

A Damped Harmonic Oscillator

David Arnold

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Abstract

In this paper we present the equation of a forced, damped, harmonic oscillator. The equation is a second order, inhomogeneous differential equation. We show how to change the second order equation into a system of two first order differential equations, then use Matlab to find and plot a solution.

The Equation

The equation for the forced, damped, harmonic oscillator is

$$y'' + 0.2y' + 3y = 4 \cos 2t. \quad (1)$$

The first step is to solve equation (1) for the highest derivative.

$$y'' = -0.2y' - 3y + 4 \cos 2t. \quad (2)$$

Next, we introduce new variables for y and y' .

$$\begin{aligned} y_1 &= y \\ y_2 &= y' \end{aligned} \quad (3)$$

The next step is to differentiate each side of the equations in (3), then substitute the result from equation (2)

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= y'' = -0.2y_2 - 3y_1 + 4 \cos 2t \end{aligned} \quad (4)$$

The final step in the conversion process is to replace each occurrence of y with y_1 , and each occurrence of y' with y_2 .

$$\begin{aligned} y_1' &= y_2 \\ y_2' &= -0.2y_2 - 3y_1 + 4 \cos 2t \end{aligned} \quad (5)$$

We now address the initial conditions. The spring is displaced -2 meters from equilibrium, then released from rest. This means that $y(0) = -2$ and $y'(0) = 0$. We translate these to y_1 and y_2 as follows.

$$\begin{aligned} y_1(0) &= y(0) = -2 \\ y_2(0) &= y'(0) = 0 \end{aligned} \quad (6)$$

Hence, $y_1(0) = -2$ and $y_2(0) = 0$.

To solve this initial value system, the first step requires that we write a function ODE file in Matlab.

```
function yprime=f(t,y)
yprime=zeros(2,1);
yprime(1)=y(2);
yprime(2)=-0.2*y(2)-2*y(1)+4*cos(2*t);
```

Save this file as `f.m`. Next, create a script file to drive the solution. First, set the initial time and conditions.

```
tspan=[0,50];
init=[-2;0];
```

We can now call the solver.

```
[t,y]=ode45(@f,tspan,init);
```

Because the goal is to plot a solution to the original second order equation (1), and y_1 represents y , we need a plot of position y_1 versus time t .

```
plot(t,y(:,1))
```

Of course, we must annotate the plot.

```
xlabel('time (s)')
ylabel('position (m)')
title('A forced, damped, harmonic oscillator')
```

Finally, we size and save the plot.

```
set(gcf,'PaperPosition',[0,0,4,3])
print -depsc2 'oscillator.eps'
```

The solution to the forced, damped, harmonic oscillator (1) is shown in Figure 1. Note in Figure 1 that we have let enough time go by to show that the oscillator achieves a final steady state.

Appendix

We present the script file in its entirety for convenience. We saved this file as `quiz3.m`.

```
% close all open figure windows
close all

% initial parameters
tspan=[0,50];
init=[-2;0];
```

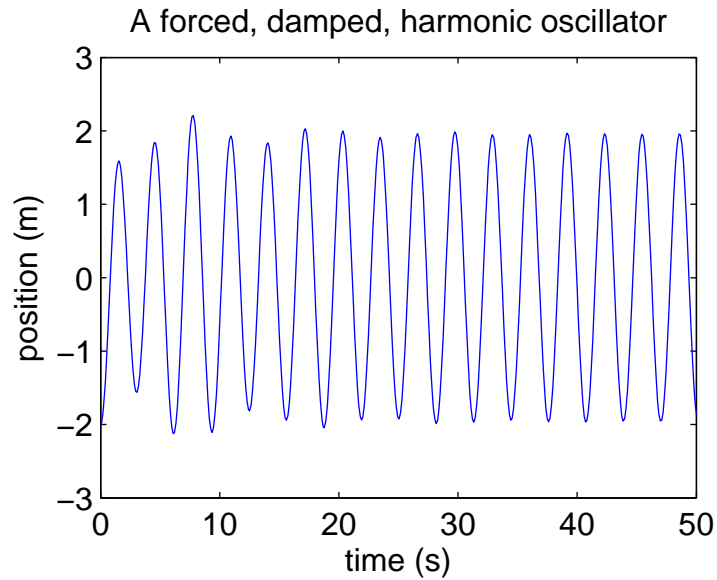


Figure 1: The position of the oscillator versus time.

```
% call the solver
[t,y]=ode45(@f,tspan,init);

% plot only the first coordinate, which is the solution to the original
% second order equation.
plot(t,y(:,1))

% annotate the plot
xlabel('time (s)')
ylabel('position (m)')
title('A forced, damped, harmonic oscillator')

% size and save the plot
set(gcf,'PaperPosition',[0,0,4,3])
print -depsc2 'oscillator.eps'
```

Here again is the function ODE file, saved as `f.m`.

```
function yprime=f(t,y)
yprime=zeros(2,1);
yprime(1)=y(2);
yprime(2)=-0.2*y(2)-2*y(1)+4*cos(2*t);
```

References

- [1] Polking and Arnold, *Using Matlab in Ordinary Differential Equations*