College of the Redwoods
Mathematics Department
Math 30—College Algebra

College Algebra Exam #4
Systems and Matrices

David Arnold
Donald Hickethier

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Instructions *This examination is open book, open notes, and untimed. You may use any resources you wish,but you must do all of the work yourself. You may not ask for, nor receive, aid from tutors, colleagues, or other teachers. With the exception of exercises 4 and 5, all calculations are to be performed by hand. Of course, you may check your work with your calculator, but you may not perform the work with your calculator. Place the solution to each of the following exercises on a separate sheet of paper. You must follow directions explicitly and show all work to receive full credit. Create a cover page with the following honor pledge.*

> I promise that all work found herein is my own. I have received no help from tutors, colleagues, or other teachers.

Arrange your papers in order, sign your honor pledge, then staple your *cover sheet to the top of your examination (10 points).*

Exercise 1. Consider the linear system

$$\begin{aligned} x + 2y - z &= 4 \\ y + z &= 8 \\ x - 2y &= 4 \end{aligned}$$

(a) Set up the augmented matrix for the system. Place the augmented matrix in row-echelon form.

(b) Write the system represented by the row-echelon form developed in part (a). Use back substitution to solve this system.

Exercise 2. Consider the linear system

$$\begin{aligned} x + y - z &= 10 \\ y + 2z &= 10 \\ x + 2y + z &= 20 \end{aligned}$$

(a) Set up the augmented matrix for the system. Place the augmented matrix in row-echlon form.

(b) Write the system represented by the row-echelon form developed in part (a). Use back substitution to solve this system.

Exercise 3. Consider the linear system

$$\begin{aligned} x + 2y + 4z &= 8 \\ x - 4y &= 4 \\ 3x + 8z &= 24 \end{aligned}$$

(a) Set up the augmented matrix for the system. Place the augmented matrix in row-echlon form.

(b) Write the system represented by the row-echelon form developed in part (a). Use back substitution to solve this system.

Exercise 4. Elizabeth has 43 coins in her purse, all nickels, dimes, or quarters. The value of the coins is \$5.70. She has 4 more quarters than dimes.

(a) Set up three equations in three unknowns that model this problem situation.

(b) State the augmented matrix for the system developed in part (a). Put this into your calculator and compute the reduced row-echelon form. Place this result on your examination paper and interpret the result.

Exercise 5. Jason invests \$5,800 in three separate funds having yearly interest rates 5%, 6%, and 6.5%, respectively. The total interest from all three investments at the end of one year is \$350. Finally, the amount invested at 6.5% is twice that invested at 6%.

(a) Set up three equations in three unknowns that model this problem situation.

(b) State the augmented matrix for the system developed in part (a). Put this into your calculator and compute the reduced row-echelon form. Place this result on your examination paper and interpret the result.

Exercise 6. Consider the system

$$2x + y = 4$$
$$3x + y = 6$$

(a) Set up a matrix equation representing the system.

(b) Use all hand calculations to find the inverse of the coefficient matrix of the matrix equation developed in part (a).

(c) Mutiliply both sides of the equation developed in part (a) by the inverse of the coefficient matrix found in part (b). Simplify and state the solution to the system.

Exercise 7. Evaluate the determinant of

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -5 \\ 1 & 1 & -2 \end{bmatrix}$$

by expanding across the second row.

Exercise 8. Use Cramer's rule to solve the system

$$2x + 3y = 6,$$
$$3x - 4y = 12,$$

Solutions to Exercises

Exercise 1(a) The augmented matrix is

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 1 & 8 \\ 1 & -2 & 0 & 4 \end{bmatrix}.$$

Multiply row one by -1 and add the result to row two.

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 1 & 8 \\ 0 & -4 & 1 & 0 \end{bmatrix}$$

Multiply row two by 4 and add the result to row three.

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 1 & 8 \\ 0 & 0 & 5 & 32 \end{bmatrix}$$

Multiply row three by 1/5.

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & 1 & 8 \\ 0 & 0 & 1 & 32/5 \end{bmatrix}$$

□

Exercise 1(b) The row-echelon form represents the following system.

$$x + 2y - z = 4 \quad (1)$$

$$y + x = 8 \quad (2)$$

$$z = 32/5 \quad (3)$$

Substitute $z = 32/5$ in equation (2).

$$y + \frac{32}{5} = 8$$

$$y = \frac{8}{5}$$

Substitute $z = 32/5$, $y = 8/5$ in equation (1).

$$x + 2\left(\frac{8}{5}\right) - \frac{32}{5} = 4$$

$$x + \frac{16}{5} - \frac{32}{5} = \frac{20}{5}$$

$$x = \frac{36}{5}$$

Thus, the system has a unique solution

$$(x, y, z) = \left(\frac{36}{5}, \frac{8}{5}, \frac{32}{5}\right).$$

□

Exercise 2(a) The augmented matrix is

$$\begin{bmatrix} 1 & 1 & -1 & 10 \\ 0 & 1 & 2 & 10 \\ 1 & 2 & 1 & 20 \end{bmatrix}.$$

Multiply row one by -1 and add the result to row three.

$$\begin{bmatrix} 1 & 1 & -1 & 10 \\ 0 & 1 & 2 & 10 \\ 0 & 1 & 2 & 10 \end{bmatrix}$$

Multiply row two by -1 and add the result to row three.

$$\begin{bmatrix} 1 & 1 & -1 & 10 \\ 0 & 1 & 2 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

□

Exercise 2(b) The row-echelon form represents the following system.

$$x + y - z = 10 \quad (4)$$
$$y + 2z = 10 \quad (5)$$

Because z is free, let $z = t$, where t is any real number. Substitute $z = t$ in equation (5).

$$y + 2t = 10$$
$$y = 10 - 2t$$

Substitute $z = t$, $y = 10 - 2t$ in equation (4).

$$x + 10 - 2t - t = 10$$
$$x + 10 - 3t = 10$$
$$x = 3t$$

Thus, the system has many solutions, represented by

$$(x, y, z) = (3t, 10 - 2t, t),$$

where t is any real number. □

Exercise 3(a) The augmented matrix is

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & -4 & 0 & 4 \\ 3 & 0 & 8 & 24 \end{bmatrix}.$$

Multiply row one by -1 and add the result to row two. Also, multiply row one by -3 and add the result to row three.

