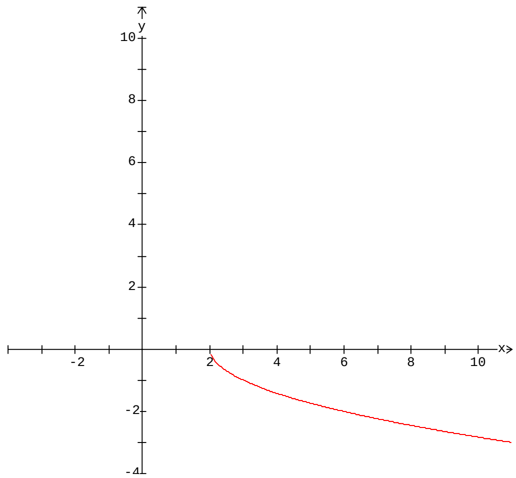
College Algebra Pretest #3 Solutions

1. $f(x) = -\sqrt{x-2}$

a.

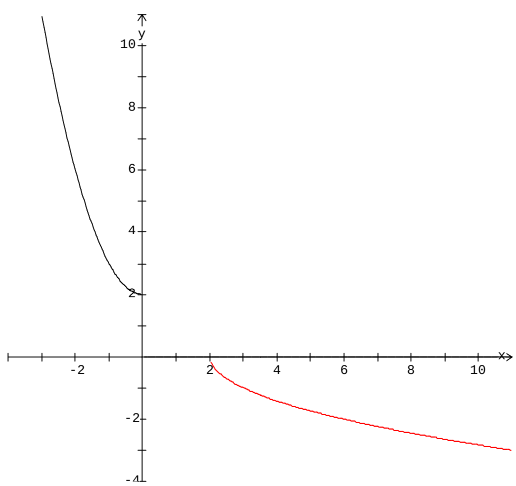


b.

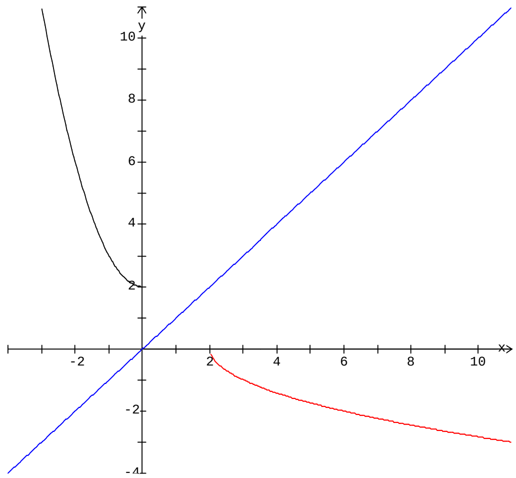
$$x = -\sqrt{y-2}$$
$$-x = \sqrt{y-2}$$
$$(-x)^2 = y-2$$
$$(-x)^2 + 2 = y$$

$f^{-1}(x) = (-x)^2 + 2 = x^2 + 2$ provided $x \le 0$.

c.



d. The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are symmetric with the line $y = x$.



2. $f(g(x))$ provided $x \neq 1$:

$$
\begin{aligned}
f(g(x)) &= f\left(\frac{x}{1-x}\right) \\
&= \frac{\frac{x}{1-x}}{\frac{x}{1-x}+1} \\
&= \frac{\frac{x}{1-x}}{\frac{x+(1-x)}{1-x}} \\
&= \frac{x}{1-x}\,\frac{1-x}{1} \\
&= x
\end{aligned}
$$

$g(f(x))$ provided $x \neq -1$:

$$
\begin{aligned}
g(f(x)) &= g\left(\frac{x}{x+1}\right) \\
&= \frac{\frac{x}{x+1}}{1-\frac{x}{x+1}} \\
&= \frac{\frac{x}{x+1}}{\frac{x+1-x}{x+1}} \\
&= \frac{x}{x+1}\,\frac{x+1}{1} \\
&= x
\end{aligned}
$$

3. In each case switch the x and y variables and solve for y.

a.

$$
\begin{aligned}
x &= \frac{y+2}{3-2y} \\
x(3-2y) &= y+2 \\
3x-2xy &= y+2 \\
-2xy-y &= 2-3x \\
-y(2x+1) &= -(3x-2) \\
f^{-1} &= \frac{3x-2}{2x+1}
\end{aligned}
$$

b.

$$\begin{aligned} x &= \sqrt{y+3} \\ x^2 &= y+3 \\ x^2 - 3 &= y \\ f^{-1}(x) &= x^2 - 3, \text{ provided } x \geq 0 \end{aligned}$$

c.

