Optimizing Chemotherapy in an HIV Model

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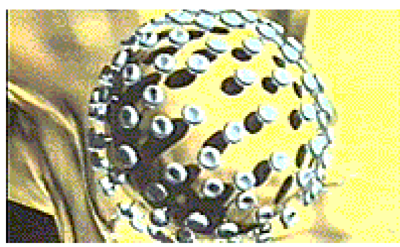
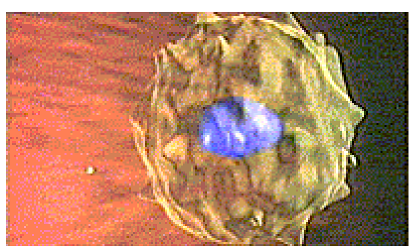
Abstract. Examination of an ordinary differential system modeling the interaction of the HIV virus and the immune system of the human body. The optimal control represents a percentage effect the chemotherapy has on the interaction of the $CD4^{+}T$ cells with the virus.

$CD4^{+}T$ cells (CD4 positive T lymphocytes, a type of white blood cell) are the cells that keep away everyday invasions on the human system. HIV infects these very cells. Essentially the HIV virus is a retrovirus and the HIV RNA is converted to DNA inside the $CD4^{+}T$ cell. The HIV DNA is duplicated in the host cell and new HIV particles bud from the host cell.

Chemotherapy for HIV(human immunodeficiency virus) has been a major treatment in recent years. Chemotherapy for HIV treatment also destroys these cells and many other "things." This model was developed to find an optimal schedule for treatment of the virus with chemotherapy using Ordinary Differential Equations and optimal control.



HIV



$CD4^{+}T$ cell

$$\frac{dT}{dt} = \frac{s}{1+V} - \mu_T T + rT\left(1 - \frac{T + T' + T''}{T_{\max}}\right) - K_1 VT \qquad 1$$

$$\frac{dT'}{dt} = K_1 VT - \mu_T T' - K_2 T' \qquad 2$$

$$\frac{dT''}{dt} = K_2 T' - \mu_b T'' \qquad 3$$

$$\frac{dV}{dt} = N\upsilon_b T'' - K_1 VT - \mu_v V \quad 4$$

Initial Conditions:

$$T(0) = T_0$$

$$T'(0) = T'_0$$

$$T''(0) = T''_0$$

$$V(0) = V_0$$

In Equation 1, $s/(1+V)$ represents the rate of generation of new $CD4^+T$ cells, while μ_T represents the death rate per cell. In Equation 2, r is the coefficient of the growth rate of $CD4^+T$ cells. r is a logistic-type growth and that ensures that the T ($CD4^+T$)cells never grow larger than $T_{\max}$.

In both Equations 1 & 2, $K_1 VT$ is the rate that free virus infects $CD4^+T$ cells. The $K_1 VT$ is subtracted from Equation 1 and added to Equation 2 because the Tcell has become latently infected.

In Equation 3, K_2 is the rate of lantently infected T cells becoming actively infected. The actively infected T cells die at a rate of μ_b per cell.

In Equation 4, N represents the number of virus formed before the host cell dies. The free virus is lost at a rate of $K_1$and $-\mu_v V$ accounts for loss of infectivity or removal from the body. In all of the equations, T represents the concentration of the uninfected $CD4^+T$cells and, T', T'' represent the conc. of latent and actively infected $CD4^+T$ cells, and V represents the conc. of free infectious virus particles.

These ODEs are then used, along with finding an optimal control, to make the equation that models the problem. After a long mathematical process, and many substitutions, the equation turns into a monster that I can’t really explain, yet.

References

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