College of the Redwoods
Mathematics Department
Math 30 — College Algebra

Exam #1
College Algebra

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Multiple Choice Questions

Directions: *In each of the following exercises, select the "best" answer and darken the corresponding oval on your scantron sheet.*

1. What is the domain of the function defined by $f(x) = \sqrt{4-x}$?

(a) $(-\infty, +\infty)$ (b) $(-\infty, 4]$ (c) $(-\infty, 4)$

(d) $(4, +\infty)$ (e) $[4, +\infty)$

2. Consider the following piecewise definition of the function f.

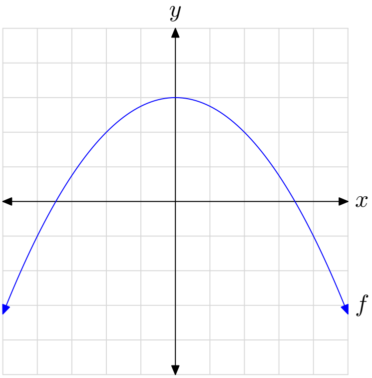
$$f(x) = \begin{cases} 3 - x, & \text{if } x < 0, \\ x^2 + 2, & \text{if } x \geq 0. \end{cases}$$

Evaluate $f(-3)$.

(a) 6 (b) 0 (c) 11

(d) -7 (e) None of these

3. Consider the following graph of f.



Evaluate $f(-2)$.

(a) -2 (b) -1 (c) 0

(d) 1 (e) 2

4. What is the range of the function pictured in Exercise 3?

(a) $(-\infty, +\infty)$ (b) $(-3, 3)$ (c) $(-\infty, -3) \cup (3, +\infty)$

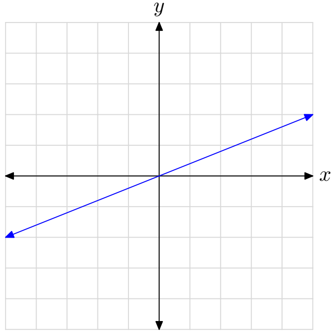
(d) $[3, +\infty)$ (e) $(-\infty, 3]$

5. What is the y-intercept of the line having standard form $2x+5y = 10$?

(a) 10 (b) 5 (c) 2

(d) $-2/5$ (e) $5/2$

6. Find the equation of the line that passes through the point $(3, -2)$ and is perpendicular to the line pictured below.



(a) $y + 2 = \frac{2}{5}(x - 3)$

(b) $y + 2 = -\frac{5}{2}(x - 3)$

(c) $y - 2 = -\frac{5}{2}(x + 3)$

(d) $y - 2 = \frac{2}{5}(x + 3)$

(e) $y + 2 = -\frac{2}{5}(x - 3)$

7. What is the midpoint of the line segment joining the points $(2, -3)$ and $(4, 5)$?

(a) $(-1, 3)$ (b) $(-4, 1)$ (c) $(1, 4)$

(d) $(3, 1)$ (e) $(3, -1)$

8. The graph of the relation defined by $x^4 + 5y^2 = 12$ is

(a) Symmetric with respect to the x-axis only

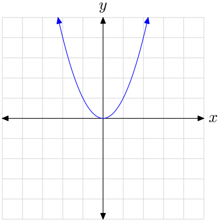
(b) Symmetric with respect to the y-axis only

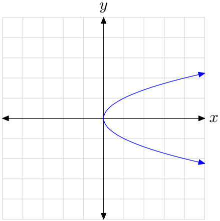
(c) Symmetric with respect to the origin only

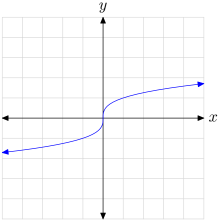
(d) Symmetric with respect to the origin, x-axis, and y-axis

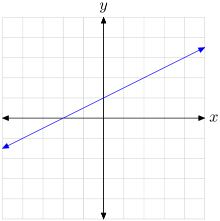
(e) None of these

9. Which of the following functions is an odd function?

(a) 

(b) 

(c) 

(d) 

(e) None of these

Essay Questions

Directions: *Place the solution to each of the following exercises on your own paper. You must follow directions explicitly and show all work to receive full credit.*

Exercise 1. Consider the function defined by $f(x) = x^2 + 2x$. Simplify the expression

$$\frac{f(x+h) - f(x)}{h}$$

as much as possible. For what values of h is your argument valid?

Exercise 2. Consider the function defined by $f(x) = |x+1| - |x-2|$.

(a) Create a piecewise definition for f.

