

College of the Redwoods
Mathematics Department

Math 55 — Differential Equations
Quiz #1

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Quiz Questions

Read Carefully! You have until Monday of next week (2/18/08) to complete the quiz. The quiz is due, in my hands, at the beginning of class.

This quiz is open notes, open book. This includes any supplementary texts or online documents. You must answer all of the exercises on your own. You are not allowed to work in groups or pairs on the quiz. You are not allowed to enlist the aid of a tutor or friend to help with the quiz. You are not allowed to read the exercises in the quiz, then seek help on similar questions. Once you open the quiz and read the questions, you may not seek any outside help of any kind.

I am not interested in reading pages and pages of calculations without accompanying narrative. It is essential that you include sound mathematical writing that both explains and justifies your solution or proof. Grammar and punctuation are important, as is the organization of your solution on the written page.

When working in the lab, please do not work at a terminal next to any other student who is also working on the quiz. For the sake of propriety, please separate yourselves when working on the quiz in the lab.

Place the solution to each exercise on a separate sheet of paper. On a good sheet of paper, write out (longhand) and sign the following honor pledge.

I promise that all work found herein is my own. I have received no help from tutors, colleagues, or other teachers. I have honored all of the quiz constraints listed in the directions.

Arrange your solutions in order, place these quiz page(s) on top of your solutions, then place the honor pledge on top of the quiz as a cover sheet. Staple. Good luck!

EXERCISE 1. Consider the initial value problem

$$y' = y(y - 5), \quad y(0) = 1. \quad (1)$$

The equation is separable. Use the technique of separating the variables to find an exact solution using pencil and paper calculations only. Show all of your work.

EXERCISE 2. Consider the initial value problem

$$y' - y = \cos t, \quad y(0) = 1. \quad (2)$$

The equation is linear. Find an exact solution using pencil and paper calculations only. Show all of your work.

EXERCISE 3. Consider the autonomous first order differential equation

$$y' = 16y - y^3. \quad (3)$$

- Sketch the right-hand side of the differential equation versus y . On the horizontal axis (the y -axis), indicate the “flow” on the phase line with arrows. Label each equilibrium point and classify its stability.
- In the y versus t plane, rotate a copy of the phase line onto the vertical axis, indicating the equilibrium points and the flow along the phase line. Sketch the equilibrium solutions. Sketch several non-equilibrium solutions, enough to indicate the overall behavior of the system.

EXERCISE 4. A chemical plant begins leaking a toxic chemical into a lake. The concentration c of the chemical in the lake is given by the differential equation

$$c' - 0.52c = 0.02, \quad (4)$$

where time is measured in years.

- Assume that the initial concentration of the chemical in the lake is 0.00. Use qualitative analysis to determine the eventual concentration of the chemical in the lake. Use **dfield** to draw this equilibrium solution, then draw the solution having $c = 0.00$ at $t = 0$. Obtain a printout.
- Use the zoom capability of **dfield** to determine how long it takes the chemical to reach a level of 0.02. Obtain carefully labeled printout of your result.