of f has a vertical asymptote at $x = 5$. The equation of the vertical asymptote is $x = 5$.

□

Solution to Question 31: The graph of f will cross the x-axis ($f(x) = 0$) when the numerator of

$$f(x) = \frac{x^2 + 6x - 16}{x+4}$$

equals zero.

$$0 = x^2 + 6x - 16$$

Factor.

$$0 = (x+8)(x-2)$$

Hence,

$$x + 8 = 0 \qquad \text{and} \qquad x - 2 = 0,$$

revealing two zeros (x-intercepts), one at $x = -8$, the other at $x = 2$.

□

Solution to Question 32: To add fractions, find equivalent fractions with a common denominator. Thus,

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd}$$

Now, add numerators, keeping the common denominator.

$$= \frac{ad + bc}{bd}$$

□

Solution to Question 33: Factor numerators and denominators.

$$\frac{y^2 - 2y - 15}{2y^2 + 6y} \cdot \frac{y^2 - 5y}{y^2 - 10y + 25} = \frac{(y-5)(y+3)}{2y(y+3)} \cdot \frac{y(y-5)}{(y-5)^2} \tag{6}$$

After cancelling,

$$= \frac{1}{2}. \tag{7}$$

□

Solution to Question 34: First,

$$\frac{f(x)-f(5)}{x-5} = \frac{\dfrac{1}{x}-\dfrac{1}{5}}{x-5}$$

Multiply numerator and denominator by $5x$ to clear fractions.

$$\begin{aligned}
&= \frac{\left(\dfrac{1}{x}-\dfrac{1}{5}\right)5x}{(x-5)5x} \\
&= \frac{\left(\dfrac{1}{x}\right)5x-\left(\dfrac{1}{5}\right)5x}{5x(x-5)} \\
&= \frac{5-x}{5x(x-5)}
\end{aligned}$$

Cancel.

$$= -\frac{1}{5x}$$

Provided, of course, that $x \neq 0, 5$.

□

Solution to Question 35: Again, addition requires equivalent fractions with a common denominator.

$$\begin{aligned}
f(x)+g(x) &= \frac{x+1}{x}+\frac{x}{x+1} \\
&= \frac{(x+1)^2}{x(x+1)}+\frac{x^2}{x(x+1)}
\end{aligned}$$

Expand numerators, then add numerators, keeping a common denominator.

$$\begin{aligned}
&= \frac{x^2+2x+1}{x(x+1)}+\frac{x^2}{x(x+1)} \\
&= \frac{2x^2+2x+1}{x(x+1)}
\end{aligned}$$

Provided, of course, that $x \neq 0, -1$.

□

Solution to Question 36: If

$$f(x) = x - \frac{3}{x},$$

then $f(x) = 2$ becomes

$$x - \frac{3}{x} = 2.$$

Multiply both sides by x.

$$\begin{aligned}
x\left(x-\frac{3}{x}\right) &= x(2) \\
x^2-3 &= 2x
\end{aligned}$$

The equation is nonlinear. Make one side zero and factor.

$$\begin{aligned}
x^2-2x-3 &= 0 \\
(x-3)(x+1) &= 0
\end{aligned}$$

Hence, the solutions are $x = 3$ or $x = -1$.

□

Solution to Question 37: Cross multiply.

$$\frac{x}{x+a} = \frac{x+b}{x}$$
$$x^2 = (x+a)(x+b)$$

Expand.

$$x^2 = x^2 + ax + bx + ab$$

Subtract x^2 from both sides.

$$0 = ax + bx + ab$$

The equation is linear, so isolate terms with x on one side of the equation.

$$ax + bx = -ab$$

Factor.

$$(a+b)x = -ab$$

Divide by $a + b$.

$$x = \frac{-ab}{a+b}$$

□

Solution to Question 38: A table summarizes given data.

	Time to paint room	Part in 1 hour	Part in 2 hours
Liz	L hr	$1/L$ hr	$2/L$
Jane	4 hr	1/4 hr	2/4
Together	-	-	1

Thus, in 2 hours, Liz finishes $2/L$ parts of the room while Jane finishes 2/4 of the room. Together, they finish the whole room, so

$$\frac{2}{L} + \frac{2}{4} = 1$$
$$\frac{2}{L} + \frac{1}{2} = 1.$$

Multiply both sides by $2L$.

$$2L\left(\frac{2}{L} + \frac{1}{2}\right) = 2L(1)$$
$$4 + L = 2L$$

Isolate L.

$$4 = 2L - L$$
$$L = 4$$

Thus, it takes Liz 4 hours, working alone.

□

Solution to Question 39: A table summarizes given data.

