College of the Redwoods
Mathematics Department
Math 30 — College Algebra

# Exam #2
# College Algebra

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## Multiple Choice Questions

**Directions**: *In each of the following exercises, select the "best" answer and darken the corresponding oval on your scantron sheet.*

**1.** Simplify $(2+3i)(3-i)$.

(a) $9+7i$ (b) $7+9i$ (c) $6-9i$

(d) $9-2i$ (e) $2-9i$

**2.** Simplify

$$\frac{1}{2+i}.$$

(a) $\frac{1}{2}+\frac{1}{2}i$ (b) $\frac{1}{2}-\frac{1}{2}i$ (c) $\frac{2}{5}-\frac{1}{5}i$

(d) $\frac{2}{5}+\frac{1}{5}i$ (e) $2-i$

**3.** If $z=a+bi$, then $z\overline{z}$ equals

(a) $a^2-b^2$ (b) $(a^2-b^2)-2abi$ (c) $a^2+ib^2$

(d) $a-ib$ (e) $a^2+b^2$

**4.** Use the quadratic formula to find one of the zeros of $p(x) = z^2 - 2x + 10$.

(a) $3 + i$ (b) $1 - 3i$ (c) $2 + 5i$

(d) $2 - 4i$ (e) $1 - i$

**5.** One of the zeros of

$$p(x) = -5(x + 2)(x - 1)(x - 3)$$

is

(a) $-1$ (b) $2$ (c) $-2$

(d) $-3$ (e) $-5$

**6.** Find $k$ so that $x - 2$ is a factor of

$$p(x) = x^3 - 2x^2 + kx - 3.$$

(a) $1/2$ (b) $-1/2$ (c) $2/3$

(d) $3/2$ (e) $-3/2$

**7.** If $p/q$ is a rational zero of the polynomial

$$a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n,$$

then $q$ divides

(a) $a_0$ (b) $a_1$ (c) $a_2$

(d) $a_n$ (e) None of these

**8.** What is the remainder when $x^3 - x + 4$ is divided by $x^2 + 1$?

(a) $4$ (b) $-4$ (c) $-2x - 4$

(d) $2x + 4$ (e) $-2x + 4$

**9.** What is the solution of the inequality

$$(x + 2)(x - 5) > 0?$$

(a) $(-2, 5)$ (b) $(-5, 2)$

(c) $(-\infty, -5) \cup (2, +\infty)$ (d) $(-\infty, -2) \cup (5, +\infty)$

(e) None of the above

## Essay Questions

**Directions**: *Place the solution to each of the following exercises on your own paper. You must follow directions explicitly and show all work to receive full credit.*

EXERCISE 1. Consider the polynomial

$$f(x) = 2x^3 + 5x^2 + 6x + 2.$$

(a) Use the Rational Zeros Theorem to list all possible zeros of this polynomial. *Note*: I am not asking for the actual zeros, but the potential zeros predicted by the theorem.

(b) Use synthetic division to show that one of the possibilities presented in part (a) is actually a zero.

(c) Use the quadratic formula to find the remaining zeros.

(d) Sketch the graph of $f$ and use the graph to solve the inequality

$$2x^3 + 5x^2 + 6x + 2 > 0.$$

Use interval notation to describe the solution of the inequality.

Exercise 2. Consider the rational function

$$f(x) = \frac{x+1}{x^2 - 5x + 6}.$$

(a) List the zeros of the function.

(b) List the equations of the vertical asymptotes.

(c) Evaluate the limit

$$\lim_{x \to \pm\infty} \frac{x+1}{x^2 - 5x + 6}$$

and interpret the result.

(d) Sketch the graph of $f$. Label the zeros with their coordinates and all asymptotes with their equations.

(e) Use the sketch in part (d) to solve the inequality

$$\frac{x+1}{x^2 - 5x + 6} > 0.$$

Use interval notation to describe the solution of the inequality.

Use interval notation to describe the solution of the inequality.

EXERCISE 3. Consider the rational function

$$f(x) = \frac{x^2 - 4x - 5}{x - 2}.$$

(a) List the zeros of the function.

(b) List the equations of the vertical asymptotes.

(c) Find the equation of the slant asymptote. Show your work.

(d) Sketch the graph of $f$. Label the intercepts with their coordinates and all asymptotes with their equations.

(e) Use the graph in part (d) to solve the inequality

$$\frac{x^2 - 4x - 5}{x - 2} \leq 0.$$

Use interval notation to describe the solution of the inequality.