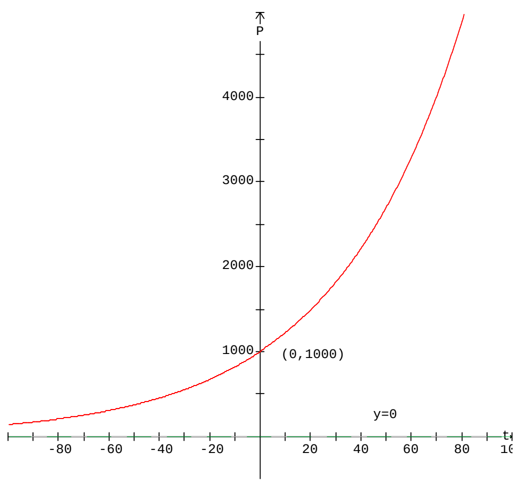
$$\begin{aligned} x &= \ln(3y+2) \\ e^x &= 3y+2 \\ e^x - 2 &= 3y \\ f^{-1}(x) &= \frac{e^x - 2}{3} \end{aligned}$$

d.

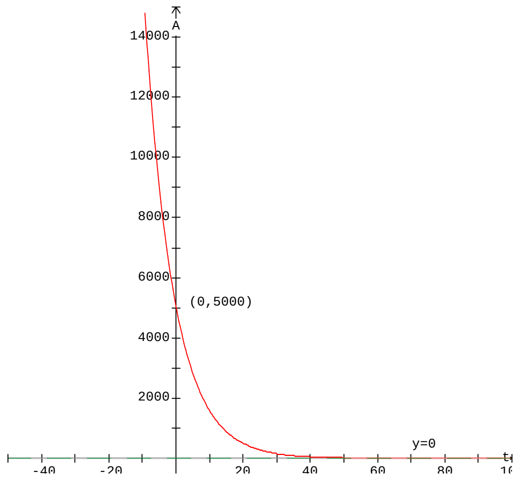
$$\begin{aligned} x &= e^{-y} + 1 \\ x - 1 &= e^{-y} \\ \ln(x-1) &= -y \\ f^{-1}(x) &= -\ln(x-1) \end{aligned}$$

