

COLLEGE ALGEBRA
PRETEST EXAM #3

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1. Consider the function defined by $f(x) = -\sqrt{x-2}$.
 - a) Sketch the graph of f on a sheet of graph paper.
 - b) Use a method of your choice to find the inverse of f .
 - c) Sketch the graph of the inverse found in part (b) on the same coordinate system which you drew the graph of f in part (a).
 - d) Sketch the graph of $y = x$ on your coordinate system. Are the graphs of f and f^{-1} symmetric with respect to the line $y = x$? If not, check your work.

2. Show that

$$f(x) = \frac{x}{x+1} \quad \text{and} \quad g(x) = \frac{x}{1-x}$$

are inverses of one another by demonstrating that $f(g(x)) = x$ and $g(f(x)) = x$.

3. Find $f^{-1}(x)$ for each of the following functions.

a) $f(x) = \frac{x+2}{3-2x}$ b) $f(x) = \sqrt{x+3}$ c) $f(x) = \ln(3x+2)$ d) $f(x) = e^{-x} + 1$

4. Sketch each of the following graphs without the aid of a calculator. Label key points with coordinates and asymptotes (if any) with their equations.

a) $P = 1000(1.02)^t$ b) $A = 5000(0.89)^t$ c) $y = e^{-x} + 3$ d) $y = \ln(x-3)$
e) $y = -\ln(-x)$ f) $N = 100e^{-0.12t}$ g) $P = 200e^{0.14t}$ h) $W = 100(1 - e^{-0.12t})$

5. Simplify each of the following expressions without the aid of a calculator.

a) $\log_2 8$ b) $\log_2 \frac{1}{4}$ c) $\log_{25} 5$ d) $\log_8 16$
e) $\log_6 4 + \log_6 9$ f) $\log_2 24 - \log_2 3$ g) $2^{\log_2 3} + 2^{\log_2 7}$ h) $3^{\log_3 18} - \log_3 2$

6. Simplify each of the following so that the resulting expression is a single logarithm of an algebraic expression.

a) $2 \log x - 3 \log y + 4 \log w - 5 \log z$
b) $2 \ln x - 3[\ln(x+2) - 2 \ln(x+1)]$
c) $\frac{1}{2} \log x - \frac{3}{2} \log(x+1)$

7. Use the change of base rule and your calculator to approximate $\log_2 5$.

8. Solve each of the following for x without the aid of a calculator. Exact answers only, please. **No decimals!**

a) $2^x = 5$ b) $\ln(2x+3) = 7$ c) $e^{3x-2} = 5$ d) $(\log x)^2 = \log x^2$

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}\text{-}\mathcal{T}\mathcal{E}\mathcal{X}$

e) $5^x = 3^{2x-1}$ f) $e^{2\ln x} = 4$ g) $\ln x - \ln(x+3) = 1$ h) $\log_3 x + \log_3(x+2) = 1$

9. The present population of Fortuna is about 10,000 people. If the town is growing at a rate of 2% per year, how long will it take the population to double?
10. On her birthday, Mary's grandmother decides to create a college fund for her new granddaughter. How much should she invest at 7.25%, compounded semiannually, so that there will be \$40,000 in the account on Mary's eighteenth birthday?
11. Suppose that \$5,000 is invested at 6.25% per year, compounded quarterly (four times a year). How long will it take the balance in the account to reach \$15,000?
12. \$2,000 is invested in an account that pays yearly interest, compounded semiannually. If the money doubles in 5 years, find the yearly interest rate.
13. A radioactive substance has a half-life of 500 years. If you start with 1000 mg of the substance, how long will it take until only 200 mg remain?
14. 500 mg of a radioactive substance decays to 200 mg in 14 days. What is the half life of the substance?
15. A fish population in a protected lake grows according to the model

$$P = \frac{1000}{10 + 90e^{-0.12t}}$$

- a) What is the initial fish population?
 - b) What is the eventual fish population?
 - c) Sketch the graph of the population without the aid of a calculator. Label key points with their coordinates and asymptotes with their equations.
16. A bacteria population grows in a laboratory and a count of the bacteria is taken every 10 days.

Days	10	20	30	40	50	60
Count	320	965	3012	9314	28913	89776

- a) Sketch the data on a sheet of graph paper.
- b) Use your calculator to fit an exponential function to the data.
- c) Use your model found in part (b) to predict the bacteria count after 45 days.
- d) When did the count reach a level of 8000?