

COLLEGE ALGEBRA
EXAM #2
STUDY PAGE

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1. Simplify each of the following complex expressions. Place your final answer in the form $a + bi$.

a) $(-1 - \sqrt{3}i)^3$ b) $\frac{2+3i}{3-4i}$ c) i^{73} d) $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^2$

2. If $z = a + bi$ and $w = c + di$, prove that $\overline{z - w} = \bar{z} - \bar{w}$.

3. Use the technique of completing the square to solve the equation $x^2 - 4x - 6 = 0$ for x .

4. Use the quadratic formula to solve each of the following equations.

a) $x^2 - 2x = 2$ b) $3x^2 + 4 = 10x$ c) $u - \sqrt{u} - 2 = 0$ d) $m^{2/3} - 2m^{1/3} = 3$

For the equations in exercises 5–9, perform each of the following tasks.

i. Place the equation in “vertex form.”

ii. Without using a calculator, find the x and y -intercepts.

iii. Sketch the graph of the parabola on graph paper. Draw the axis of symmetry and label it with its equation. Label the vertex, x and y -intercepts with their coordinates.

5. $y = x^2 - 2x - 3$

6. $y = -x^2 - 4x + 5$

7. $f(x) = 2x^2 - 4x - 4$

8. $f(x) = -3x^2 + 3x - 6$

9. The demand for an article, as a function of its unit price, is given by the equation $x = 50 - p$, where x is the number of articles purchased and p is the unit price. What unit price will maximize a shopkeeper’s revenue from the sales of this article?

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10. A baseball is hit 3 feet above ground at a velocity of 100 feet per second and at an angle of 45 degrees with respect to level ground. The path of the baseball is given by the function $h = -0.0032x^2 + x + 3$, where h is the height of the baseball (in feet) and x is the distance from home plate (in feet). What is the maximum height reached by the baseball?
11. A rectangular field is to be fenced off along the bank of a river where no fence is required along the bank. If the material for the fence costs \$12 per running foot for the two ends and \$18 per running foot for the side parallel to the river, find the dimensions of the field of largest possible area that can be enclosed with \$5400 worth of fence. (*Hint: Let x represent the width of the field and y the length of the field parallel to the river. The area of the field is given by $A = xy$. Find another relationship between x and y and use the relationship to express A as a function of x alone.*)
12. Find c so that $f(x) = \frac{1}{2}x^2 - cx - 8$ has a minimum value of -14 . (*Note: There are two answers.*)
13. State the division algorithm.
14. Divide $x^4 - 3x^3 + 7x^2 + 21x - 26$ by $x^2 + x - 2$.
15. State the remainder theorem.
16. Find all zeros, both real and complex, of $p(x) = x^3 - 1$.
17. State the factor theorem.
18. State the rational zero theorem.
19. Use the rational zero theorem to state the possible rational zeros of the polynomial function $f(x) = 2x^3 + 2x^2 - 16x - 24$. Find all of the zeros of f and express $f(x)$ as a product of *linear* factors.
20. State the fundamental theorem of algebra.
21. Complete the following Theorem: If $p(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ is a polynomial with real coefficients, then if $a + bi$ is a zero . . .
22. Find a polynomial of minimal degree, having rational coefficients, and zeros -2 and $3 + i$.
23. if $-1 + i$ is a zero of $p(x) = x^4 - 4x^2 - 8x - 4$, find the remaining zeros of p and express $p(x)$ as a product of linear factors.

In Exercises 24–27, perform each of the following tasks.

- i. Find the vertical asymptotes.
- ii. Find the x -intercepts or the zeros of the function.
- iii. Identify the horizontal or slant asymptotes.
- iv. Sketch the graph of the function on graph paper. Label intercepts with coordinates, asymptotes with their equations.

24. $f(x) = \frac{x+3}{4-2x}$

25. $f(x) = \frac{x+1}{2x^2-4x}$

26. $f(x) = \frac{x^2}{x-3}$

27. $f(x) = \frac{2x^2-5x-3}{x-1}$

28. The rational function

$$f(x) = \frac{x^2-9}{x-3}$$

has a removable singularity. Sketch the graph of f on graph paper.

Solve each of the inequalities in Exercises 29-32. Use interval notation to describe your solution.

29. $x^2 - 4x > 5$

30. $2x^3 - 5x^2 - x + 6 \leq 0$

31. $\frac{2x+3}{x-1} \leq 0$

32. $\frac{x+1}{x-3} \leq \frac{x}{x+1}$