4. Sketch without the aid of a calculaotr.

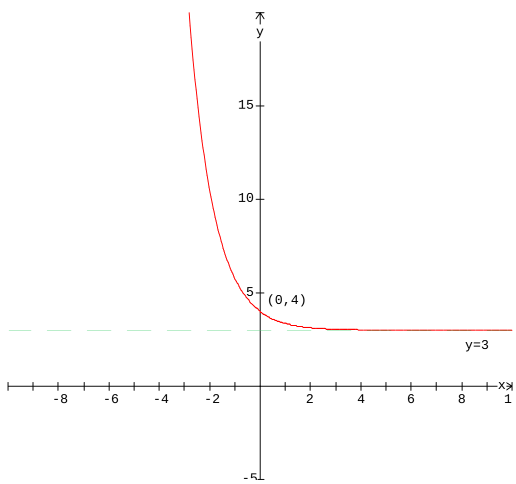
a. $P = 1000(1.20)^t$



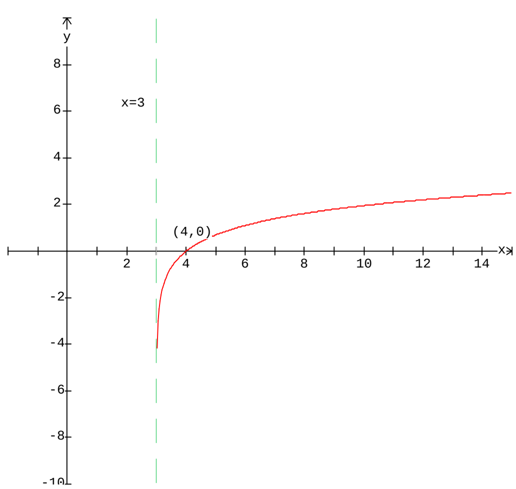
b. $A = 5000(0.89)^t$



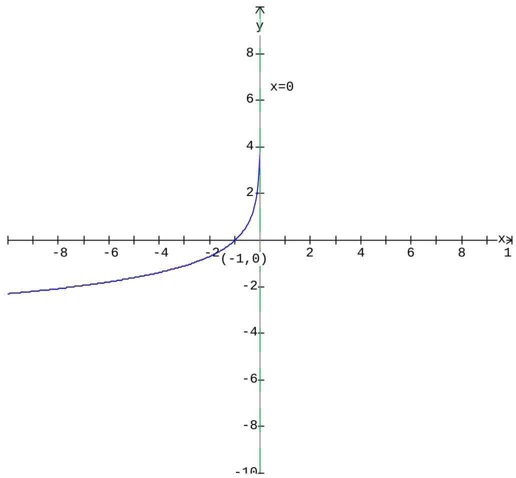
c. $y = e^{-x} + 3$



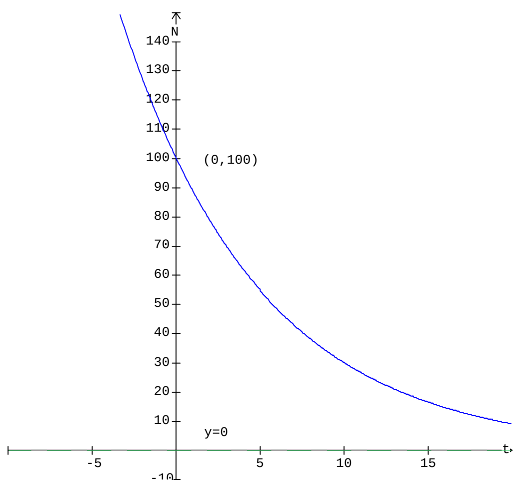
d. $y = \ln(x - 3)$



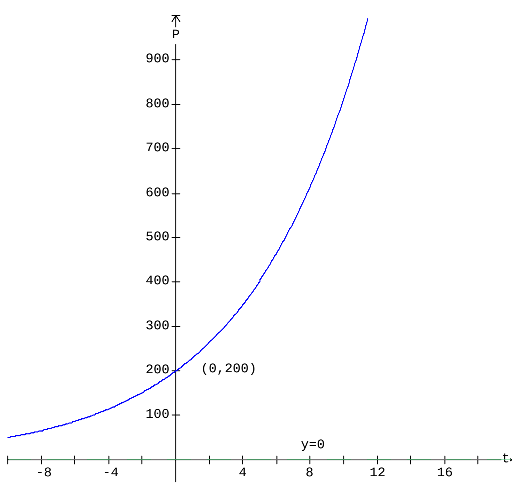
e. $y = -\ln(-x)$



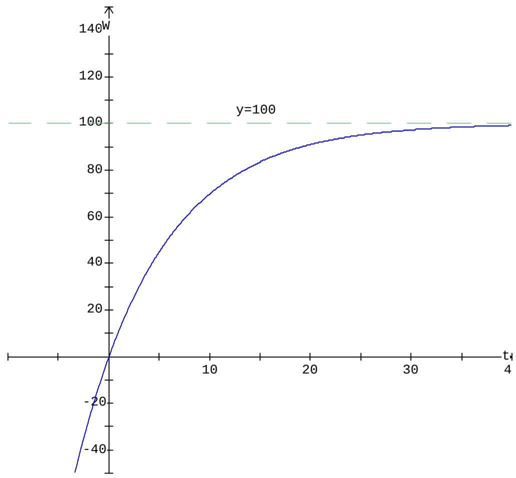
f. $N = 100e^{-0.12t}$



g. $P = 200e^{0.14t}$



h. $W = 100(1 - e^{-0.12t})$



5. Simplify without the aid of a calculator.

a.

$$\log_2(8) = n \Leftrightarrow 2^n = 8$$
$$n = 3$$

b.

$$\log_2\left(\frac{1}{4}\right) = n \Leftrightarrow 2^n = \frac{1}{4}$$
$$2^n = 2^{-2}$$
$$n = -2$$

c.

$$\log_{25}(5) = n \Leftrightarrow 25^n = 5$$
$$(5^2)^n = 5$$
$$5^{2n} = 5^1$$
$$2n = 1$$
$$n = \frac{1}{2}$$

d.

$$\log_8(16) = n \Leftrightarrow 8^n = 16$$
$$(2^3)^n = 2^4$$
$$2^{3n} = 2^4$$
$$3n = 4$$
$$n = \frac{4}{3}$$

e.

