College Algebra Chapter 1 Exam Pretest Solutions

1. A relation is a **function** if and only if each member of the domain corresponds to *exactly one* member of the range.

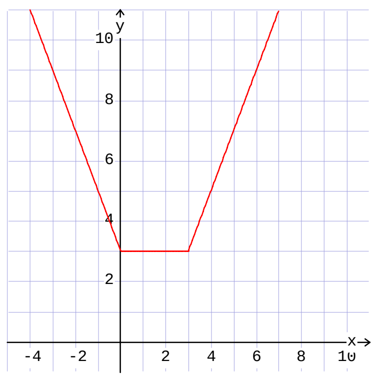
2.

- **a.** $\text{Dom} f = \{x : x \neq 4\}$
- **b.** $\text{Dom}\, g = \{x : x \leq 3\} = (-\infty, 3]$

3. $f(x) = |x| + |x - 3|$

$\text{Dom} f = (-\infty, \infty)$

$\text{Range} f = [3, \infty)$



4.

- **a.** Given $g(x) = x^2 + 2x$, then $g(1) = 3$.

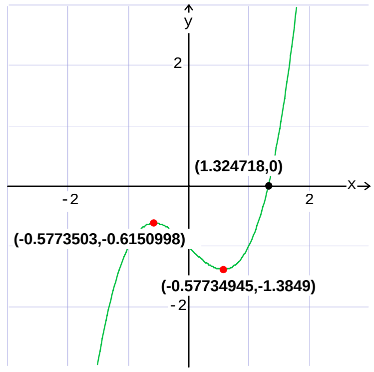
$$\begin{aligned}\frac{g(x) - g(1)}{x - 1} &= \frac{x^2 + 2x - (3)}{x - 1} \\ &= \frac{(x + 3)(x - 1)}{x - 1} \\ &= x + 3 \text{ provided } x \neq 1\end{aligned}$$

- **b.**

$$\begin{aligned}\frac{g(x + h) - g(h)}{h} &= \frac{(x + h)^2 + 2(x + h) - (x^2 + 2x)}{h} \\ &= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h} \\ &= \frac{2xh + h^2 + 2h}{h} \\ &= 2x + h + 2\end{aligned}$$

5. $f(x) = x^3 - x - 1$

- **a.**



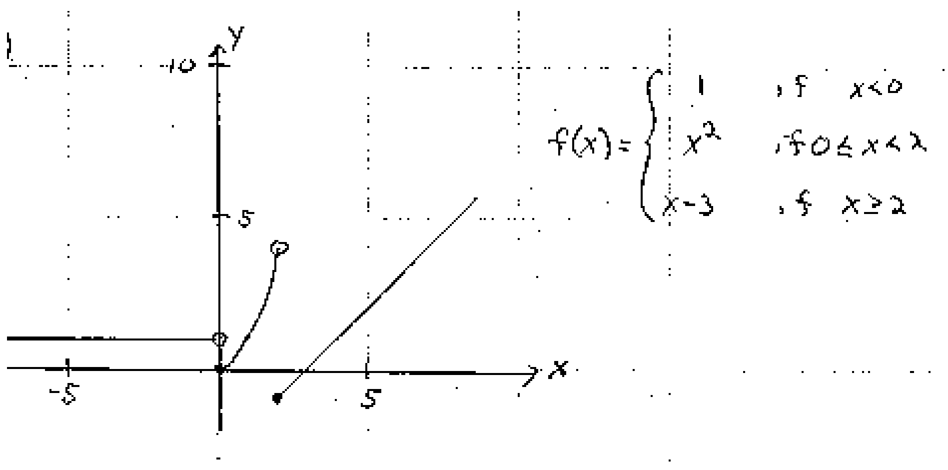
b. There is only one zero at $x = 1.324718$.

c. A local maximum is located at $(-.5773503, -.6150998)$ and a local minimum is located at $(.57734945, -1.3849)$.

d. The function is increasing on the intervals $(-\infty, -.5773503) \cup (.57734945, \infty)$.

e. And the function is decreasing on the interval $(-.5773503, .57734945)$.

