The Force of a Falling Chain

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Abstract

The purpose of the experiment is to determine the magnitude of the force exerted on a force sensor by a falling chain. A mathematical model of the force was used to find the theoretical values. Then, the experimental values were determined using Science Workshop and compared to the theoretical values.

1. Introduction

Imagine a long chain with a length L and mass M, made up of many links. One end is attached to a force sensor, and the other is held directly above the force sensor so that the chain exerts no force on the sensor to begin with, see Figure 1.

Upon release of the chain, there is a force (which exceeds the weight force) exerted on the sensor, until the last link falls and the chain comes to a rest.

The question is: what is that force exerted on the sensor by the chain as a function of x (the distance that the top of the chain has fallen) during the time the chain is falling. Through a series of derivations, by using the laws of physics, the total force that the chain exerts on the sensor is found.

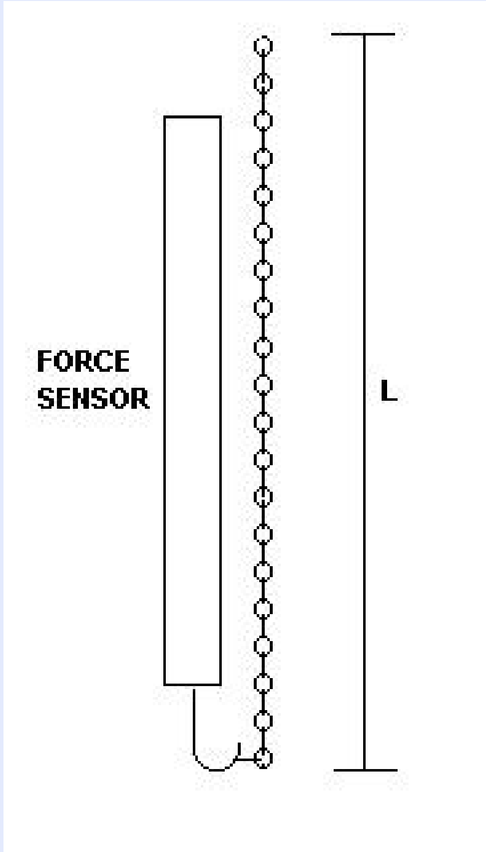

Figure 1: The set-up

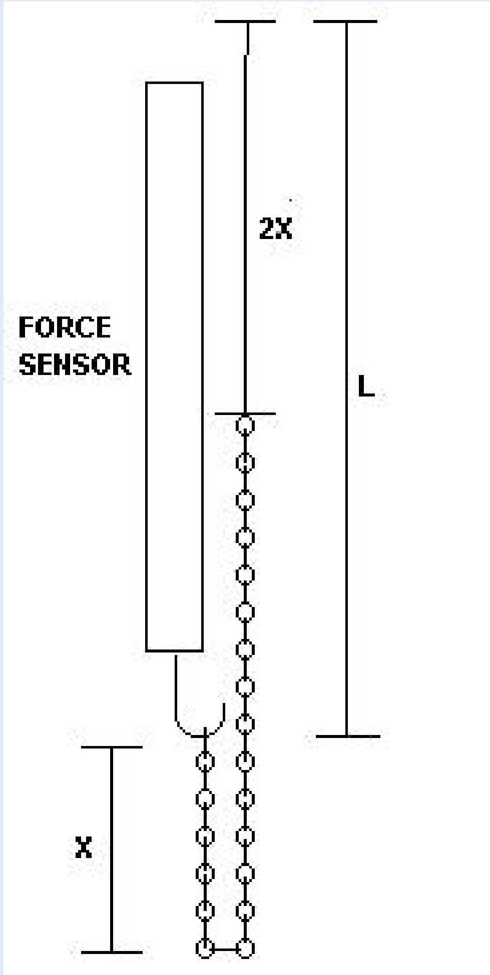

Figure 2: Diagram of Chain Falling

Home Page
Title Page
Contents
◀◀ ▶▶
◀ ▶
Page 5 of 16
Go Back
Full Screen
Close
Quit