(b) Use the piecewise definition found in part (a) to help sketch the graph of f. Place your plot on graph paper. Label and scale each axis. *Note: This graph should be drawn without the aid of a calculator. No credit will be given for graphs that are simply copied from a calculator screen. However, it is all right to check your solution with your calculator.*

EXERCISE 3. Amy and Jim perform a physics experiment by hanging various masses to the end of a spring and measuring the vertical distance that the spring stretches from its equilibrium point. Here is their data.

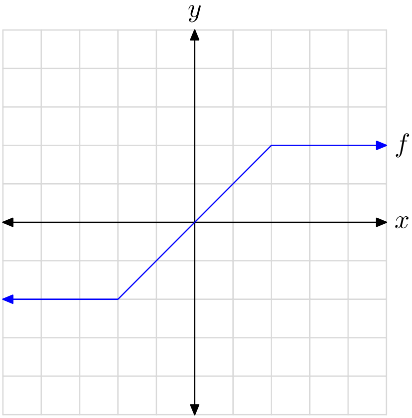
m (grams)	10	20	30	40	50
x (centimeters)	1.3	3.3	5.4	6.3	8.1

(a) Set up a coordinate system on a sheet of graph paper. Label and scale each coordinate axis. Plot the data from the table on your coordinate system.

(b) Draw a line that "fits" the data on the plot developed in part (a). Your line should just be a rough guess, but you should attempt to balance the errors, with some data points appearing above the line, others below. Select two points *on the line* and use them to compute the slope of the line. Using hand calculations only, what is the equation of your "line of best fit?" Record this answer on the plot developed in part (a) and clearly indicate that it is your hand-calculated solution.

(c) Enter the data in your calculator. Use your calculator to find the equation of the line of best fit. Record this answer on the plot developed in part (a) and clearly indicate that this is your calculator solution.

Exercise 4. The graph of $y = f(x)$ is drawn in the following figure.



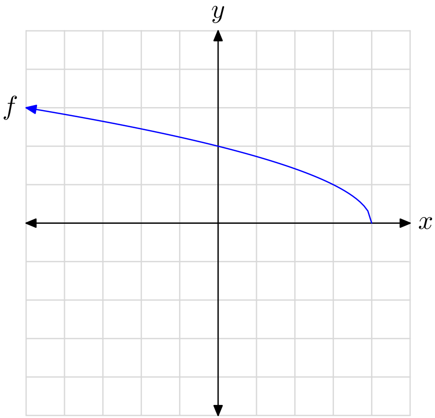
Sketch the graph of $y = -f(x-2) - 1$ on a sheet of graph paper. Be sure to correctly plot all key points.

Solutions to Multiple Choice Questions

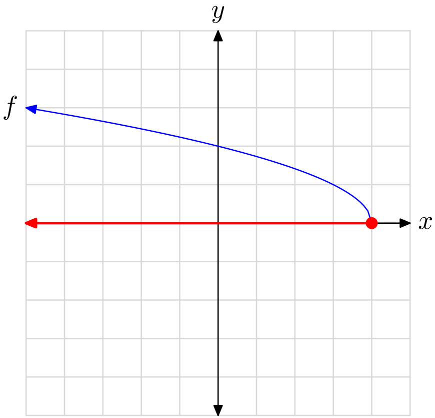
Solution to Question 1: If $f(x) = \sqrt{4-x}$, you can find the domain by noting that you cannot take the square root of a negative number. Therefore,

$$\begin{aligned} 4 - x &\geq 0 \\ -x &\geq -4 \\ x &\leq 4 \end{aligned}$$

Alternatively, you can sketch the graph of f



and project all points on the graph of f onto the x-axis.



Either way, the domain of f is

$$D = \{x : x \leq 4\} = [4, +\infty)$$

□

Solution to Question 2: If

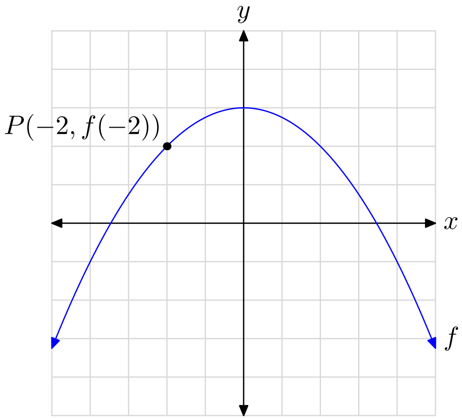
$$f(x) = \begin{cases} 3 - x, & \text{if } x < 0, \\ x^2 + 2, & \text{if } x \geq 0, \end{cases}$$