	Distance	Speed	Times
Upstream	24 mi	$5+c$ mi/hr	$\dfrac{24}{5+c}$ hr
Downstream	24 mi	$5-c$ mi/hr	$\dfrac{24}{5-c}$ hr
Totals	-	-	10 hr

Note how we used the fact that distance equals spped times time.

$$d = st$$
$$t = \frac{d}{s}$$

to find the time to go upstream, divide the distance by the net speed, $24.(5+c)$. Similarly, the time to go downstream is $24/(5-c)$. The toatl time of the trip is 10 hours, so

$$\frac{24}{5+c} + \frac{24}{5-c} = 10$$

Multiply both sides by $(5+c)(5-c)$.

$$\begin{aligned}(5+c)(5-c)\left[\frac{24}{5+c} + \frac{24}{5-c}\right] &= 10(5+c)(5-c)\\ 24(5-c) + 24(5+c) &= 10(5+c)(5-c)\end{aligned}$$

Expand and simplify.

$$\begin{aligned}120 - 24c + 120 + 24c &= 10(25-c^2)\\ 240 &= 250 - 10c^2\end{aligned}$$

Thus,

$$\begin{aligned}10c^2 &= 10\\ c^2 &= 1\\ c &= 1\end{aligned}$$

Hence, the speed of the current is 1 mile per hour.

□

Solution to Question 40: Start with

$$\begin{aligned}x + 2y &= a \quad (1)\\ 2x - y &= b. \quad (2)\end{aligned}$$

Multiply equation (2) by 2.

$$\begin{aligned}x + 2y &= a \quad (1)\\ 4x - 2y &= 2b \quad (3)\end{aligned}$$

Add equation (1) and (3).

$$5x = a + 2b$$

Divide by 5.

$$x = \frac{a+2b}{5}$$

□

Solution to Question 41: Because $\sqrt{x^2}$ calls for the nonnegative square root of x^2, we must wrap the answer in absolute value bars to insure a nonnegative answer.

$$\sqrt{x^2} = |x|$$

□

Solution to Question 42: You cannot take the square root of a negative number. Hence, the radicand of

$$f(x) = \sqrt{2x+3}$$

must be nonnegative.

$$2x + 3 \geq 0$$

Solve for x.

$$\begin{aligned} 2x &\geq -3 \\ x &\geq -\frac{3}{2} \end{aligned}$$

Thus, the domain of f is

$$[-3/2, +\infty) = \{x : x \geq -3/2\}.$$

□

Solution to Question 43: Factor out a perfect square.

$$\begin{aligned} \sqrt{25x^3} &= \sqrt{25x^2}\sqrt{x} \\ &= |5x|\sqrt{x} \end{aligned}$$

However, $x \geq 0$, so $|5x| = 5x$. Thus,

$$\sqrt{25x^3} = 5x\sqrt{x}.$$

□

Solution to Question 44: Multiply numerator and denominator by $\sqrt[3]{9a}$.

$$\begin{aligned} \frac{1}{\sqrt[3]{3a^2}} &= \frac{1}{\sqrt[3]{3a^2}} \cdot \frac{\sqrt[3]{9a}}{\sqrt[3]{9a}} \\ &= \frac{\sqrt[3]{9a}}{\sqrt[3]{27a^3}} \end{aligned}$$

However, $\sqrt[3]{27a^3} = 3a$, so

$$= \frac{\sqrt[3]{9a}}{3a}.$$

□

Solution to Question 45: Multiply numerator and denominator by $\sqrt{x} + \sqrt{a}$.

$$\begin{aligned} \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}-\sqrt{a}} &= \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}-\sqrt{a}} \cdot \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}} \\ &= \frac{(\sqrt{x}+\sqrt{a})^2}{(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})} \end{aligned}$$

Expand both numerator and denominator.

$$= \frac{x + 2\sqrt{xa} + a}{x - a}$$

□

Solution to Question 46: The exponent is $-2/3$. The 2 means "square," the 3 is "cube root," and the minus sign requires that we invert. So,

$$(a+1)^{-2/3} = \frac{1}{\sqrt[3]{(a+1)^2}}.$$

□

Solution to Question 47: If

$$f(x) = a - \sqrt{x^2+a^2},$$

then $f(x) = -1$ becomes

$$a - \sqrt{x^2+a^2} = -1.$$

Isolate the radical.

$$a+1 = \sqrt{x^2+a^2}$$

Square both sides.

$$\begin{aligned}(a+1)^2 &= \left(\sqrt{x^2+a^2}\right)^2\\ a^2+2a+1 &= x^2+a^2\end{aligned}$$

Subtract a^2 from both sides.

$$2a+1 = x^2$$

Take the square roots.

$$x = \pm\sqrt{2a+1}$$

□

Solution to Question 48: Set $y = 0$ to find the x-intercept.