2. Theoretical

The force exerted by the chain on the sensor is a combination of two forces. The first force, F_1, is the impulse force (momentum) of each link as it is stopped by the link above it. The second force, F_2, is the weight force of the links already hanging from the hook. The total force F_t exerted on the sensor is:

$$F_t = F_1 + F_2 \tag{1}$$

Looking at the process of the falling chain, we see that the chain is made up of links that have individual mass elements dm, which are associated with their length increments dx, see Figure 3.This ratio is comparable to the ration of the total mass to the total length of the chain. Using this relationship we come up with the equation:

$$\frac{dm}{dx} = \frac{M}{L} \tag{2}$$

After a little rearrangement we are left with:

$$dm = \frac{M}{L} dx \tag{3}$$

Before continuing further on the derivation of the equations for the total force the chain exerts on the sensor, we must understand the concept of center of mass. The center of mass is defined to be, "the point at which we assume all the mass of an object is concentrated in order to determine its motion in response to external forces."[2] We find the center of mass by adding up all of the mass increments times each corresponding x coordinate and dividing this sum by the total mass of the chain (which stays constant).

$$X_c = \frac{\sum m_i x_i}{M} \tag{4}$$

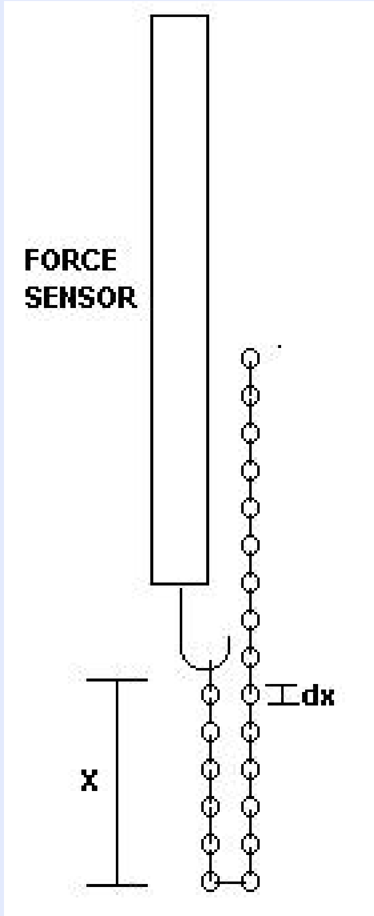

Figure 3: This diagram shows the relationship between x and dx

Home Page
Title Page
Contents
◀◀ ▶▶
◀ ▶
Page 7 of 16
Go Back
Full Screen
Close
Quit

By letting the number of elements approach infinity we can find the exact value for the center of mass. By replacing this limit with an integral we come up with:

$$X_c = \frac{1}{M}\int_0^L x\,dm \tag{5}$$

The relationship between position and velocity are put into play by using the fact that the velocity of the center of mass is equal to the derivative of the position of the center of mass with respect to t. This allows us to state the relationship that the sum of the mass increments times the velocities of those mass increments, divided by the total mass of the chain is equal to the velocity of the center of mass.

$$V_c = \frac{dX_c}{dt} = \frac{1}{M}\sum m_i v_i \frac{dx_i}{dt} = \frac{\sum m_i v_i}{M} \tag{6}$$

Knowing that momentum is defined in physics to be equal to the mass times the volume, we rearranged Equation 6 to show that:

$$MV_c = \sum m_i v_i = \sum p_i = P \tag{7}$$

We also know that there will be an acceleration of the center of mass which is equal to the derivative of the velocity of the center of mass with respect to time. This tells us that the acceleration of the center of mass is equal to the sum of the masses times the accelerations of each of these mass increments, divided by the entire mass of the chain. Giving us the equation:

$$A_c = \frac{dV_c}{dt} = \frac{1}{M}\sum m_i \frac{dv_i}{dt} = \frac{1}{M}\sum m_i a_i \tag{8}$$

