

COLLEGE ALGEBRA
PRETEST #4

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April 14, 1999

1. For each of the following systems, set up an augmented matrix representing the system, then place the augmented matrix in *row echelon form*. Write out the system of equations represented by the row echelon form of the augmented matrix and use back substitution to find the solution. Show all work. Pencil and paper calculations only!

a) $x + 2y = 4$
 $3x - y = 6$

b) $x + 2y - z = 4$
 $y + 2z = 8$
 $2x - y - 3z = 9$

c) $x + y - 2z = 4$
 $2x - y + z = 8$
 $3x - z = 12$

In problems 2–5, set up a system of equations that models the problem situation. State the augmented matrix for your system. Enter the augmented matrix in your calculator and place it in *reduced row echelon form*. Report this matrix on your paper and state the final solution of the word problem.

2. Josephine's Candy Store makes a nut mix containing peanuts and cashews. Peanuts costs \$1.75 per pound, while cashews cost \$3.25 per pound. Peanuts and cashews are mixed to produce 25 pounds of mixed nuts costing \$2.05 per pound. How many pounds of each nut produces such a mix?
3. Jamal inherits \$11,000 and proceeds to invest all of it in two investments, part in a mutual fund paying 5% per year, the remainder in a certificate of deposit, paying 6% per year. If the combined yearly interest from both investments is \$623, how much did Jamal invest in each account?
4. Jamie has \$6.10 in her pocket, all in nickels, dimes, and quarters, 41 coins in all. If the number of quarters is four more than the number of dimes, how many nickel, dimes, and quarters does Jamie have?
5. One year, Don invested a total of \$40,000, part at 12%, part at 13%, and the rest at $14\frac{1}{2}\%$. The total amount of interest received on the investments was \$5370. The interest received on the $14\frac{1}{2}\%$ investment was \$1050 more than the interest received on the 13% investment. How much was invested at each rate?
6. Consider the matrices:

$$A = \begin{pmatrix} 1 & 2 \\ -3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 3 \\ 0 & 5 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} -1 & -1 \\ 2 & 6 \end{pmatrix}.$$

Simplify each of the following expressions without the aid of a calculator.

- | | |
|---------------------|----------------|
| a) AB | b) BA |
| c) $(B + C)(B - C)$ | d) $B^2 - C^2$ |
| e) $A(2B)$ | f) $2(AB)$ |
| g) $A(B + C)$ | h) $AB + AC$ |

7. Given the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix},$$

find a non-zero matrix

$$B = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$$

so that

$$AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

8. Use hand calculations (no calculators) to multiply the following two matrices together.

$$\begin{pmatrix} 1 & -1 & 0 & 2 \\ 2 & -2 & 4 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 2 & 0 \\ -1 & -2 & 4 \\ 4 & 5 & -2 \end{pmatrix}$$

9. Use hand calculations (no calculators) to find the inverse of the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}.$$

10. In each of the following problems, set up a matrix equation representing the system. Multiply each side of the matrix equation by the inverse of the coefficient matrix to provide a solution of the system. Hand calculations only, please!

a) $x + 2y = 4$
 $2x + 5y = 10$

b) $x - y - z = 1$
 $x + 2z = 4$
 $2x - 3y + z = 0$

11. Evaluate each of the following determinants by hand. No calculators!

a) $\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix}$

b) $\begin{vmatrix} 1 & 2 & -3 \\ 1 & 4 & -1 \\ -2 & 2 & 2 \end{vmatrix}$

c) $\begin{vmatrix} 1 & 1 & -1 & 1 \\ 2 & -2 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 2 & -2 & 0 & 1 \end{vmatrix}$

12. Use Cramer's rule to solve the following system of equations. Hand calculations, please. No calculators.

$$3x - 5y = 12$$

$$2x + 9y = 18$$

13. Use Cramer's rule to setup, but do **not** solve, the solution for x , y , and z in the system

$$x - y - z = 1$$

$$x + 2z = 4$$

$$2x - 3y + z = 0$$

14. Sketch the solution set of each of the following systems of inequalities on a sheet of graph paper.

a) $x + 2y \geq 4$
 $y > x$

b) $x + 2y \leq 12$
 $2x + y \leq 12$
 $x \geq 0$
 $y \geq 0$

c) $y \geq x^2$
 $y \leq 2x + 3$