then we evaluate $f(-3)$ by substituting -3 for x in the first piece $(-3 < 0)$. Thus,

$$\begin{aligned} f(-3) &= 3 - (-3), \\ &= 3 + 3, \\ &= 6. \end{aligned}$$

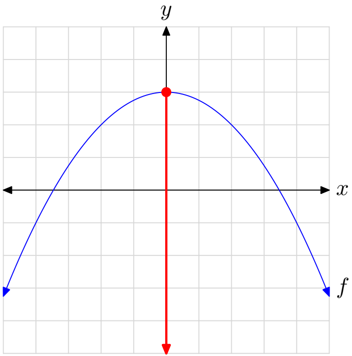
□

Solution to Question 3: Locate the point $(-2, f(02))$ on the graph of f.



Because $f(-2)$ is the y-value of point P, $f(-2) = 2$. □

Solution to Question 4: Project all points on the graph of f onto the y-axis.



Thus, the range of f is

$$R = \{y : y \leq 3\} = (-\infty, 3].$$

□

Solution to Question 5: If you let $x = 0$, then

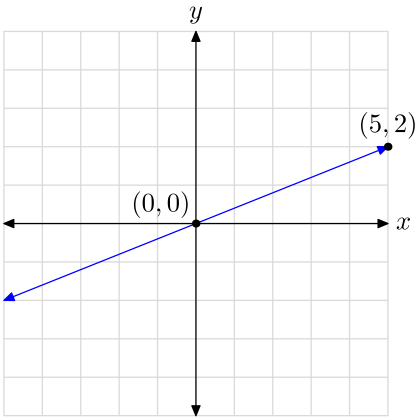
$$\begin{aligned} 2x + 5y &= 10, \\ 2(0) + 5y &= 10, \\ 5y &= 10, \\ y &= 2. \end{aligned}$$

Alternatively, place the equation in the form $y = mx + b$ as follows:

$$\begin{aligned} 2x + 5y = 10,& \\ 5y &= -x + 10, \\ y &= -\frac{2}{5}x + 2. \end{aligned}$$

Thus, the y-intercept is 2. □

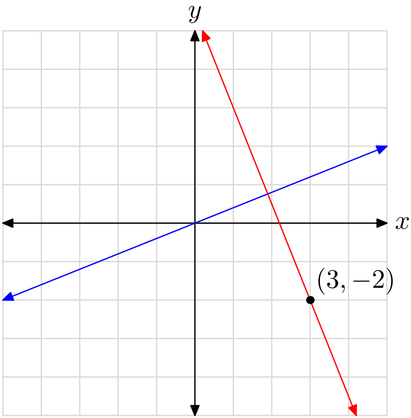
Solution to Question 6: Note that the line



passes through the point $(0,0)$ and $(5,2)$. Thus, the line has slope

$$m = \frac{2-0}{5-0}$$
$$m = \frac{2}{5}$$

If two lines are perpendicular, then the product of their slopes is -1 Thus, the line we seek has slope $-5/2$ and passes through the point $(3,-2)$.

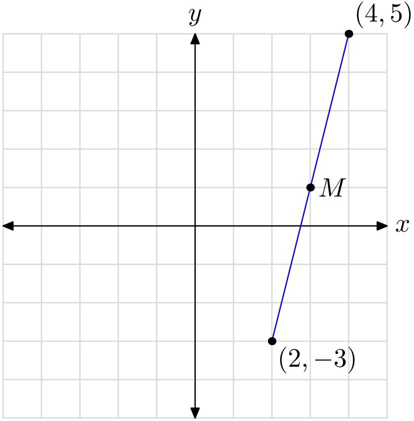

y
x
(3, −2)

Thus, the equation of the line is

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - (-2) &= -\frac{5}{2}(x - 3) \\ y + 2 &= -\frac{5}{2}(x - 3) \end{aligned}$$

□

Solution to Question 7: The midpoint of the segment joining $(2, -3)$ and $(4, 5)$ is



$$m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$m = \left(\frac{2 + 4}{2}, \frac{-3 + 5}{2}\right)$$

$$m = (3, 1)$$

□

Solution to Question 8: If you replace x with $-x$, then you get the same equation back.

$$x^4 + 5y^2 = 12$$
$$(-x)^4 + 5y^2 = 12$$
$$x^4 + 5y^2 = 12$$

Therefore, the graph is symmetric with respect to the y-axis. However, if you replace y with $-y$, you again get the same equation back.