Rearranging Equation 8 we find that the total mass of the chain times the acceleration of the center of mass is equal to the sum of all the mass increments times the accelerations of each of these mass increments. This leaves us with Newton's second law which states that the sum of the forces F_1 is equal to the sum of the mass increments times the accelerations of the mass increments.

$$MA_c = \sum m_i a_i = \sum F_i = F_1 \tag{9}$$

A force on a particle can include both external and internal forces. Taking Newton's third law into consideration, where the force of particle A on particle B is equal to the force of particle B on particle A, we can see that when summing all of the internal forces, that they cancel in pairs and we are left only with external forces. We then use Newton's second law and Newton's third law to show that the sum of the external forces is equal to the total mass of the chain times the acceleration of the center of mass. Then by taking the derivative of this with respect to t(knowing that the mass of the system is changing and the velocity is constant), and knowing that momentum is equal to the velocity times the mass, we can show that:

$$F_1 = \frac{dP}{dt} = v\frac{dm}{dt} \tag{10}$$

In order to find F_1 in terms of variables that we know, we need to substitute Equation 3, into Equation 10. We then arrive at the conclusion that:

$$F_1 = v\frac{M}{L}(\frac{dx}{dt}) = \frac{M}{L}v^2 \tag{11}$$

The only variable left in Equation 11 that we do not know is the velocity. By using one of the kinematic equations we can find a substitution for the velocity.

$$v_f^2 = v_0^2 + 2a\Delta x \tag{12}$$

In order to simplify Equation 12, we have known information that can be of use.

$$\begin{aligned} a &= g, \quad \text{(acceleration due to gravity)} \\ \Delta x &= 2x \\ v_0 &= 0 \end{aligned} \tag{13}$$

Equation 13 is true because as x amount of chain has fallen at the bottom of the sensor, two times that amount has fallen from the top. Knowing this new information provided in Equation 13we have a new velocity equation defined as:

$$v_f^2 = 2g(2x) = 4gx \tag{14}$$

This new equation for the velocity (in terms of known variables) is substituted into our F_1 equation.

$$F_1 = \frac{M}{L}(4gx) = 4Mg\left(\frac{x}{L}\right) \tag{15}$$

Now that we know one of the forces that is exerted on our sensor, we have one additional force to define. In addition to the force of the falling link, the chain already hanging on the sensor exerts a force equal to its weight force.

$$F_2 = Mg\frac{x}{L} \tag{16}$$

As shown in Equation 1, by adding together F_1 and F_2 we can find the total force. Using our equations derived for F_1 and F_2 we find that the total force is

Home Page
Title Page
Contents
◀◀ ▶▶
◀ ▶
Page 10 of 16
Go Back
Full Screen
Close
Quit

equal to:

$$F_t = 4Mg\frac{x}{L} + Mg\frac{x}{L} = 5Mg\frac{x}{L} \tag{17}$$

This equation tells us a lot about the total force exerted by a falling chain. Just as the final link drops, $x = L$ and we can see that the total force is equal to five times its weight.

$$F_t = 5mg \tag{18}$$

In order to compare theoretical results to those in the actual experiment, we must express force in relation to time (t). Since we have Equation 17 in terms of x, we now need to find it in terms of t. The following is one of Newton's kinematic equations that defines position in terms of time.

