College of the Redwoods
Mathematics Department
Math 30—College Algebra

Exam #1—Chapter 1

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Multiple Choice Questions

Directions: *In each of the following exercises, select the "best" answer and darken the corresponding oval on your scantron sheet.*

1. Given the equation $f(x) = x^2$, simplify $\dfrac{f(x) - f(a)}{x - a}$.

(a) $x - a$ (b) $1/(x - a)$ (c) $x + a$

(d) $1/(x + a)$ (e) None of these

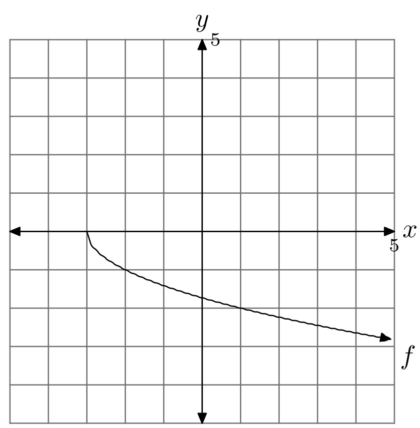
2. Consider the piecewise definition

$$f(x) = \begin{cases} -2 & \text{if } x < -1, \\ 3 - 2x, & \text{if } -1 \le x < 3, \\ 4, & \text{if } x \ge 3 \end{cases}.$$

What is $f(4)$?

(a) -2 (b) -5 (c) 5 (d) 4 (e) None of these

Use the following graph to answer questions 3–5.



3. What is $f(-2)$?

(a) -2 (b) 0 (c) -3 (d) 1 (e) None of these

4. What is the domain of f?

(a) $(-\infty, 0]$ (b) $[0, -\infty)$ (c) $(+\infty, 0]$

(d) $[-3, +\infty)$ (e) None of these

5. What is the range of f?

(a) $(-\infty, 0]$ (b) $[0, -\infty)$ (c) $(+\infty, 0]$

(d) $[-3, +\infty)$ (e) None of these

6. What is the distance between points A$(2, 3)$ and B$(3, -4)$?

(a) $2\sqrt{5}$ (b) $5\sqrt{2}$ (c) $3\sqrt{2}$

(d) $4\sqrt{5}$ (e) $2\sqrt{3}$

7. What is the midpoint of the segment joining points A$(2, -3)$ and B$(4, -1)$?

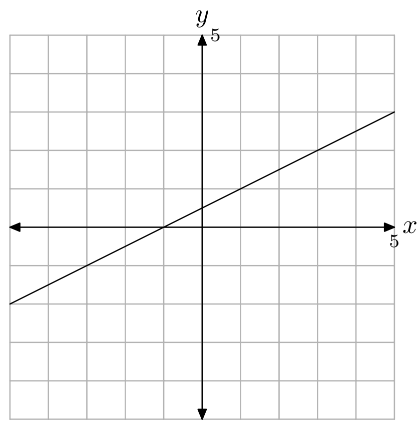
(a) $(3, -2)$ (b) $(-2, 3)$ (c) $(-1, -1)$

(d) $(1, 1)$ (e) None of these

8. What is the equation of the circle with center at C$(1, -2)$ that passes through the point $(4, 4)$?

(a) $(x-1)^2 + (y+2)^2 = 29$
(b) $(x+1)^2 + (y-2)^2 = 45$
(c) $(x-4)^2 + (y-4)^2 = \sqrt{45}$
(d) $(x-1)^2 + (y+2)^2 = 45$
(e) $(x-1)^2 + (y-2)^2 = 5$

Use the following figure to answer questions 9–10.



9. What is the slope of the line in the figure?

(a) $1/2$
(b) 2
(c) $3/2$
(d) $-1/2$
(e) -2

10. What is the equation of the line that passes through the point $(-1, 1)$ that is *perpendicular* to the line given in the figure?

(a) $y - 1 = (-1/2)(x+1)$
(b) $y - 1 = 2(x+1)$
(c) $y - 1 = -2(x+1)$
(d) $y + 1 = -2(x-1)$
(e) None of these

11. The graph of the relation defined by $x^3 + y^4 = 36$ is

(a) Symmetric with respect to the y-axis only.
(b) Symmetric with respect to the x-axis only.
(c) Symmetric with respect to the origin only.
(d) Symmetric with respect to the origin, the x-axis, and the y-axis.
(e) None of these

12. Given $f(x) = \sqrt{x}$ and $g(x) = 2x - 3$, then $(g \circ f)(x)$ equals

(a) $\sqrt{2x-3}$
(b) $x\sqrt{2x-3}$
(c) $2\sqrt{x} - 3$
(d) $2\sqrt{x-3}$
(e) None of these

Essay Questions

Directions: *Place the solution to each of the following exercises on your own paper. You must follow directions explicitly and show all work to receive full credit.*

EXERCISE 1. Consider the function defined by $f(x) = |x+1| + |x-3|$.

a) Create a piecewise definition for f.

b) Use the piecewise definition found in part (a) to help sketch the graph of f. Place your plot on graph paper. Label and scale each axis. *Note: This graph should be drawn without the aid of a calculator. No credit will be given for graphs that are simply copied from a calculator screen. However, it is all right to check your solution with your calculator.*

EXERCISE 2. Amy and Jim perform a physics experiment by releasing an object from rest. Each data point in the following table measures the speed s (in meters per second) of the object t seconds after it was released.