6. The graph of $f(x)$ is



7. $f(x) = |x+2| + |x-4|$

a. $|x+2| = \begin{cases} -(x+2) & \text{if } x < -2 \\ x+2 & \text{if } x \geq -2 \end{cases}$

$|x-4| = \begin{cases} -(x-4) & \text{if } x < 4 \\ x-4 & \text{if } x \geq 4 \end{cases}$

The previous absolute values divide the number line into 3 pieces; $x < -2$, $-2 \leq x < 4$, and $x \geq 4$. Now consider the sum of the absolute values on each interval.

For $x < 2$

$$\begin{aligned} |x+2| + |x-4| &= -(x+2) + (-(x-4)) \\ &= -x - 2 - x + 4 \\ &= -2x + 2 \end{aligned}$$

For $-2 \leq x < 4$

$$\begin{aligned} |x+2| + |x-4| &= x + 2 + (-(x-4)) \\ &= x + 2 - x + 4 \\ &= 6 \end{aligned}$$

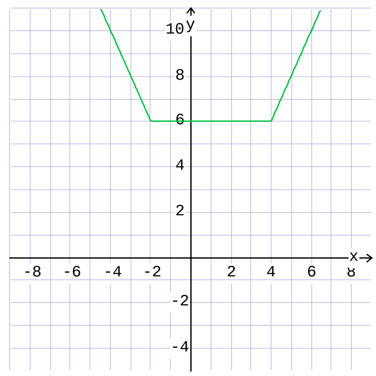
For $x \geq 4$

$$|x+2|+|x-4| = x+2+(x-4)$$
$$= 2x-2$$

Combining all 3 interval defines the absolute value equation as the piecewise function

$$|x+2|+|x-4| = \begin{cases} -2x+2 & \text{if } x < -2 \\ 6 & \text{if } -2 \le x < 4 \\ 2x-2 & \text{if } x \ge 4 \end{cases}$$

b.



8.

a. Given the point $(2,-3)$ on the line and parallel to the line with slope $m = \frac{-2-(-3)}{1-(-2)} = \frac{1}{3}$ use the point-slope form of the line to get

$$y-(-3) = \frac{1}{3}(x-2)$$
$$y+3 = \frac{1}{3}x - \frac{2}{3}$$
$$y = \frac{1}{3}x - \frac{11}{3}$$

b. Given the point $(2,-3)$ on the line and perpendicular to the line $y = -\frac{2}{3}x+2$, use the point-slope form of the line to get

$$y-(-3) = \frac{3}{2}(x-2)$$
$$y+3 = \frac{3}{2}x-3$$
$$y = \frac{3}{2}x-6$$

9.

a. Cost varies directly as its weight implies $C(W) = kW$ for some constant k.

$$C(.45) = .45k = .75$$
$$k = \frac{.75}{.45} = \frac{5}{3}$$

So, $C(W) = \frac{5}{3}W$.

b. For $W = 0.56$, $C(0.56) = \frac{5}{3}(0.56) = \frac{14}{15} \approx \$0.93$

10. Wind resistance varies directly as the square of the velocity has equation $R(v) = kv^2$, where R is the wind resistance, v is the velocity, and k is a constant.

$$R(50) = k(50)^2 = 60$$
$$k = \frac{60}{2500} = \frac{3}{125}$$

Then $R(v) = \frac{3}{125}v^2$.

11. $V(t) = mt + b$, where V is the value of the copy machine, t is the number of years since the purchase, b is the initial cost, and m is the rate of depreciation.

a. With a purchase price of \$50000, $b = 50,000$.

$$V(5) = m(5) + 50,000 = 20,000$$

$$m = \frac{20,0000 - 50,000}{5} = -6,000$$

So, $V(t) = -6,000t + 50,000$

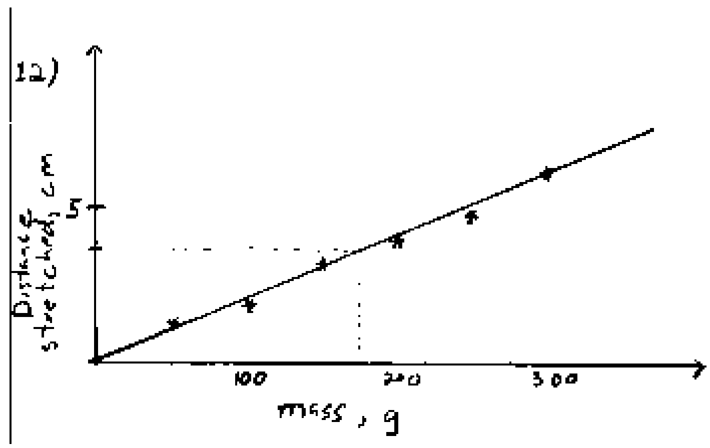
b.

$$V(7) = -6000(7) + 50,000$$
$$= 8,000$$

The value of the copy machine after 7 years will be \$8,000.