$$x_f = x_0 + v_0 t + \frac{1}{2}at^2 \tag{19}$$

In our example, x_0 and v_0 are equal to zero and a is the acceleration due to gravity (g), leaving us with:

$$x_f = \frac{1}{2}gt^2 \tag{20}$$

This new information in Equation 20 allows us to define the total force in terms of t:

$$F_t = \frac{3Mg^2t^2}{L} \tag{21}$$

This new equation is a quadratic equation that shows us that the forces maximum value is at five times the chains weight. This is until the last link fall. Once the last link has fallen, the total force that the chain exerts on the sensor will return to being solely the chains weight, which is defined as:

$$F = mg \tag{22}$$

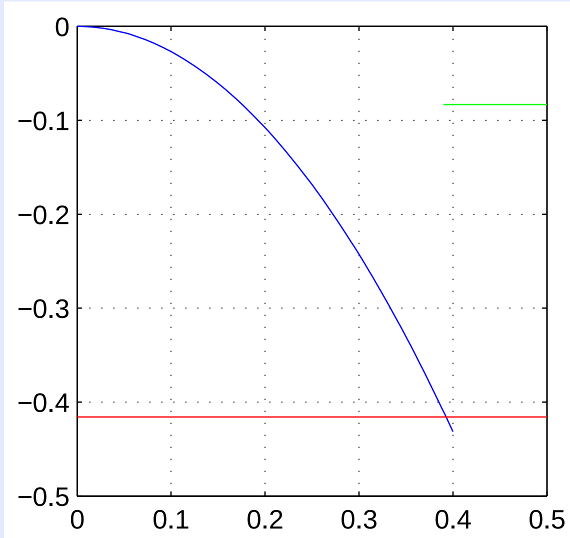

Figure 4: A Graph of Theoretical Values

Figure 4 is a theoretical graph of the force exerted on the sensor versus time. The blue line is a graph of the theoretical falling force, the Red line is what we calculated the maximum Force exerted on the chain to be, and the Green line is what the weight force of the chain.

Introduction
Theoretical
Experimental
Conclusion

Home Page
Title Page
Contents
Page 12 of 16
Go Back
Full Screen
Close
Quit

3. Experimental

In order to prove our derivations the experiment was performed, and the data found was compared to the expected data. The experiment was performed using the PASCO scientific Model CI-6537 $\pm 50\,\mathrm{N}$ Force Sensor and the Science Workshop 2.3.3 Software. A chain was suspended from a horizontal bar by a piece of thread, so that the final link was attached to the hook of the force sensor without exerting a force on it (see Figure 1. The following was data was pertinent to the chain used in the experiment,

$$\begin{aligned} L &= 0.7645\,m \\ M &= 8.479\,g \end{aligned} \tag{23}$$

The force sensor was plugged into the signal interface which was connected to the computer. The experiment was performed with the sensor's original calibration. The string holding the chain suspended was cut. As the chain fell, data was collected and then plotted directly into Scientific Workshop. The software provided a graph of the force exerted by the chain versus time, which is shown in Figure 4. Since the theoretical mass depends on a long flexible chain, it is important that the chain used has links that are very small in relationship to it's total length. It is also necessary that the chain falls as vertically as possible to minimize any oscillations (noise) from pendulum type motion during the time data is being collected. It is also important that the force sensor itself is securely mounted in order to prevent more sever oscillations than necessary from being included in the data.

When inputting our data for the mass of the chain into our derived equations, the following data was expected:

