Humboldt State University
Mathematics Department
Math 115—Precalculus

Exam #3—Triangle Trigonometry

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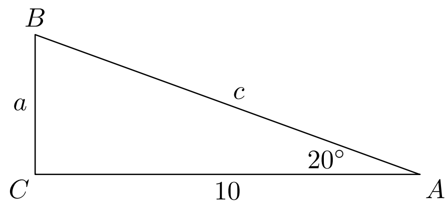
Multiple Choice Questions

Directions: *In each of the following exercises, select the "best" answer and darken the corresponding oval on your scantron sheet.*

1. Given right triangle $\triangle ABC$, estimate the length of side a.

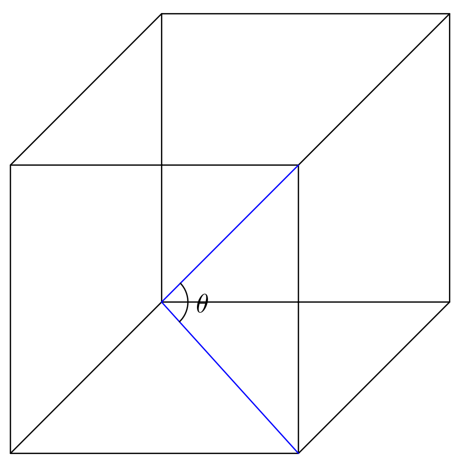


(a) 3.53 (b) 3.59 (c) 3.64
(d) 3.72 (e) 3.95

2. A ladder 16 feet long leans against the side of a house. Find the height h from the top of the ladder to the ground if the angle of elevation of the ladder is 74°.

(a) 13.2 (b) 13.8 (c) 14.2
(d) 14.7 (e) 15.4

3. Find the tangent of angle θ, the angle between the diagonal of a cube and the diagonal of the base of the cube.



(a) $\frac{\sqrt{3}}{3}$ (b) $\frac{\sqrt{2}}{2}$ (c) $\frac{\sqrt{3}}{2}$
(d) $\frac{\sqrt{2}}{3}$ (e) $\frac{1}{2}$

4. Given triangle $\triangle ABC$, with $A = 150°$, $C = 20°$, and $a = 200$, find the length of side b.

(a) 66.8 (b) 67.5 (c) 68.3
(d) 69.5 (e) 71.2

5. How many triangles $\triangle ABC$ can be constructed, given that $A = 58°$, $a = 4.5$, and $b = 12.8$?

(a) one (b) two (c) none

6. Given $a = 75.4$, $b = 52$, and $c = 52$, find angle A of triangle $\triangle ABC$.

(a) 89.0° (b) 92.9° (c) 94.2°
(d) 96.1° (e) 99.0°

7. If triangle $\triangle ABC$ has sides measuring $a = 5$, $b = 7$, and $c = 10$, use Heron's formula to compute the area.

(a) $6\sqrt{11}$ (b) $11\sqrt{6}$ (c) $2\sqrt{66}$

(d) $12\sqrt{6}$ (e) none of these

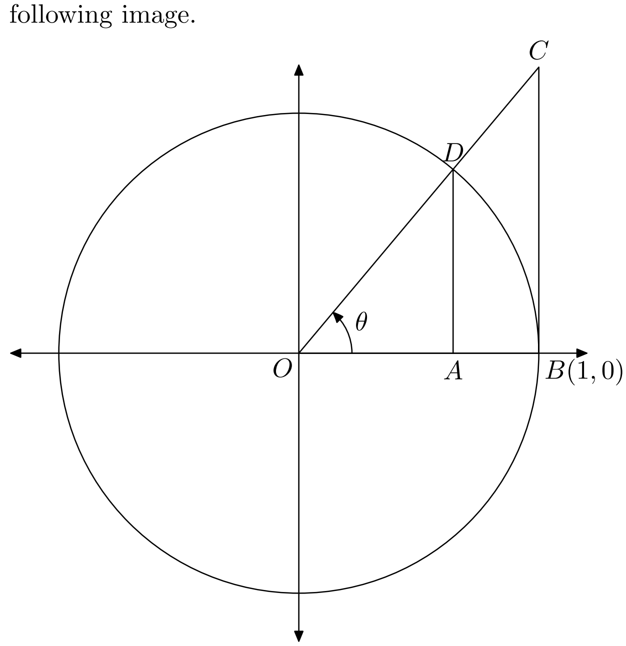
Essay Questions

Directions: *Place the solution to each of the following exercises on your own paper. You must follow directions explicitly and show all work to receive full credit. You are encouraged to draw pictures to scale, but you will not receive full credit for measured solutions. You must show the analysis, either using right triangle trigonometry, the law of sines, or the law of cosines to receive full credit.*

Exercise 1. A plane flies 810 miles from A to B with a bearing $N\,75^\circ\,E$. Then it flies 648 miles from B to C with a bearing $N\,32^\circ\,E$. Draw a figure that visually represents the problem, then find the straight line distance from C to A.