12.

a.



b. An estimated best fit line is drawn on the graph.

c. From the estimated best fit line, the spring stretches about 3.6 cm when a mass of 175 g is hung from the spring.

d. Using LinReg $(ax + b)$ gives

$$y = ax + b$$
$$a = 0.0197714286$$
$$b = 0.7333333333$$

e. If $x = 175$ g, then $y = 3.53333333$ cm.

13.

a.

$$d(A,B) = \sqrt{(2-(-6))^2 + (-3-5)^2}$$
$$= \sqrt{64+64}$$
$$= \sqrt{128}$$
$$= 8\sqrt{2}$$

b. $M = \left(\frac{2+(-6)}{2}, \frac{-3+5}{2}\right) = (-2,1)$

c. Graphing the points A and B one sees to find the point 7/8 the distance from A to B

$$x = 2 + \frac{7}{8}(-6 - 2) = -5$$

$$y = -3 + \frac{7}{8}(5 - (-3)) = 4$$

The point $(-5, 4)$ is 7.8 the distance from A to B.

d. $(x - 2)^2 + (y + 3)^2 = 68$

14. Analytic Test for x-axis symmetry: Replace y with $-y$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric about the x-axis.

$$2x + 4(-y)^2 = 9$$
$$2x + 4y^2 = 9$$

15. Analytic Test for y-axis symmetry: Replace x with $-x$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric about the y-axis.

$$4y - (-x)^2 = 3$$
$$4y - x^2 = 3$$

16. Analytic Test for symmetry about the origin: Replace x with $-x$ and y with $-y$ in the equation and simplify. If an equivalent equation results, the graph of the equation is symmetric about the origin.

$$(-x)^2 + 4(-y)^2 = 9$$
$$x^2 + 4y^2 = 9$$

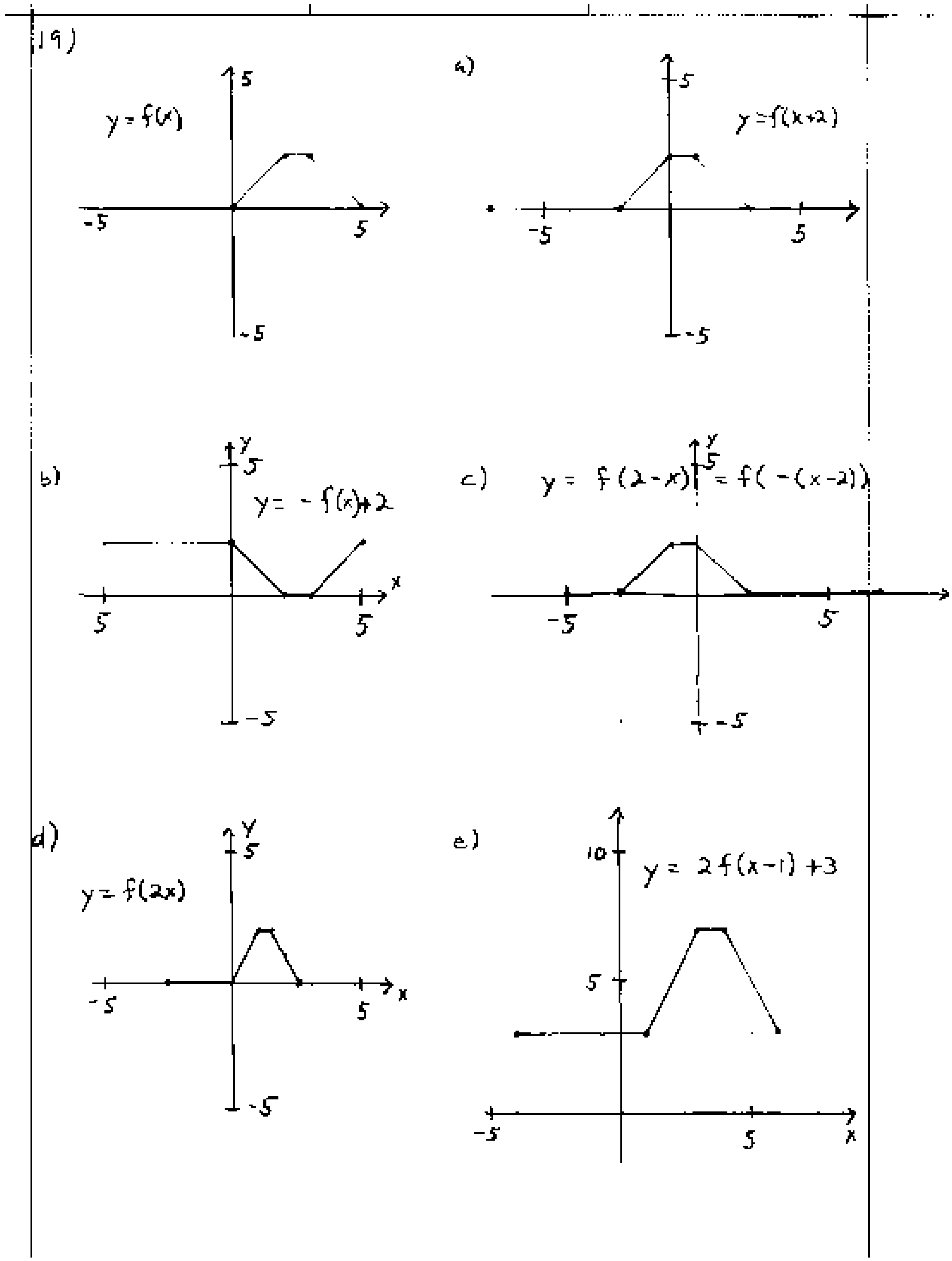
17. The function f is an **even** function if and only if $f(-x) = f(x)$.