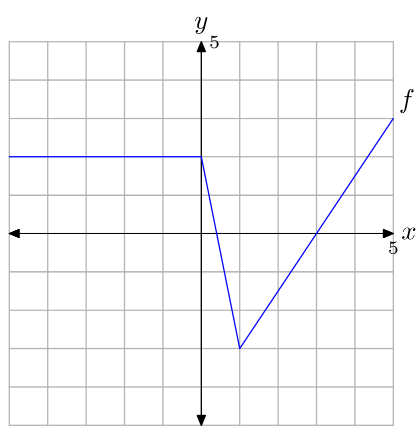
t	0	1	2	3	4
s	0	11.0	19.4	29.2	39.4

a) Set up a coordinate system on a sheet of graph paper. Label and scale each coordinate axis. Plot the data from the table on your coordinate system.

b) Draw a line that "fits" the data on the plot developed in part (a). Select two points *on the line* and use them to compute the slope of the line. What is the equation of your hand-drawn "line of best fit?" Record this answer on the plot developed in part (a) and clearly indicate that it is your hand-calculated solution.

c) Enter the data in your calculator. Use your calculator to find the line of best fit. Record this answer on the plot developed in part (a) and clearly indicate that this is your calculator solution.

EXERCISE 3. The graph of $y = f(x)$ is drawn in the following figure.



Sketch the graph of $y = -f(x+3) + 2$ on a sheet of graph paper. Be sure to accurately plot all key points.

Solutions to Quizzes

Solution to Question 1:

$$\begin{aligned}\frac{f(x)-f(a)}{x-a} &= \frac{x^2-a^2}{x-a} \\ &= \frac{(x+a)(x-a)}{x-a} \\ &= x+a, \text{ provided } x \neq a\end{aligned}$$

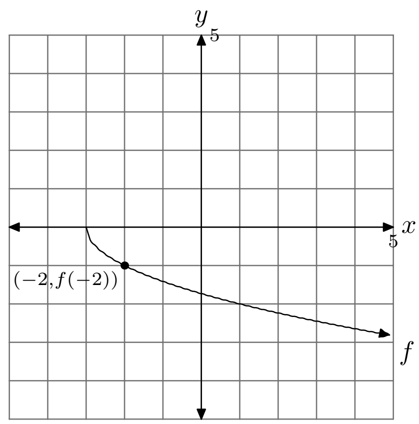
□

Solution to Question 2: Because $4 \geq 3$, $f(4)$ is computed by using the third piece of

$$f(x) = \begin{cases} -2 & \text{if } x < -1, \\ 3-2x, & \text{if } -1 \leq x < 3, \\ 4, & \text{if } x \geq 3 \end{cases}.$$

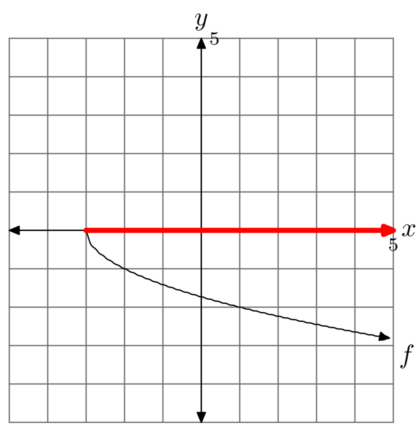
Consequently, $f(4) = 4$. □

Solution to Question 3: In the following figure, we've labeled the point $(-2, f(-2))$.



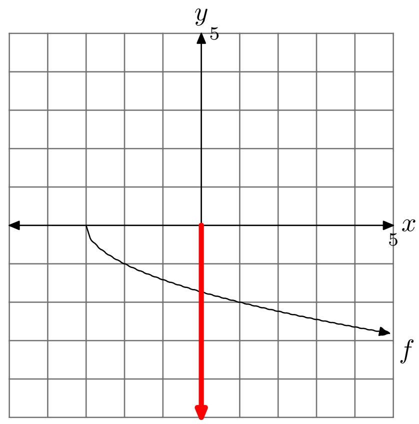
The y-value of the point $(-2, f(-2))$ equals $f(-2)$. Consequently, $f(-2) = -1$. □

Solution to Question 4: The domain is found by projecting each point of the graph onto the x-axis, as shown in the following figure.