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & -6 & -4 & -4 \\ 0 & -6 & -4 & 0 \end{bmatrix}$$

Multiply row two by $-1/6$.

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & -6 & -4 & 0 \end{bmatrix}$$

Multiply row two by 6 and add the result to row three.

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Multiply row three by 1/4.

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 2/3 & 2/3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

□

Exercise 3(b) The row-echelon form represents the following system.

$$x + 2y + 4z = 8 \tag{6}$$

$$y + \frac{2}{3}z = \frac{2}{3} \tag{7}$$

$$0x + 0y + 0z = 1 \tag{8}$$

Because equation (8) has no solutions, neither does the system. The system is *inconsistent*. □

Exercise 4(a) Let's first organize part of the information in a table. Let n, d, and q represent the number of nickels, dimes, and quarters in Elizabeth's purse, respectively.

Coin	Number	Value
Nickels	n	$5n$
Dimes	d	$10d$
Quarters	q	$25q$
Totals	43	570

The second and third columns of the table give us two equations. The fact that the number of quarters is 4 more than the number of dimes gives us $q = 4 + d$.

$$\begin{aligned} n + d + q &= 43 \\ 5n + 10d + 25q &= 570 \\ -d + q &= 4 \end{aligned}$$

□

Exercise 4(b) The augmented matrix is

$$\begin{bmatrix} 1 & 1 & 1 & 43 \\ 5 & 10 & 25 & 570 \\ 0 & -1 & 1 & 4 \end{bmatrix}.$$

The reduced row-echelon form of this matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 15 \end{bmatrix}.$$

Of course, this represents the system

$$n = 17,$$
$$d = 11,$$
$$q = 15,$$

which is the solution to the problem. Elizabeth has 17 nickels, 11 dimes, and 15 quarters. □

Exercise 5(a) Again, organize the information in tabular form. Let x, y, and z represent the dollar amount invested at 5%, 6%, and 6.5%, respectively.

Rate	Amount invested	Interest
5%	x	$0.05x$
6%	y	$0.06y$
6.5%	z	$0.065z$
Totals	5800	350

The second and third columns of the table give us two equations. A third equation is constructed from the fact that the amount invested at 6.5% is twice the amount invested at 6%; i.e., $z = 2y$.

$$\begin{aligned} x + y + z &= 5800 \\ 0.05x + 0.06y + 0.065z &= 350 \\ -2y + z &= 0 \end{aligned}$$

□

Exercise 5(b) The augmented matrix is

$$\begin{bmatrix} 1 & 1 & 1 & 5800 \\ 0.05 & 0.06 & 0.065 & 350 \\ 0 & -2 & 1 & 0 \end{bmatrix}.$$

The reduced row-echelon form of this matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 1300 \\ 0 & 1 & 0 & 1500 \\ 0 & 0 & 1 & 3000 \end{bmatrix}.$$

Of course, this represents the system

$$\begin{aligned} x &= 1300, \\ y &= 1500, \\ z &= 3000, \end{aligned}$$

which is the solution to the problem. Jason invested \$1300, \$1500, and \$3000, respectively. □

Exercise 6(a)

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

□

Exercise 6(b) Set up the augmented matrix $[A : I]$.

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

We need to place this matrix in reduced row-echelon form. Multiply the first row by 1/2.

$$\begin{bmatrix} 1 & 1/2 & 1/2 & 0 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

Multiply row one by -3 and add the result to row two.

$$\begin{bmatrix} 1 & 1/2 & 1/2 & 0 \\ 0 & -1/2 & -3/2 & 1 \end{bmatrix}$$

Multiply row two by -2.

$$\begin{bmatrix} 1 & 1/2 & 1/2 & 0 \\ 0 & 1 & 3 & -2 \end{bmatrix}$$

Multiply row two by $-1/2$ and add the result to row one.

$$\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 3 & -2 \end{bmatrix}$$

Therfore,

$$\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix}.$$

□

Exercise 6(c)

$$\begin{aligned} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 6 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} -1 & 1 \\ 3 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \end{bmatrix} \\ \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 2 \\ 0 \end{bmatrix} \end{aligned}$$

Thus, $x = 2$ and $y = 0$. □

Exercise 7.

$$\begin{aligned} \det A &= -3\begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} - (-5)\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}, \\ &= -3(2-2) + 1(-2-2) + 5(1-(-1)), \\ &= 0 - 4 + 10, \\ &= 6, \end{aligned}$$

Exercise 7

Exercise 8. Solving for x,

$$x = \frac{\begin{vmatrix} 6 & 3 \\ 12 & -4 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & -4 \end{vmatrix}} = \frac{-24 - 36}{-8 - 9} = \frac{60}{17}.$$

Solving for y,

$$y = \frac{\begin{vmatrix} 2 & 6 \\ 3 & 12 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 3 & -4 \end{vmatrix}} = \frac{24 - 18}{-8 - 9} = -\frac{6}{17}.$$

Exercise 8