$$\begin{aligned}y &= x^2-2x-4\\ 0 &= x^2-2x-4\end{aligned}$$

Compare with

$$0 = ax^2+bx+c$$

and note that

$$\begin{aligned}a &= 1\\ b &= -2\\ c &= -4.\end{aligned}$$

Thus, the x-intercepts are provided by the quadratic formula.

$$\begin{aligned}x &= \frac{-b\pm\sqrt{b^2-4ac}}{2a}\\ x &= \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-4)}}{2(1)}\\ x &= \frac{2+\sqrt{4+16}}{2}\\ x &= \frac{2\pm\sqrt{20}}{2}\end{aligned}$$

But, $\sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$, so

$$x = \frac{2 \pm 2\sqrt{5}}{2}$$

Dividing both terms in the numerator by 2,

$$x = \frac{2}{2} \pm \frac{2\sqrt{5}}{5}$$
$$x = 1 \pm \sqrt{5}.$$

These are the x-intercepts.

□

Solution to Question 49: The parabola

$$y = -3x^2 + 9x + 12$$

has a vertical axis of symmetry through its vertex. The x-value of the vertex is

$$x = \frac{-b}{2a} = \frac{-9}{2(-3)} = \frac{3}{2}.$$

Thus, the equation of the axis of symmetry is

$$x = \frac{3}{2}.$$

□

Solution to Question 50: The parabola

$$y = x^2 - 4x - 6$$

opens upward and has a minimum y-value at its vertex. The x-coordinate of the vertex is

$$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2.$$

Thus, the minimum y-value is

$$\begin{aligned} y_{\text{min}} &= (2)^2 - 4(2) - 6 \\ &= 4 - 8 - 6 \\ &= -10. \end{aligned}$$

□

Solution to Question 51: The equation

$$S = 2\pi r^2 + 2\pi rh$$

is quadratic (2^{nd} degree) in r. Make one side zero.

$$0 = 2\pi r^2 + 2\pi rh - S$$

Compare this with

$$0 = ar^2 + br + c$$

and note that

$$\begin{aligned} a &= 2\pi \\ b &= 2\pi h \\ c &= -S. \end{aligned}$$

Write the quadratic formula. We're solving for r.

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the given values of a, b, and c.

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-S)}}{2(2\pi)}$$

$$r = \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8\pi S}}{4\pi}$$

We can factor a 4 out of the radical.

$$r = \frac{-2\pi h \pm \sqrt{4}\sqrt{\pi^2 h^2 + 2\pi S}}{4\pi}$$

$$r = \frac{-2\pi h \pm 2\sqrt{\pi^2 h^2 + 2\pi S}}{4\pi}$$

We can reduce. Divide numerator and denominator by 2.

$$r = \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2\pi S}}{2\pi}$$

We want a positive radius, so we select

$$r = \frac{-\pi h + \sqrt{\pi^2 h^2 + 2\pi S}}{2\pi}.$$

□

Solution to Question 52: Let x and y represent two numbers whose sum is 10.

$$x + y = 10 \tag{1}$$

The sum of the squares is

$$S = x^2 + y^2 \tag{2}$$

Solve equation (1) for y.

$$y = 10 - x \tag{3}$$

Substitute equation (3) in (2).

$$S = x^2 + (10 - x)^2$$

Expand and simplify.

$$S = x^2 + 100 - 20x + x^2$$

$$S = 2x^2 - 20x + 100$$

This is a parabola that opens up. S is a minimu at

$$x = \frac{-b}{2a} = \frac{-(-20)}{2(2)} = 5.$$

Thus, one number is $x = 5$, the other is found by subbing $x = 5$ in (3).

$$y = 10 - 5$$

$$y = 5$$

□

Solution to Question 53: Write the population as an exponential function of t, the number of years measured from the present.

$$P(t) = 12,000(1 + 0.042)^t$$

At $t = 5$ years, the population will be

$$P(5) = 12,000(1.042)^5,$$

or, approximately,

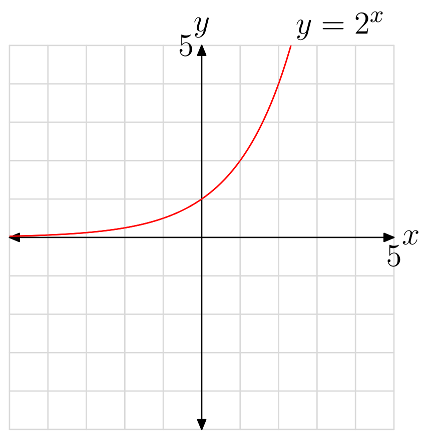
$$P(5) \approx 14,740.$$

□

Solution to Question 54: The function

$$y = 2^x$$

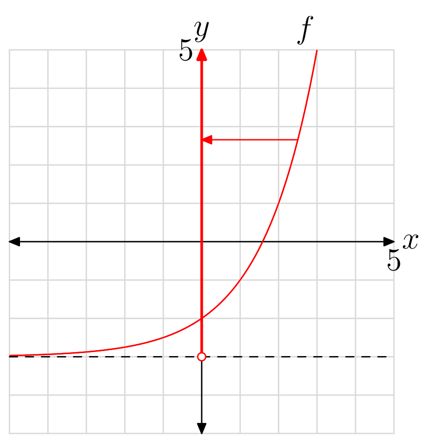
grows exponentially as you move from left to right. As you move from right to left, it decays asymptotically to zero.