Exercise 2. The angles of elevation to an airplane from two points A and B on level ground are 51° and 68°, respectively. The points A and B are 2.5 miles apart, and the plane is east of both points in the same vertical plane. Find the altitude of the plane.

Exercise 3. Consider the following image.



If the circle pictured above has radius 1, find formulas for each of the following lengths in terms of θ.

- OA
- AD
- BC

Solutions to Quizzes

Solution to Question 1: Use the tangent.

$$\tan A = \frac{\text{opp}}{\text{hyp}}$$
$$\tan 20^\circ = \frac{a}{10}$$

Cross multiply.

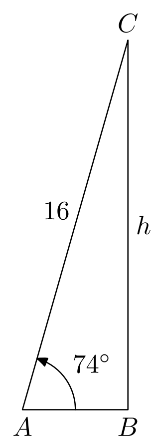
$$a = 10 \tan 20^\circ$$

Use a calculator to estimate.

$$a \approx 3.64$$

□

Solution to Question 2: Draw a figure representing a 16 foot ladder leaning against the house with angle of elevation 74°.



Use the sine.

$$\sin A = \frac{\text{opp}}{\text{hyp}}$$
$$\sin 74^\circ = \frac{h}{16}$$

Cross multiply.

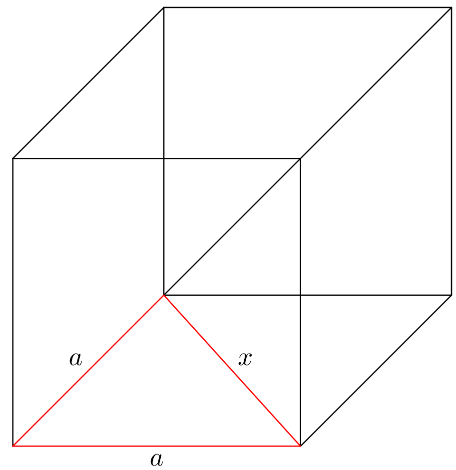
$$h = 16 \sin 74^\circ$$

Use a calculator to estimate h.

$$h \approx 15.4 \text{ ft}$$

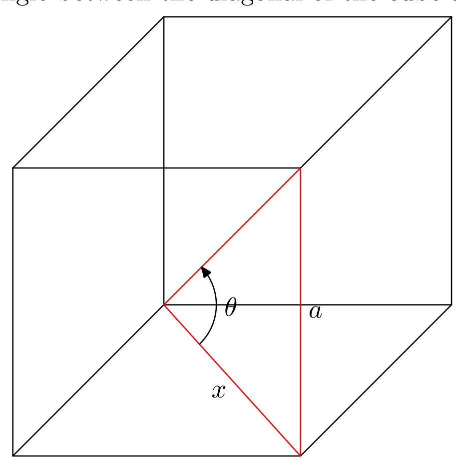
□

Solution to Question 3: First, use the Pythagorean Theorem to determine the length of the base diagonal.



$$x^2 = a^2 + a^2$$
$$x^2 = 2a^2$$
$$x = \sqrt{2a^2}$$
$$x = \sqrt{2}a$$

Find the tangent of angle θ, the angle between the diagonal of the cube and the diagonal of its base.



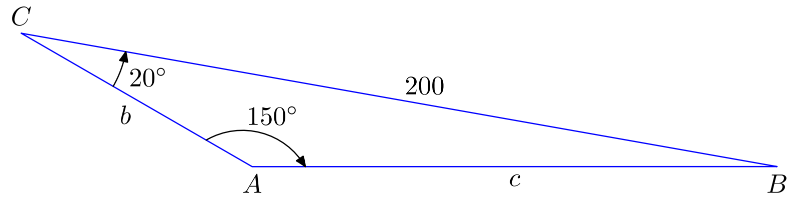
$$\tan\theta = \frac{\text{opp}}{\text{hyp}}$$
$$\tan\theta = \frac{a}{x}$$

But $x = \sqrt{2}a$, so

$$\tan\theta = \frac{a}{\sqrt{2}a},$$
$$\tan\theta = \frac{1}{\sqrt{2}},$$
$$\tan\theta = \frac{\sqrt{2}}{2}.$$

□

Solution to Question 4: Draw the figure.