$$x^4 + 5y^2 = 12$$
$$x^4 + 5(-y)^2 = 12$$
$$x^4 + 5y^2 = 12$$

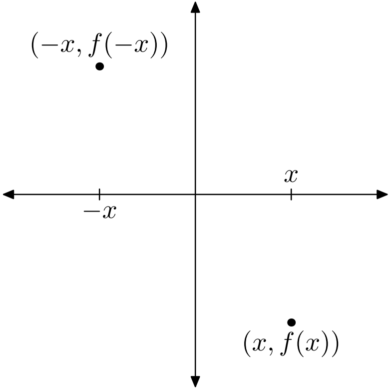
Thus, the graph is also symmetric with respect to the x-axis. Finally, if you replace x with $-x$ and y with $-y$, you get the same equation back.

$$\begin{aligned} x^4 + 5y^2 &= 12 \\ (-x)^4 + 5(-y)^2 &= 12 \\ x^4 + 5y^2 &= 12 \end{aligned}$$

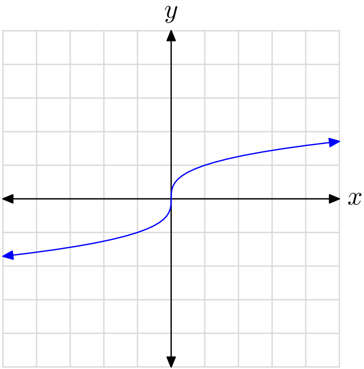
Therefore, the graph is also symmetric with respect to the origin.

□

Solution to Question 9: The function f is odd if and only if $f(-x) = -f(x)$ for all x. That is, if and only if the y-value at $-x$ is the negative of the y of the y-value at x.



Thus, f is odd if and only if the graph of f is symmetric with respect to the origin. Thus, the only odd graph is (c).


y
x

□

Solutions to Exercises

Exercise 1. If $f(x) = x^2 + 2x$, then

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{[(x+h)^2 + 2(x+h)] - [x^2 + 2x]}{h} \\
&= \frac{x^2 + 2hx + h^2 + 2x + 2h - x^2 - 2x}{h} \\
&= \frac{2xh + h^2 + 2h}{h} \\
&= \frac{h(2x + h + 2)}{h} \\
&= 2x + h + 2
\end{aligned}$$

Of course, this argument is valid only if $h \neq 0$ (otherwise, the denominator of $[f(x+h) - f(x)]/h$ is zero).

Exercise 1

Exercise 2(a) If $f(x) = |x+1| - |x-2|$, then we construct a number line analysis as follows.

	$x < -1$	$-1 \le x < 2$	$x \ge 2$
$\lvert x+1 \rvert$	$-x-1$	$x+1$	$x+1$
$\lvert x-2 \rvert$	$-x+2$	$-x+2$	$x-2$
	-1	2	
$x+1$	$-$	$+$	$+$
$x-2$	$-$	$-$	$+$

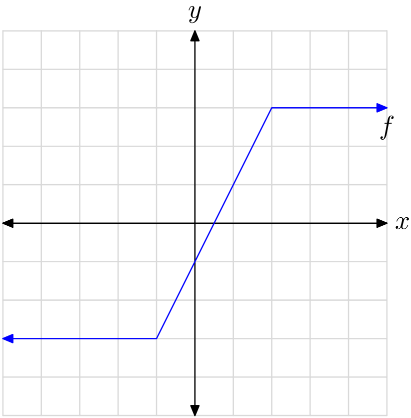
Using the information on the number line, craft a piecewise function.

$$f(x) = \begin{cases} (-x-1) - (-x+2), & \text{if } x < -1 \\ (x+1) - (-x+2), & \text{if } -1 \le x < 2 \\ (x+1) - (x-2), & \text{if } x \ge 2 \end{cases}$$

$$f(x) = \begin{cases} -3, & \text{if } x < -1 \\ 2x-1, & \text{if } -1 \le x < 2 \\ 2, & \text{if } x \ge 2 \end{cases}$$

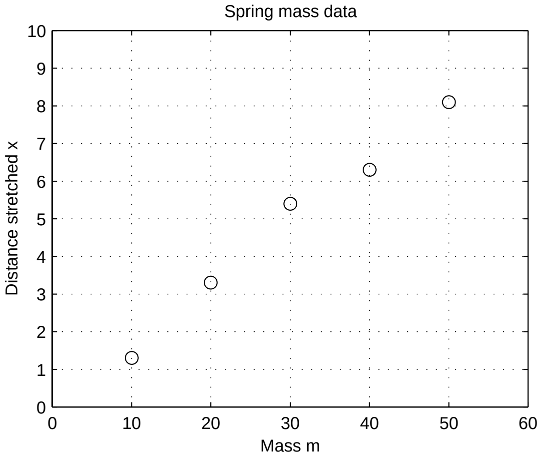
□

Exercise 2(b) Using the piecewise definition in part (a), we draw the graph of f.