$$\log_6(4) + \log_6(9) = \log_6(36)$$
$$\log_6(36) = n \Leftrightarrow 6^n = 36$$
$$n = 2$$

f.

$$\log_2(24) - \log_2(3) = \log_2\left(\frac{24}{3}\right) = \log_2(8)$$
$$\log_2(8) = n \Leftrightarrow 2^n = 8$$
$$n = 3$$

g.

$$\log_2(3) + 2\log_2(7) = \log_2(3) + \log_2(7^2)$$
$$= \log_2(3 * 7^2)$$
$$2^{\log_2(3)+2\log_2(7)} = 2^{\log_2(147)} = 147$$

h.

$$\log_3(18) - \log_3(2) = \log_3\left(\frac{18}{2}\right)$$
$$3^{\log_3(18)-\log_3(2)} = 3^{\log_3(9)} = 9$$

6. Simplify each of the following to an algebraic expression with a single logarithm.

a.

$$2\log x - 3\log y + 4\log w - 5\log z = \log x^2 - \log y^3 + \log w^4 - \log z^5$$
$$= \log\frac{x^2}{y^3} + \log w^4 - \log z^5$$
$$= \log\frac{x^2w^4}{y^3} - \log z^5$$
$$= \log\frac{x^2w^4}{y^3z^5}$$

b.

$$2\ln x - 3[\ln(x+2) - 2\ln(x+1)] = \ln x^2 - 3\left[\ln(x+2) - \ln(x+1)^2\right]$$
$$= \ln x^2 - 3\ln\frac{(x+2)}{(x+1)^2}$$
$$= \ln x^2 - \ln\left(\frac{(x+2)}{(x+1)^2}\right)^3$$
$$= \ln\left(\frac{x^2}{\frac{(x+2)^3}{(x+1)^6}}\right)$$
$$= \ln\left(\frac{x^2(x+1)^6}{(x+2)^3}\right)$$

c.

$$\frac{1}{2}\log x - \frac{3}{2}\log(x+1) = \log x^{\frac{1}{2}} - \log(x+1)^{\frac{3}{2}}$$
$$= \log\frac{x^{\frac{1}{2}}}{(x+1)^{\frac{3}{2}}}$$
$$= \log\left(\frac{x}{(x+1)^3}\right)^{\frac{1}{2}}$$

7.

$$\log_2(5) = \frac{\log 5}{\log 2} \approx 2.3219$$

8. Solve without the aid of a calculator.

a.

$$2^x = 5 \Leftrightarrow x = \log_2(5)$$

b.

$$\begin{aligned} \ln(2x+3) &= 7 \\ 2x+3 &= e^7 \\ 2x &= e^7 - 3 \\ x &= \frac{1}{2}(e^7 - 3) \end{aligned}$$

c.

$$\begin{aligned} e^{3x-2} &= 5 \\ 3x - 2 &= \ln 5 \\ 3x &= \ln 5 + 2 \\ x &= \frac{1}{3}(\ln 5 + 2) \end{aligned}$$

d.

$$\begin{aligned} (\log x)^2 &= \log x^2 \\ (\log x)^2 - 2\log x &= 0 \\ \log x(\log x - 2) &= 0 \end{aligned}$$

$\log x = 0 \Rightarrow \quad x = 10^0 = 1$

or

$\log x = 2 \Rightarrow \quad x = 10^2 = 100$

e.

$$\begin{aligned} 5^x &= 3^{2x-1} \\ \ln(5^x) &= \ln(3^{2x-1}) \\ x\ln 5 &= (2x-1)\ln 3 \\ x\ln 5 &= 2(\ln 3)x - \ln 3 \\ x\ln 5 - x\ln 3^2 &= -\ln 3 \\ x\ln\left(\frac{9}{5}\right) &= -\ln 3 \\ x &= \frac{-\ln 3}{\ln\left(\frac{9}{5}\right)} \end{aligned}$$

f.

$$
\begin{aligned}
e^{2\ln x} &= 4 \\
e^{\ln x^2} &= 4 \\
x^2 &= 4 \\
x &= \pm 2
\end{aligned}
$$

$\ln x$ is only defined for positive x –values, so $x = 2$ is the solution.

g.

$$
\begin{aligned}
\ln x - \ln(x+3) &= 1 \\
\ln\left(\frac{x}{x+3}\right) &= 1 \\
\frac{x}{x+3} &= e^1 \\
x &= e(x+3) \\
x &= ex + 3e \\
x - ex &= 3e \\
x(1-e) &= 3e \\
x &= \frac{3e}{1-e}
\end{aligned}
$$

Since $\ln x$ is defined for positive x –values and $1 - e$ is less than zero, there is no solution.

h.

$$
\begin{aligned}
\log_3 x + \log_3(x+2) &= 1 \\
\log_3(x(x+2)) &= 1 \\
x^2 + 2x &= 3^1 \\
x^2 + 2x - 3 &= 0 \\
(x+3)(x-1) &= 0
\end{aligned}
$$

The quadratic has two solutions, $x = -3$ and $x = 1$. But since $\log_3 x$ is defined for positive x –values, $x = 1$ is the only solution.