# Solutions to Multiple Choice Questions

**Solution to Question 1:** Multiply.

$$\begin{aligned}(2+3i)(3-i) &= 6-2i+9i-3i^2\\ &= 6-2i+9i-3(-1)\\ &= 6-2i+9i+3\\ &= 9+7i\end{aligned}$$

□

**Solution to Question 2:** Multiply both numerator and denominator by the conjugate of $2 + i$.

$$\begin{aligned} \frac{1}{2+i} &= \frac{1}{2+i} \cdot \frac{2-1}{2-1} \\ &= \frac{2-i}{4-i^2} \end{aligned}$$

But, $i^2 = -1$, so

$$\begin{aligned} \frac{1}{2+i} &= \frac{2-i}{4+1} \\ &= \frac{2-i}{5} \\ &= \frac{2}{5} - \frac{1}{5}i \end{aligned}$$

$\square$

**Solution to Question 3:** If $z = a + bi$, then the conjugate of $z$ is $\overline{z} = a - bi$. Thus,

$$\begin{aligned} z\overline{z} &= (a+bi)\overline{(a+bi)}, \\ &= (a+bi)(a-bi), \\ &= a^2 - b^2 i^2. \end{aligned}$$

However, $i^2 = -1$, so

$$\begin{aligned} z\overline{z} &= a^2 - b^2(-1), \\ &= a^2 + b^2. \end{aligned}$$

□

**Solution to Question 4:** The solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Thus, the solutions of $x^2 - 2x + 10 = 0$ are

$$z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2},$$

$$z = \frac{2 \pm \sqrt{4 - 40}}{2},$$

$$z = \frac{2 \pm \sqrt{-36}}{2},$$

$$z = \frac{2 \pm 6i}{2},$$

$$z = 1 \pm 3i.$$

□

**Solution to Question 5:** By the Factor Theorem, if $x - r$ is a factor of $p(x)$, then $r$ is a zero. Because factors of

$$p(x) = -5(x+2)(x-1)(x-3)$$

are $x+2$, $x-1$, and $x-3$, the zeros are $-2$, 1, and 3. □

**Solution to Question 6:** Because $x-2$ is to be a factor, we need 2 to be a zero. Set up and perform synthetic division, dividing by $x-2$.

$$\begin{array}{r|rrrr} \boxed{2} & 1 & -2 & k & -3 \\ & & 2 & 0 & 2k \\ \hline & 1 & 0 & k & 2k-3 \end{array}$$

If $x-2$ is a factor, then 2 is a zero, so the remainder must equal zero.

$$\begin{aligned} 2k-3 &= 0 \\ 2k &= 3 \\ k &= \frac{3}{2} \end{aligned}$$

□

**Solution to Question 7:** This problem simply requires that you know the hypothesis and conclusion of the Rational Zeros Theorem.

*Rational Zeros Theorem*: If

$$p(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$$

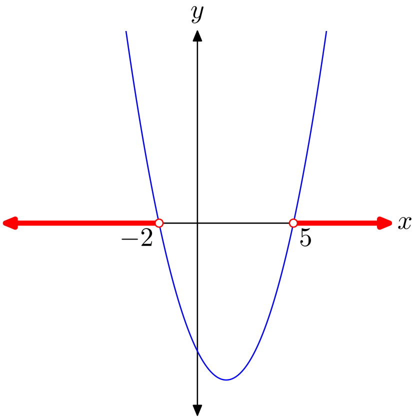
is a polynomial with integer coefficients, then in $p/q$ is a rational zero, then $p$ provides $a_0$ and $q$ divides $a$. □

**Solution to Question 8:** Use long division to find the remainder.

$$
\begin{array}{rrrrrrrrrrr}
 & & & x & & & & & & & \\
\cline{4-11}
x^2 & + & 1 & )x^3 & + & 0x^2 & - & x & + & 4 \\
 & & & x^3 & & & + & x & & \\
\cline{4-11}
 & & & & & & - & 2x & + & 4
\end{array}
$$

Thus, the remainder is $-2x + 4$. □

**Solution to Question 9:** The graph of the polynomial $p(x) = (x + 2)(x - 5)$ is a parabola opening upward, that passes through $(-2, 0)$ and $(5, 0)$.