If we subtract 3 from all of the y-values, as in

$$y = 2^x - 3,$$

the graph shifts downward 3 units. More importantly, the horizontal asymptote shifts to the line $y = -3$. Projecting onto the y-axis reveals the range.



Thus, the range is

$$(-3, +\infty) = \{x : x > -3\}.$$

□

Solution to Question 55: We use the formula for interest compounded a discrete number of times each year.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Fill in the given values.

$$\begin{aligned} P &= 10,000 \\ r &= 0.06 \\ n &= 4 \text{ quarterly} \\ t &= 10 \text{ years} \end{aligned}$$

Thus,

$$\begin{aligned} A &= 10,000\left(1 + \frac{0.06}{4}\right)^{4(10)} \\ A &\approx \$18,140.18 \end{aligned}$$

□

Solution to Question 56: Use the formula for interest compounded continuously,

$$A = Pe^{rt},$$

and substitute the given

$$\begin{aligned} A &= 10,000 \\ r &= 0.06 \\ t &= 5. \end{aligned}$$

That is,

$$10,000 = Pe^{0.06(5)}$$

Divide both sides by $e^{0.06(5)}$.

$$\begin{aligned} P &= \frac{10,000}{e^{0.06(5)}} \\ P &\approx \$7,408.18 \end{aligned}$$

□

Solution to Question 57: Evaluate inner parentheses first.

$$g(f(x)) = g(2x + 3)$$

Substitute $2x + 3$ for x in the "squaring function" g.

$$\begin{aligned} &= (2x + 3)^2 \\ &= 4x^2 + 12x + 9 \end{aligned}$$

□

Solution to Question 58: Consider an object dropped into the function machine defined by

$$f(x) = 3x + 4.$$

First, the object gets multiplied by 3, then 4 is added to the result. The inverse function must "undo" these operations in inverse order. That is, the inverse must

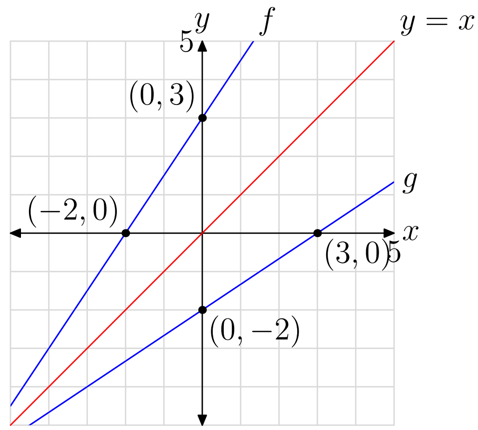
- subtract r, then
- divide by 3.

Starting with x, this yields an inverse function

$$g(x) = \frac{x-4}{3}.$$

□

Solution to Question 59: The graph of the inverse must be a reflection of the graph of f across the line $y = x$.



It is also helpful to note that the inverse simply reverses the order of ordered pairs. Hence,

$$(0,3) \to (3,0)$$
$$(-2,0) \to (0,-2).$$

□

Solution to Question 60: The domain of the natural logarithm is

$$(0, +\infty) = \{x : x > 0\}.$$

That is, you can take logarithms of positive numbers only. Thus, in order that

$$f(x) = \ln(x+4)$$

be defined, we must have

$$x + 4 > 0.$$

Solving for x, the domain of $f(x) = \ln(x+4)$ is

$$(-4, +\infty) = \{x : x > -4\}.$$

□

Solution to Question 61: Recall that $y = \log_a x$ if and only if $a^y = x$. Change

$$\log_x 9 = -2$$

into exponential form.

$$x^{-2} = 9$$

Now, $x^{-2} = 1/x^2$, so

$$\frac{1}{x^2} = 9.$$

Inverting,

$$x^2 = \frac{1}{9}$$
$$x = \frac{1}{3}.$$

□

Solution to Question 62: Recall that $y = \log_a x$ if and only if $a^y = x$. Change

$$\log_9 27 = x$$

into exponential form.

$$9^x = 27$$

Now, use a common base. That is, note that $3^2 = 9$ and $3^3 = 27$.

$$(3^2)^x = 3^3$$

Multiply exponents ($(a^m)^n = a^{mn}$).

$$3^{2x} = 3^3$$

Equate exponents.

$$2x = 3$$

Solve for x.

$$x = \frac{3}{2}$$

□