$$\begin{aligned} f(-x) &= (-x)^4 + 8(-x)^2 - 16 \\ &= x^4 + 8x^2 - 16 \\ &= f(x) \end{aligned}$$

18. The function f is an **odd** function if and only if $f(-x) = -f(x)$.

$$\begin{aligned} f(-x) &= (-x)^5 - 2(-x)^3 \\ &= -x^5 - 2(-x^3) \\ &= -x^5 + 2x^3 \\ &= -(x^5 - 2x^3) \\ &= -f(x) \end{aligned}$$

19.



20.

a.

$$
\begin{aligned}
(f+g)(x) &= f(x) + g(x) \\
&= \frac{x+1}{x-1} + \frac{2x}{x+3} \\
&= \frac{(x+1)(x+3) + 2x(x-1)}{(x-1)(x+3)} \\
&= \frac{x^2 + 4x + 3 + 2x^2 - 2x}{(x-1)(x+3)} \\
&= \frac{3x^2 + 2x + 3}{(x-1)(x+3)}
\end{aligned}
$$

b.

$$
\begin{aligned}
(f \circ g)(x) &= f\left(\frac{2x}{x+3}\right) \\
&= \frac{\frac{2x}{x+3} + 1}{\frac{2x}{x+3} - 1} \\
&= \frac{\frac{2x}{x+3} + \frac{x+3}{x+3}}{\frac{2x}{x+3} - \frac{x+3}{x+3}} \\
&= \frac{3x+3}{x-3} \\
&= \frac{3(x+1)}{x-3}
\end{aligned}
$$

c.

$$
\begin{aligned}
(g \circ f)(x) &= g\left(\frac{x+1}{x-1}\right) \\
&= \frac{2\left(\frac{x+1}{x-1}\right)}{\left(\frac{x+1}{x-1}\right) + 3} \\
&= \frac{2\left(\frac{x+1}{x-1}\right)}{\frac{x+1+3(x-1)}{x-1}} \\
&= \frac{2(x+1)}{4x-2} \\
&= \frac{x+1}{2x-1}
\end{aligned}
$$

21. Let $f(x) = \sqrt{x}$ and $g(x) = 2x + 3$. Then $(f \circ g)(x) = f(g(x)) = f(2x+3) = \sqrt{2x+3}$