9. With initial population $P_0 = 10,000$ and a growth rate of $k = 0.02$ the equation for the population is $P(t) = 10,000e^{0.02t}$. To find the time for the population to double we must solve $20000 = 10000e^{0.02t}$.

$$
\begin{aligned}
20000 &= 10000e^{0.02t} \\
2 &= e^{0.02t} \\
\ln 2 &= 0.02t \\
t &= \frac{\ln 2}{0.02}
\end{aligned}
$$

With a growth rate of 2%, Fortuna will double its population in $t = \frac{\ln 2}{0.02} \approx 34.657$ years.

10. Given the final amount, $A = \$40,000$, after $t = 18$ years invested at $r = 7.25\%$ compounded semianually, we need to find the inital amount needed to invest.

$$40,000 = P_0\left(1 + \frac{0.0725}{2}\right)^{2*18}$$
$$P_0 = \frac{40000}{\left(1 + \frac{0.0725}{2}\right)^{36}}$$
$$P_0 = \$11,100$$

11. Given $P_0 = 5000$ and an interest rate of 6.25% compounded quarterly, we want an ending balance of $15,000.

$$15000 = 5000\left(1 + \frac{0.0625}{4}\right)^{4t}$$
$$3 = \left(1 + \frac{0.0625}{4}\right)^{4t}$$
$$\ln 3 = 4t\ln\left(1 + \frac{0.0625}{4}\right)$$
$$t = \frac{\ln 3}{4\ln\left(1 + \frac{0.0625}{4}\right)} \approx 17.715$$

After approximately 17.715 years the balance will be $15,000.

12. Given a doubling time of 5 years with semiannual compounding and an initial deposit of $2,000, we need to find the yearly interest rate.

$$4000 = 2000\left(1 + \frac{r}{2}\right)^{2*5}$$
$$2 = \left(1 + \frac{r}{2}\right)^{10}$$
$$\pm\sqrt[10]{2} = 1 + \frac{r}{2}$$
$$\frac{r}{2} = \sqrt[10]{2} - 1 \quad \text{choosing only the positive rate}$$
$$r = 2\left(\sqrt[10]{2} - 1\right) \approx 0.14355$$

The yearly interest rate will be 14.36%.

13. With a half-life of 500 years, we need to find the time needed for a substance starting with 1000 mg to decay to 200 mg. First we need to find the rate of decay.

$$\frac{1}{2}P_0 = P_0e^{-500k}$$
$$\ln\left(\frac{1}{2}\right) = -500k$$
$$k = \frac{\ln\left(\frac{1}{2}\right)}{-500}$$

Now we can solve the problem. To simplify the notation we will substitute for k at the end of the problem.

$$
\begin{aligned}
200 &= 1000e^{-kt} \\
\frac{1}{5} &= e^{-kt} \\
\ln\left(\frac{1}{5}\right) &= -kt \\
t &= \frac{\ln\left(\frac{1}{5}\right)}{-k} \\
t &= \frac{500\ln(5^{-1})}{\ln(2^{-1})} = \frac{500\ln 5}{\ln 2}
\end{aligned}
$$

It will take about 1161.0 years for the substance to decay from 1000 mg to 200mg.

14. Given 500 mg decays to 200 mg in 14 days we can find the rate of decay.

$$
\begin{aligned}
200 &= 500e^{-14k} \\
\frac{2}{5} &= e^{-14k} \\
\ln\left(\frac{2}{5}\right) &= -14k \\
k &= \frac{\ln\left(\frac{2}{5}\right)}{-14}
\end{aligned}
$$

Now we can find the half-life. To simplify the notation we will substitute for k at the end of the problem.

$$
\begin{aligned}
\frac{1}{2}P_0 &= P_0e^{-kH} \\
\ln\left(\frac{1}{2}\right) &= -kH \\
H &= \frac{\ln(2^{-1})}{-k} = \frac{14\ln(2^{-1})}{\ln\left(\frac{2}{5}\right)}
\end{aligned}
$$

The substance has a half-life of about 10.591 days.