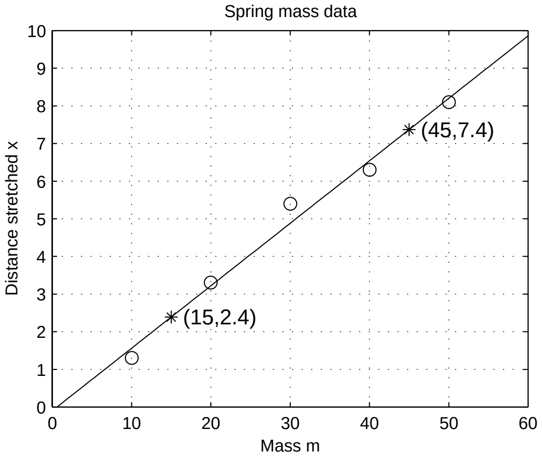
□

Exercise 3(a)


Spring mass data
Distance stretched x
Mass m
0
1
2
3
4
5
6
7
8
9
10
0
10
20
30
40
50
60

□

Exercise 3(b)



Note that two points on the line are estimated to be $(15, 2.4)$ and

$(45, 7.4)$. Using these points, we calculate the slope.

$$m = \frac{7.4 - 2.4}{45 - 15}$$
$$m = 0.1667$$

Use the point slope formula to find the equation of the line.

$$y - y_0 = m(x - x_0)$$
$$y - 2.4 = 0.1667(x - 15)$$
$$y - 2.4 = 0.1667x - 2.5005$$
$$y = 0.1667x - 0.1005$$

Note that we used the point $(15, 2.4)$, but we could just as easily use the other point, $(45, 7.4)$.

Of course, your axes are labelled x and m, not y and x, so it is more appropriate to write

$$x = 0.1667m - 0.1005,$$

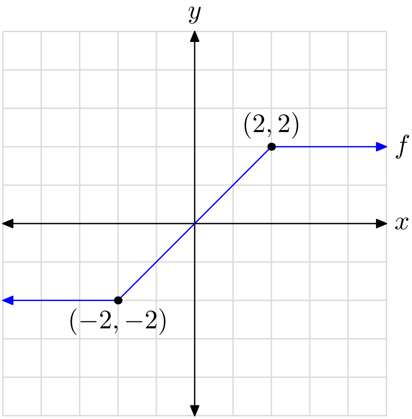
which gives the distance stretched as a function of the mass. □

Exercise 3(c) Using a calculator, the line of best fit is generated.

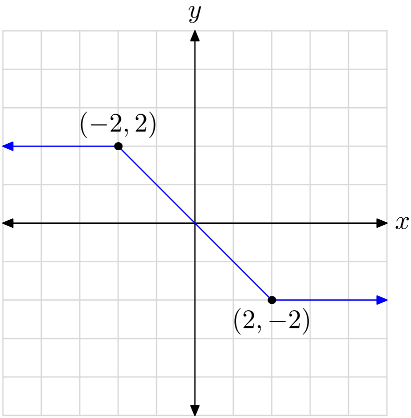
$$y = ax + b$$
$$y = 0.1660x - 0.1000$$

Note that this is quite close to our approximation in part (b). □

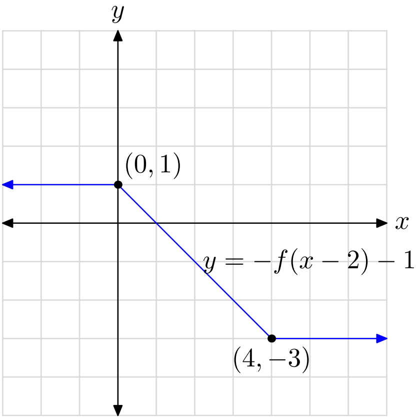
Exercise 4. We begin with the graph of f.



The graph of $y = -f(x)$ is a reflection of the graph of f across the x-axis.



Finally, to draw the graph of $y = -f(x-2) - 1$, take the graph of $y = -f(x)$ and shift it 2 units to the right and 1 unit down.



Exercise 4