The solution of $(x + 2)(x - 5) > 0$ is found by noting where the graph of $p$ is above the $x$-axis. Thus, the solution is

$$\{x : x < -2 \text{ or } x > 5\} = (-\infty, -2) \cup (5, +\infty)$$

□

# Solutions to Exercises

**Exercise 1(a)** Since the polynomial has integer coefficients, if $p/q$ is a rational zero, then $p$ divides 2 and $q$ divides 2. Thus,

$$p = \pm 1, \pm 2,$$
$$q = \pm 1, \pm 2,$$
$$p/q = \pm 1, \pm 1/2, \pm 2.$$

□

**Exercise 1(b)** Because

$$\begin{array}{c|cccc} \boxed{\text{-1/2}} & 2 & 5 & 6 & 2 \\ & & -1 & -2 & -2 \\ \hline & 2 & 4 & 4 & 0 \end{array}$$

we have that $-1/2$ is a zero. $\square$

**Exercise 1(c)** Because $-1/2$ was found as a zero in part (b) provides the following factorization.

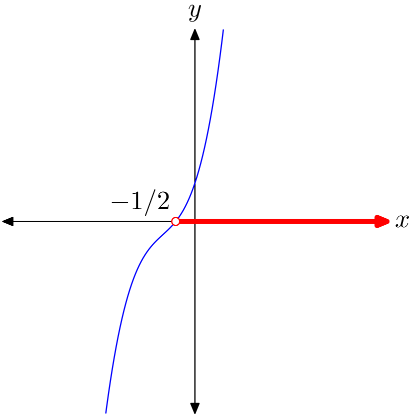
$$f(x) = \left(x + \frac{1}{2}\right)(2x^2 + 4x + 4)$$
$$= 2\left(x + \frac{1}{2}\right)(x^2 + 2x + 2)$$

Factoring out a 2 as we've done will save some time normally spent in simplifying the upcoming zeros. To complete the solution, use the quadratic formula to find the zeros of $x^2 + 2x + 2 = 0$.

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{-2 \pm \sqrt{4 - 8}}{2}$$
$$x = \frac{-2 \pm \sqrt{-4}}{2}$$
$$x = \frac{-2 \pm \sqrt{2i}}{2}$$
$$x = -1 \pm i$$

Thus, the remaining zeros are $-1 + i$ and $-1 - i$. □

**Exercise 1(d)** The leading term $2x^3$, so the graph's end-behavior matches that of its leading term. Because it has odd degree, the graph of the polynomial must rise from negative infinity, wiggle through its real zeros, then rise to positive infinity.



The graph of $f$ is *above* the $x$-axis to the right of 1/2. Thus, the solution of $2x^3 + 5x^2 + 6x + 2 > 0$ is

$$\left\{ x : x > \frac{1}{2} \right\} = \left( \frac{1}{2}, +\infty \right).$$

□

**Exercise 2(a)** Since $f(x)$ is reduced to lowest terms,

$$f(x) = \frac{x+1}{(x-3)(x-2)},$$

whatever makes the numerator zero is a zero of the function. Thus, $x = -1$ is the only zero of the function. □

**Exercise 2(b)** Whatever makes the denominator of a rational function zero introduces a vertical asymptote. Thus, there are two vertical asymptotes of

$$f(x) = \frac{x+1}{(x-3)(x-2)},$$

namely, $x = 3$ and $x = 2$. □

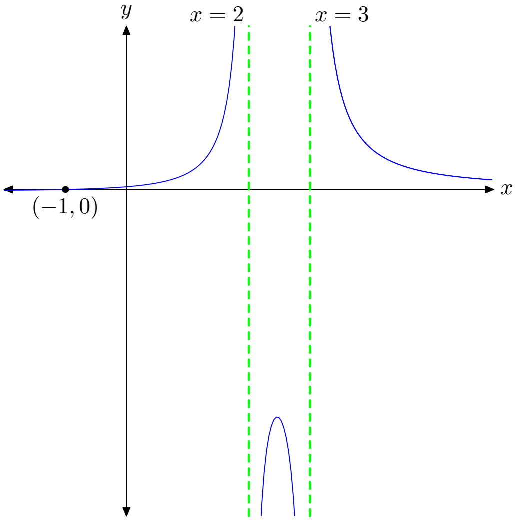
**Exercise 2(c)** Because the degree of the numerator is less than or equal to the degree of the denominator, divide both numerator and denominator by the largest power of $x$ present.

$$\begin{aligned}\lim_{x\to\pm\infty}\frac{x+1}{x^2-5x+6} &= \lim_{x\to\pm\infty}\frac{\dfrac{x}{x^2}+\dfrac{1}{x^2}}{\dfrac{x^2}{x^2}-\dfrac{5x}{x^2}+\dfrac{6}{x^2}}\\ &= \lim_{x\to\pm\infty}\frac{\dfrac{1}{x}+\dfrac{1}{x^2}}{1-\dfrac{5}{x}+\dfrac{6}{x^2}}\\ &= \frac{0+0}{1-0+0}\\ &= \frac{0}{1}\\ &= 0\end{aligned}$$

Therefore, the graph has a horizontal asymptote with equation $y = 0$.