15. A fish population is modeled by the function

$$P(t) = \frac{1000}{10 + 90e^{-0.12t}}$$

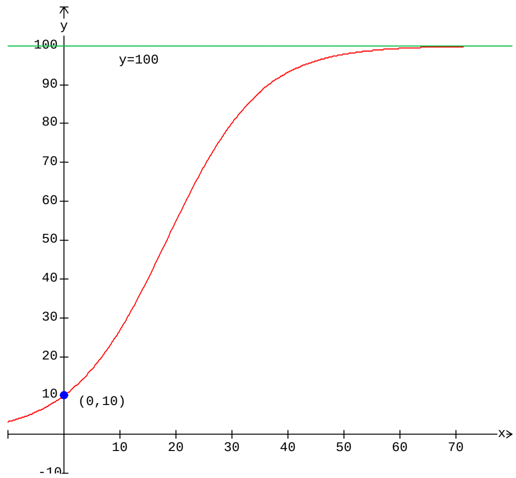
a. The initial population occurs at time $t = 0$.

$$
\begin{aligned}
P_0 &= P(0) = \frac{1000}{10 + 90e^{-0.12(0)}} \\
P_0 &= \frac{1000}{10+90} = \frac{1000}{100} \\
P_0 &= 10
\end{aligned}
$$

b. To find the eventual fish population let $t \to \infty$ and recall $\lim\limits_{t\to\infty} e^{-x} = 0$.

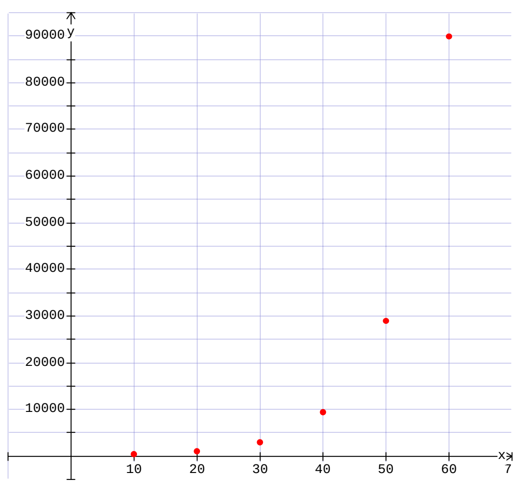
$$
\begin{aligned}
\lim_{t\to\infty} \frac{1000}{10 + 90e^{-0.12t}} &= \frac{1000}{10 + 90 * (0)} \\
&= \frac{1000}{10} \\
&= 10
\end{aligned}
$$

c.



16. Bacteria Population

a.



b. Using ExpReg on the calculator gives

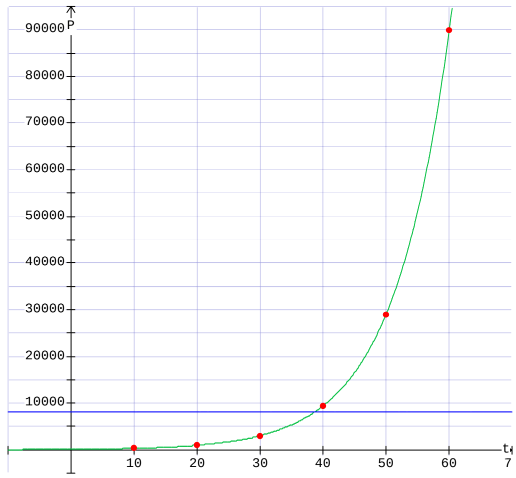
$$y = ab^x$$
$$a = 102.2498869$$
$$b = 1.11950221$$

c. Rounding a nad b to $a = 102.25$ and $b = 1.12$ and letting $t = 45$ gives a population of $P(45) = 102.25 * 1.12^{45} = 16768.$

d.

$$8000 = 102.25 * 1.12^t$$
$$\frac{8000}{102.25} = 1.12^t$$
$$\ln\left(\frac{8000}{102.25}\right) = t\ln(1.12)$$
$$t = \frac{\ln\left(\frac{8000}{102.25}\right)}{\ln(1.12)}$$

After approximately 38.47 days the population count reached 8000.



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90000
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10000
10
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60
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