Consequently, the domain is $[-3, +\infty)$. □

Solution to Question 5: The range is found by projecting each point of the graph onto the y-axis, as shown in the following figure.



Consequently, the domain is $(-\infty, 0]$. □

Solution to Question 6: Use the distance formula.

$$\begin{aligned} d &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{(2-3)^2 + (3-(-4))^2} \\ &= \sqrt{1+49} \\ &= \sqrt{50} \\ &= \sqrt{25}\sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

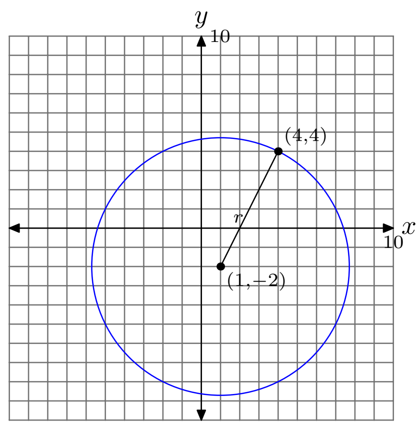
□

Solution to Question 7: Use the midpoint formula.

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2+4}{2}, \frac{-3+(-1)}{2} \right) \\ &= (3, -2) \end{aligned}$$

□

Solution to Question 8: Draw the circle.



Compute the radius with the distance formula.

$$\begin{aligned} r &= \sqrt{(4-1)^2 + (4-(-2))^2} \\ &= \sqrt{9+36} \\ &= \sqrt{45} \end{aligned}$$

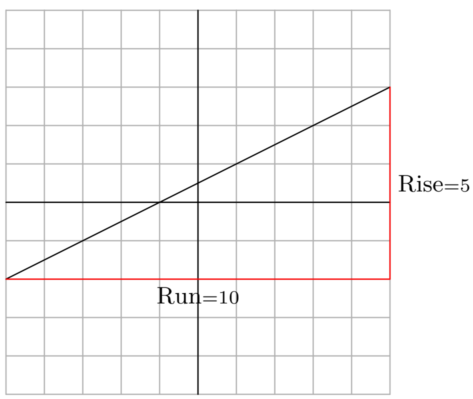
The equation of the circle with center at (h,k) and radius r is

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 \\ (x-1)^2 + (y-(-2))^2 &= \sqrt{45}^2 \\ (x-1)^2 + (y+2)^2 &= 45 \end{aligned}$$

□

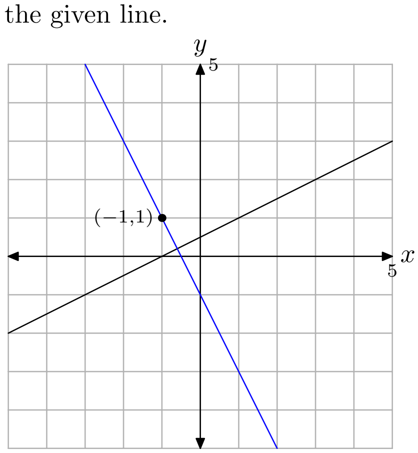
Solution to Question 9: The slope is calculated by dividing the rise by the run; i.e.,

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{5}{10} = \frac{1}{2}$$



□

Solution to Question 10: The line through $(-1,1)$ perpendicular to the given line must have a slope that is the negative reciprocal of that of the given line.



Consequently, the slope is -2 and the equation of the line is

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 1 &= -2(x-(-1)) \\ y - 1 &= -2(x+1) \end{aligned}$$

□

Solution to Question 11: Replace y with $-y$.

$$x^3 + y^4 = 36$$
$$x^3 + (-y)^4 = 36$$
$$x^3 + y^4 36$$

Because we get the same equation back, the graph must be symmetric with respect to the x-axis. □

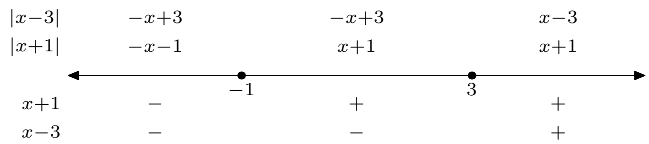
Solution to Question 12:

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x}) \\ &= 2\sqrt{x} - 3\end{aligned}$$

□

Solutions to Exercises

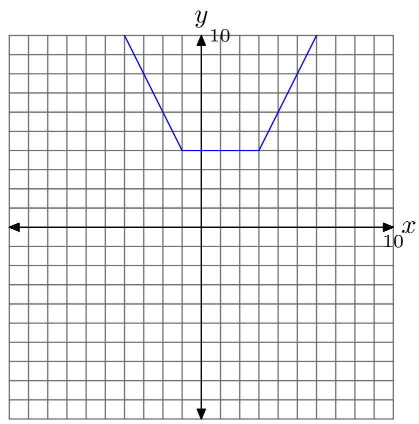
Exercise 1. With the aid of the following construct,



we can construct a piecewise function.