The sum of the angles of a triangle equals 180°.

$$A + B + C = 180^\circ$$
$$150^\circ + B + 20^\circ = 180^\circ$$
$$B = 10^\circ$$

Now that we know that $B = 10^\circ$, we can use the Law of Sines to find side b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 10^\circ} = \frac{200}{\sin 150^\circ}$$

Cross multiply.

$$b \sin 150^\circ = 200 \sin 10^\circ$$

Divide both sides by 150°.

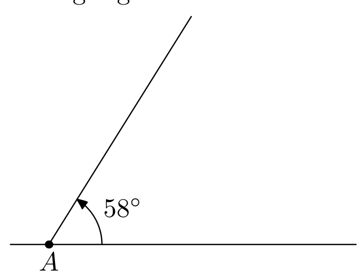
$$b = \frac{200 \sin 10^\circ}{\sin 150^\circ}$$

Use a calculator to approximate b.

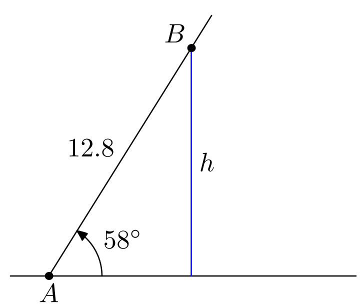
$$b \approx 69.5$$

□

Solution to Question 5: Begin by drawing a guideline and marking point A.



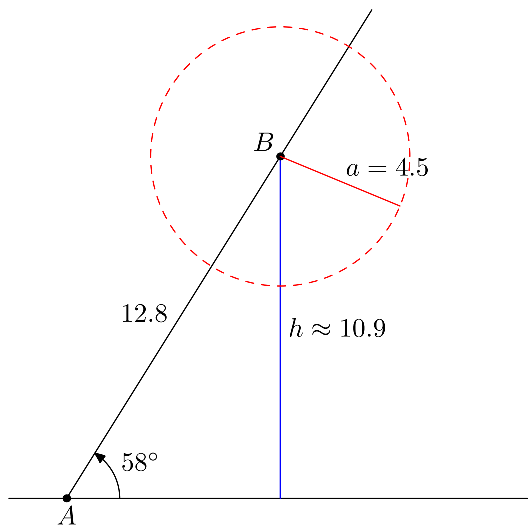
At point A, use a protractor to mark a 58° angle and draw another guideline. Along this new guideline, mark a length 12.8 and mark point B.



Compute h, the shortest distance from point B to the original guideline containing point A.

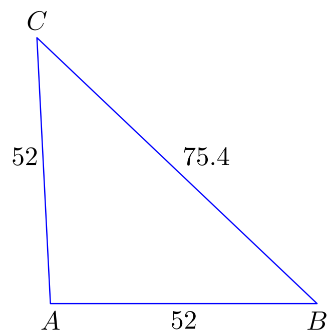
$$\begin{aligned}
\sin A &= \frac{\text{opp}}{\text{hyp}} \\
\sin 58^\circ &= \frac{h}{12.8} \\
h &= 12.8 \sin 58^\circ \\
h &\approx 10.9
\end{aligned}$$

Because side a is given as 4.5, and $h \approx 10.9$, side a is too short to complete $\triangle ABC$.



Therefore, no triangles can be constructed with the given information. □

Solution to Question 6: Sketch the figure.



Use the Law of Cosines.

$$\begin{aligned}
a^2 &= b^2 + c^2 - 2bc \cos A \\
75.4^2 &= 52^2 + 52^2 - 2(52)(52) \cos A
\end{aligned}$$

Solve for $\cos A$.

$$\begin{aligned}
2(52)(52) \cos A &= 52^2 + 52^2 - 75.4^2 \\
\cos A &= \frac{52^2 + 52^2 - 75.4^2}{2(52)(52)}
\end{aligned}$$

Use a calculator to approximate $\cos A$.

$$\cos A \approx -0.05125$$

Take the inverse cosine to find angle A.

$$A \approx \cos^{-1}(-0.05125)$$
$$A \approx 92.9^\circ$$

□

Solution to Question 7: The semiperimeter is

$$s = \frac{a+b+c}{2},$$
$$s = \frac{5+7+10}{2},$$
$$s = \frac{22}{2},$$
$$s = 11.$$

