5pts

(1)

$$\begin{bmatrix} 1 & 0 & 2 & 4 & 1 \\ 0 & 1 & -2 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

/35

a) Particular Solution: $\vec{p} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 0 \end{bmatrix}$

b) $\mathcal{B}_{N(A)} = \left\{ \begin{bmatrix} -2 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

c) $\vec{x} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} -2 \\ 2 \\ 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -4 \\ -1 \\ 0 \\ 1 \end{bmatrix}$,

where α and β are any real numbers

5pts
②

$V = [\vec{v}_1, \vec{v}_2, \vec{v}_3]$

$$V = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 0 & 4 \\ 3 & 4 & 9 \end{bmatrix} \xrightarrow{rref} R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

There are no free variables. The vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are linearly independent.

5pts
③

$$A = \begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}$$

$$0 = \det(A - \lambda I)$$

$$0 = \det\left(\begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$0 = \det \begin{bmatrix} 5-\lambda & 3 \\ -6 & -4-\lambda \end{bmatrix}$$

$$0 = (5-\lambda)(-4-\lambda) + 18$$

$$0 = -20 - 5\lambda + 4\lambda + \lambda^2 + 18$$

$$0 = \lambda^2 - \lambda - 2$$

$$0 = (\lambda - 2)(\lambda + 1)$$

$$\lambda = 2, -1$$

10 pts

(4)

$$x' = x(4-2x) - xy$$
$$y' = y(4-2y) - xy$$

$0 = x(4-2x) - xy \qquad 0 = y(4-2y) - xy$

$0 = x[4-2x-y] \qquad 0 = y[4-2y-x]$

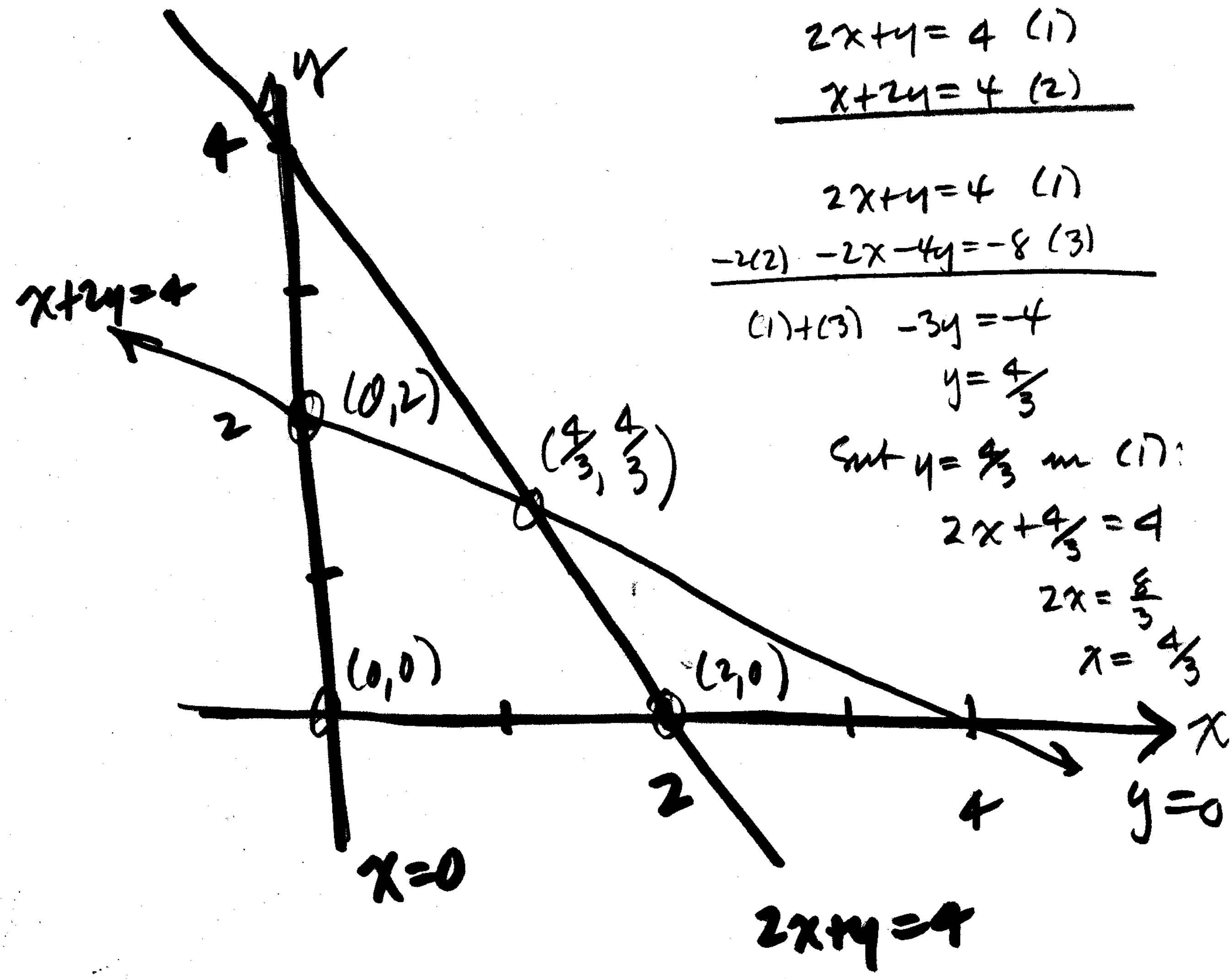
$x=0$ or $2x+y=4 \qquad y=0$ or $x+2y=4$