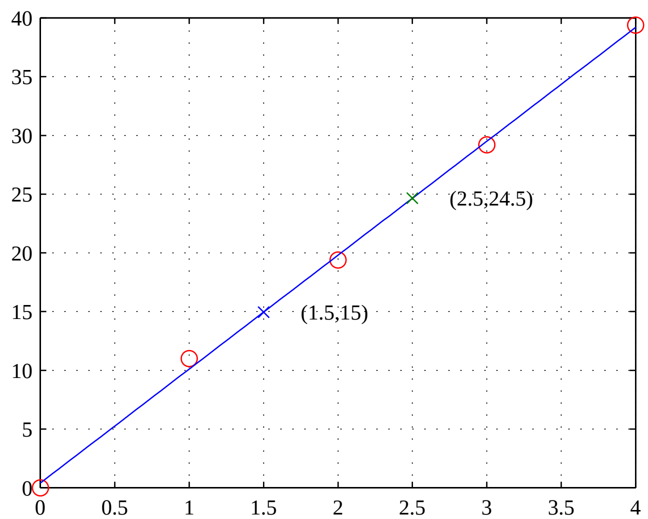
$$f(x) = \begin{cases} -2x+2 & \text{if } x < -1, \\ 4, & \text{if } -1 \le x < 3, \\ 2x-2, & \text{if } x \ge 3 \end{cases}$$

This leads to the following graph.



Exercise 1

Exercise 2. Plot the data and draw a "line of best fit."



I've selected two points on the line, $(1.5, 15)$ and $(2.5, 24.5)$. The slope of the line through these two points is calculated as follows:

$$m = \frac{24.5 - 15}{2.5 - 1.5}$$
$$m \approx 9.5$$

I estimate the y-intercept from the graph. It is approximately $b \approx 0.5$. Consequently, the equation of the line is

$$y = mx + b$$
$$y = 9.5x + 0.5$$

Of course, the vertical axis gives the displacement s, and the horizontal axis records the time, t. Consequently, the equation of the line is given as

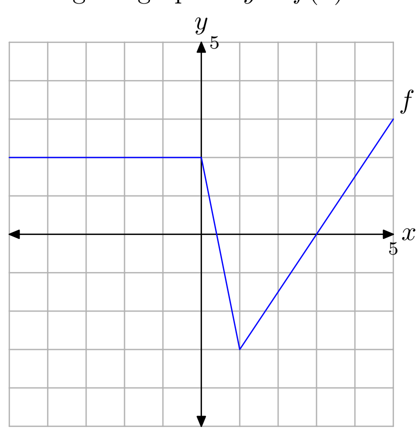
$$s = 9.5t + 0.5.$$

Enter the data from the table in a calculator.Use the linear regression feature of the calculator to generate the following result:

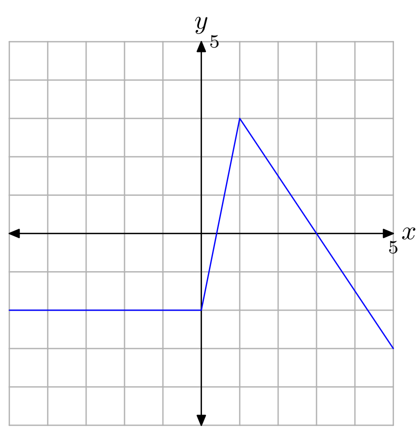
$$s = 9.7t + 0.4$$

Exercise 2

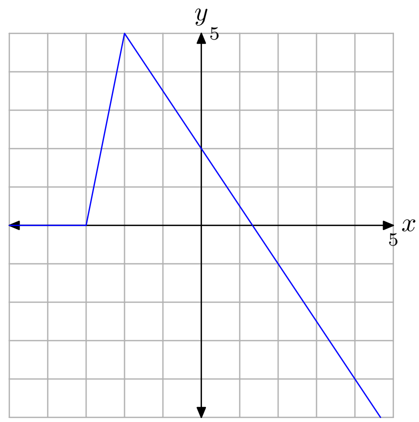
Exercise 3. For reference, here is the original graph of $y = f(x)$.



The graph of $y = -f(x)$ is reflected across the x-axis.



To draw the graph of $y = -f(x + 3) + 2$, take the last graph and shift it left 3 units, then up 2 units.



Exercise 3