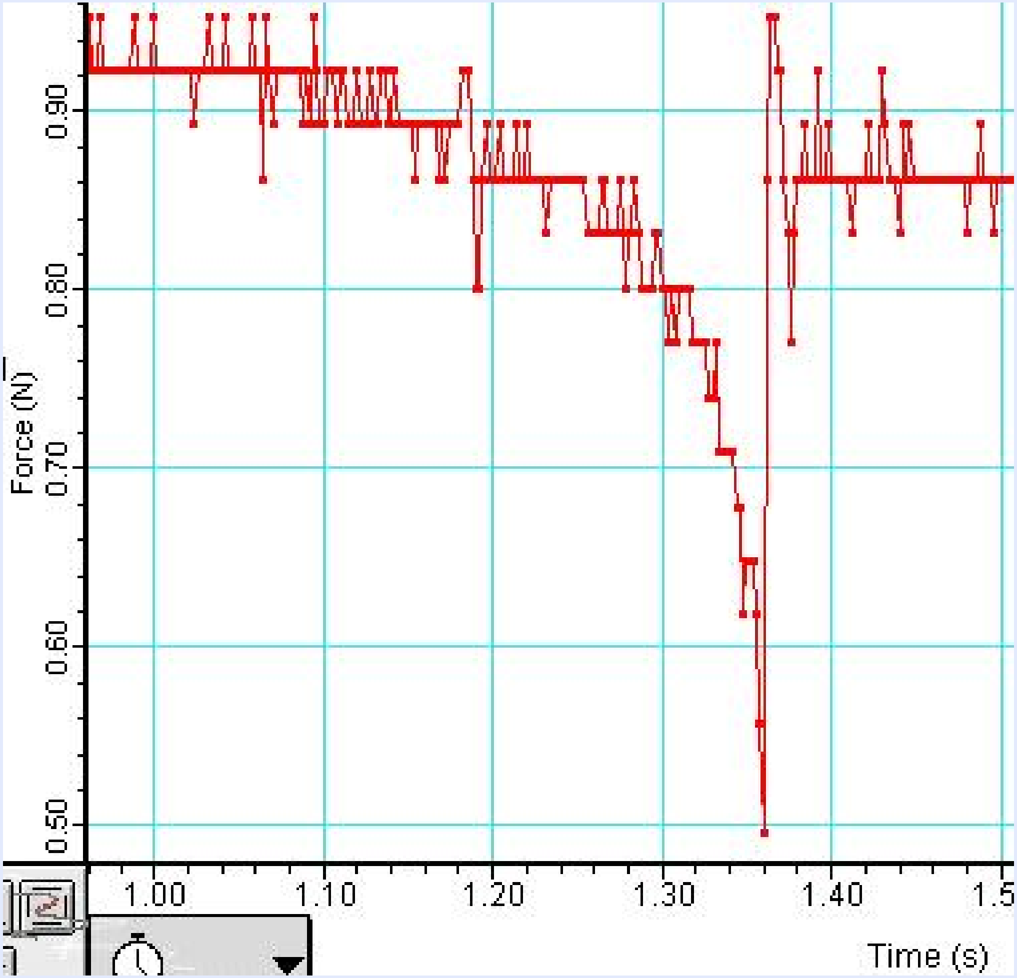

Figure 5: A graph of force exerted by chain versus time

Weight of chain	0.0832 N
Five times the weight force	0.04159 N

By looking at Figure 5 you can see the following,

$$F_0 \approx 0.9250\,N$$
$$F_{\max} \approx 0.5050\,N$$

This leaves the total force on the sensor, at any maximum displacement, as the final link falls to be,

$$F_T \approx 0.9250\,N - 0.5050\,N$$
$$F_T \approx 0.4200\,N$$

Home Page
Title Page
Contents
◀◀ ▶▶
◀ ▶
Page 15 of 16
Go Back
Full Screen
Close
Quit

4. Conclusion

The force measured in the experiment was very close to the theoretical value varying by only,

$$\%\ \text{difference} = \frac{0.4200\,N - 0.4159\,N}{0.4159\,N} = 1.0\,\%$$

The instantaneous force of a falling chain (or anything that acts in a similar manner: rope, cable, etc.) is much greater than the simple weight force of the object itself. The force due to its momentum contributes a great deal to the actual force it exerts on the device holding it. In actuality, the force exerted is five times the weight force of the chain itself. The facts surrounding this experiment point to many engineering applications in which this type of behavior is observed. In any place that a long flexible object is dropped, attached to a device that suspends it from above, the additional force cause by the momentum of the flexible object must be considered. The real world applications are numerous for the problem of a falling chain.

Introduction
Theoretical
Experimental
Conclusion

Home Page

Title Page

Contents

Page 16 of 16

Go Back

Full Screen

Close

Quit

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