□

**Exercise 2(d)**



A table of points helps to make the following plot.

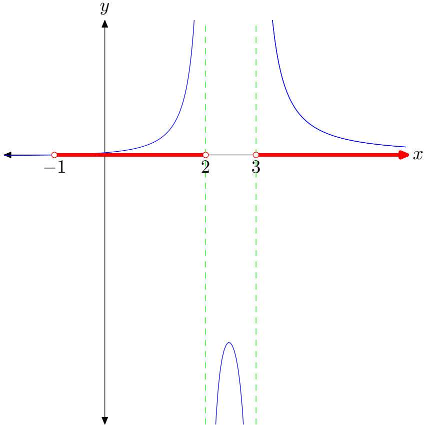
| $x$ | $-2$ | $0$ | $2.5$ | $4$ |
|---|---|---|---|---|
| $y$ | $-1/20$ | $1/6$ | $-14$ | $2.5$ |

□

**Exercise 2(e)** To find the solutions of

$$\frac{x+1}{x^2-5x+6} > 0,$$

shade those $x$-values on the $x$-axis where the graph of the rational function is *above* the $y$-axis.




Thus, the solution is

$$\{x : -1 < x < 2 \text{ or } x > 3\} = (-1, 2) \cup (3, +\infty).$$

□

**Exercise 3(a)** Factor the numerator.

$$f(x) = \frac{x^2 - 4x - 5}{x - 2} \quad = \frac{(x - 5)(x + 1)}{x - 2}$$

The zeros of $f$ are those values of $x$ that make the numerator zero; namely $x = 5$ and $x = -1$. □

**Exercise 3(b)** The vertical asymptotes are found by identifying the values of $x$ that make the denominator of

$$f(x) = \frac{(x-5)(x+1)}{x-2}$$

equal to zero. Thus, we have a vertical asymptote with equation $x = 2$. □

**Exercise 3(c)** The degree of the numerator of

$$f(x) = \frac{x^2 - 4x - 5}{x - 2}$$

is larger than the degree of the denominator. Divide.

$$\begin{array}{r|rrr} \boxed{2} & 1 & -4 & -5 \\ & & 2 & -4 \\ \hline & 1 & -2 & -9 \end{array}$$

Thus,

$$f(x) = x - 2 - \frac{9}{x - 2}$$

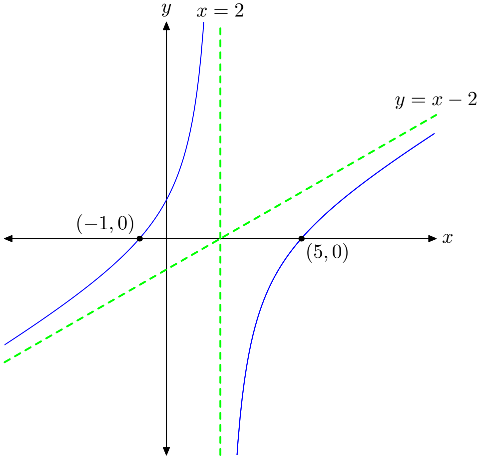
and, as $x \to \pm\infty$,

$$f(x) \approx x - 3.$$

Therefore, the graph has a *slant* asymptote with equation $y = x - 2$.

□

**Exercise 3(d)** There is enough information to craft the following sketch.

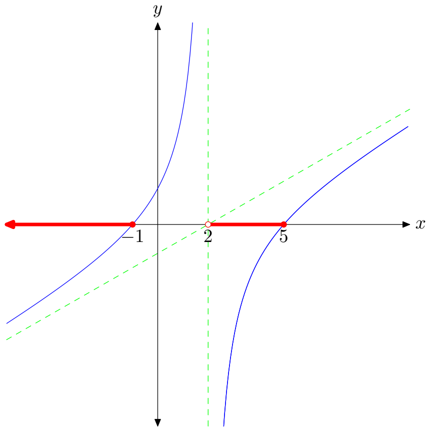



□

**Exercise 3(e)** To find the solution of

$$\frac{x^2 - 4x - 5}{x - 2} \geq 0,$$

Note where the graph is *below* or *on* the $x$-axis and mark the associated $x$-values on the $x$-axis.




Therefore, the solution is

$$\{x : x \leq -1 \text{ or } 2 < x \leq 5\} = (-\infty, -1] \cup (2, 5].$$

□