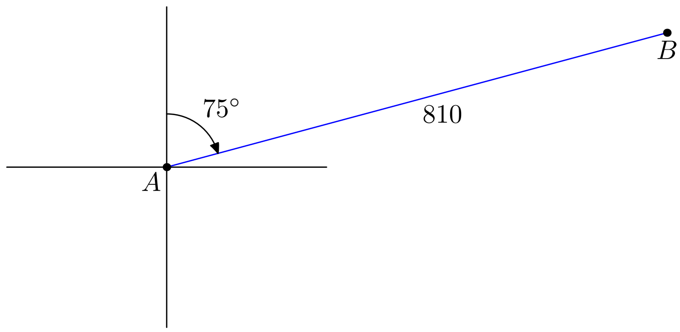
By Heron's formula, the area is

$$K = \sqrt{s(s-a)(s-b)(s-c)},$$
$$K = \sqrt{11(11-5)(11-7)(11-10)},$$
$$K = \sqrt{11(6)(4)(1)},$$
$$K = \sqrt{264},$$
$$K = \sqrt{4}\sqrt{66},$$
$$K = 2\sqrt{66}.$$

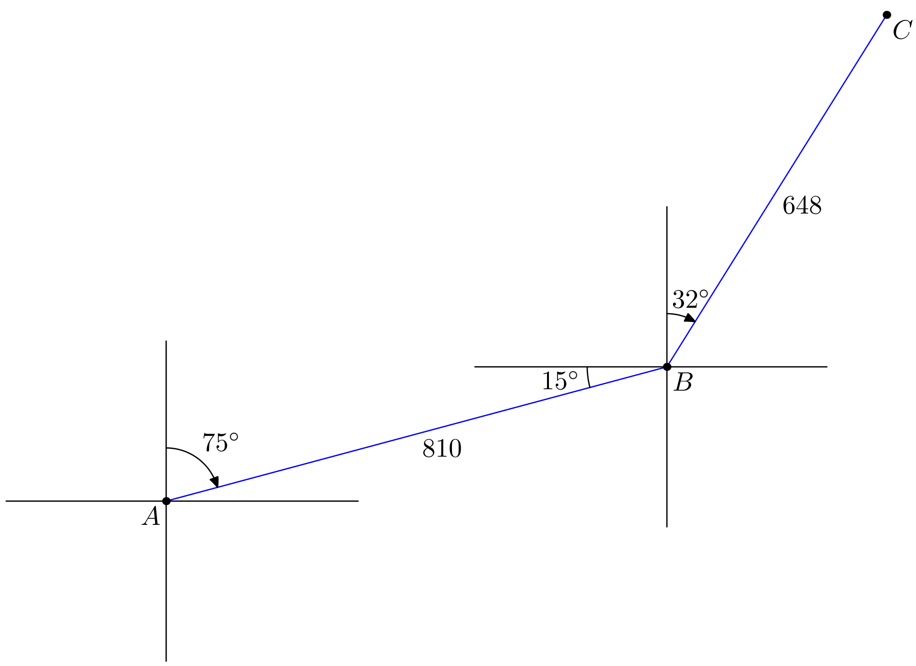
□

Solutions to Exercises

Exercise 1. Use a protractor to mark an angle, 75° east of north. Draw a guideline, measure 810 miles, then mark a point at B.



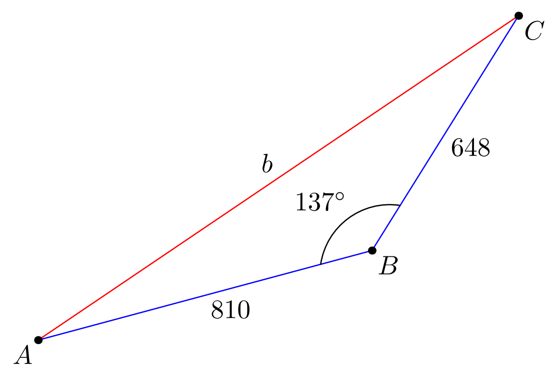
Use a protractor to mark an angle at B, 32° east of north. Draw a guideline, measure 648 miles, then mark a point at C.



The angle at B is

$$B = 15^\circ + 90^\circ + 32^\circ,$$
$$B = 137^\circ.$$

And we may draw the following $\triangle ABC$.