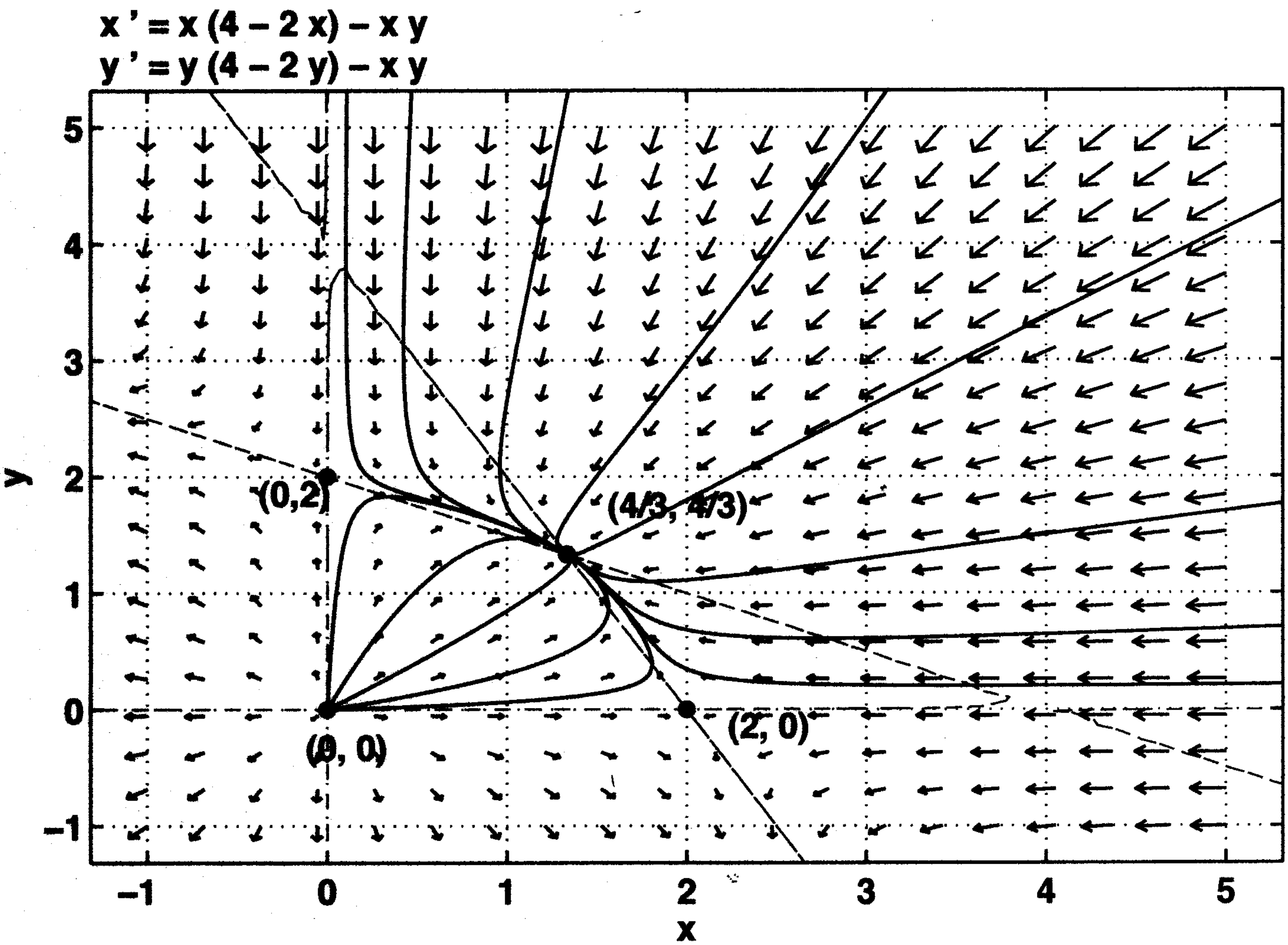
$2x+y=4 \quad (1)$
$x+2y=4 \quad (2)$

$2x+y=4 \quad (1)$
$-2(2) \quad -2x-4y=-8 \quad (3)$

$(1)+(3) \quad -3y=-4$
$y = \frac{4}{3}$

Sub $y=\frac{4}{3}$ in (1):
$2x+\frac{4}{3}=4$
$2x=\frac{8}{3}$
$x=\frac{4}{3}$





x ' = x (4 – 2 x) – x y
y ' = y (4 – 2 y) – x y
(0,2)
(4/3, 4/3)
(0, 0)
(2, 0)
x
y
–1
0
1
2
3
4
5

~~10 pts~~ 5

$$x' = \begin{bmatrix} 7 & 10 \\ -5 & -8 \end{bmatrix} x$$

a) $\vec{x}_1 = e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$\vec{x}_1' = -3e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 10 \\ -5 & -8 \end{bmatrix} \vec{x}_1 = \begin{bmatrix} 7 & 10 \\ -5 & -8 \end{bmatrix} e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = e^{-3t} \begin{bmatrix} -3 \\ 3 \end{bmatrix} = -3e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Hence, $\vec{x}_1$ is a solution

$\vec{x}_2 = e^{2t} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$$\vec{x}_2' = 2e^{2t} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 10 \\ -5 & -8 \end{bmatrix} \vec{x}_2 = \begin{bmatrix} 7 & 10 \\ -5 & -8 \end{bmatrix} e^{2t} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = e^{2t} \begin{bmatrix} 4 \\ -2 \end{bmatrix} = 2e^{2t} \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Hence, $\vec{x}_2$ is a solution.

b) $\vec{x}_1(0) = e^{-3(0)} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\vec{x}_2(0) = e^{2(0)} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\vec{x}_1(0)$ and $\vec{x}_2(0)$ are linearly independent. Hence, $\vec{x}_1(t)$ and $\vec{x}_2(t)$ are linearly independent for all t. Hence, the general solution is

$$\vec{x}(t) = c_1 \vec{x}_1(t) + c_2 \vec{x}_2(t)$$

$$\vec{x}(t) = c_1 e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

The initial condition is

$$\vec{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Hence,

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Hence, $c_1 = -3$ and $c_2 = 2$, and the soln of the IVP is

$$\vec{x}(t) = -3e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 2e^{2t} \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$