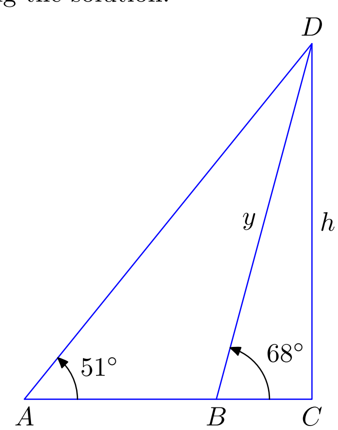


Using the Law of Cosines,

$$b^2 = 810^2 + 648^2 - 2(810)(648)\cos 137°,$$
$$b = \sqrt{810^2 + 648^2 - 2(810)(648)\cos 137°},$$
$$b \approx 1358 \text{ miles}.$$

Exercise 1

Exercise 2. Draw a figure representing the solution.



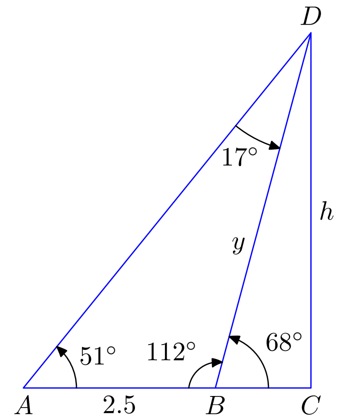
The angle $\angle ABC$ is supplementary to angle $\angle DBC$.

$$\angle ABD + \angle DBC = 180°$$
$$\angle ABD + 68° = 180°$$
$$\angle ABD = 112°$$

In $\triangle ABD$,

$$\angle A + \angle ABD + \angle BDA = 180°,$$
$$51° + 112° + \angle BDA = 180°,$$
$$\angle BDA = 17°.$$

Mark these angles in the figure.



Use the Law of Sines in $\triangle ABD$ to find y.

$$\begin{aligned}\frac{\sin 51^\circ}{y} &= \frac{\sin 17^\circ}{2.5}\\ y\sin 17^\circ &= 2.5\sin 51^\circ\\ y &= \frac{2.5\sin 51^\circ}{\sin 17^\circ}\\ y &\approx 6.6452\end{aligned}$$

Use triangle $\triangle BCD$ to compute the height h of the plane.

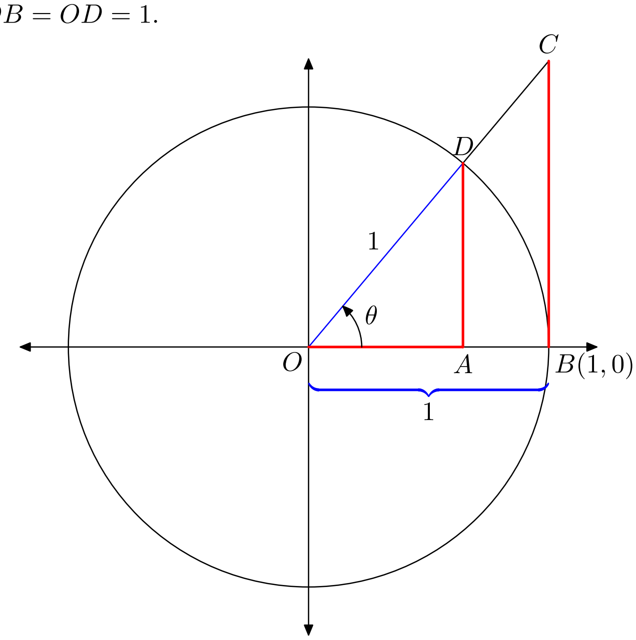
$$\begin{aligned}\sin 68^\circ &= \frac{h}{y}\\ h &= y\sin 68^\circ\end{aligned}$$

But, $y \approx 6.6452$, so

$$\begin{aligned}h &\approx 6.6452\sin 68^\circ,\\ h &\approx 6.16 \text{ miles.}\end{aligned}$$

Exercise 2

Exercise 3. Note that $OB = OD = 1$.



In right triangle $\triangle OAD$,

$$\cos\theta = \frac{\text{adj}}{\text{hyp}},$$
$$\cos\theta = \frac{OA}{1},$$
$$OA = \cos\theta.$$

In right triangle $\triangle OAD$,

$$\sin\theta = \frac{\text{opp}}{\text{hyp}},$$
$$\sin\theta = \frac{AD}{1},$$
$$AD = \sin\theta.$$

In right triangle $\triangle OBC$,

$$\tan\theta = \frac{\text{opp}}{\text{hyp}},$$
$$\tan\theta = \frac{BC}{1},$$
$$BC = \tan\theta.$$

Exercise 3