The Asteroid: Special Plane Curves

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Figure 1: The man who found the Asteroid.

Double Generation

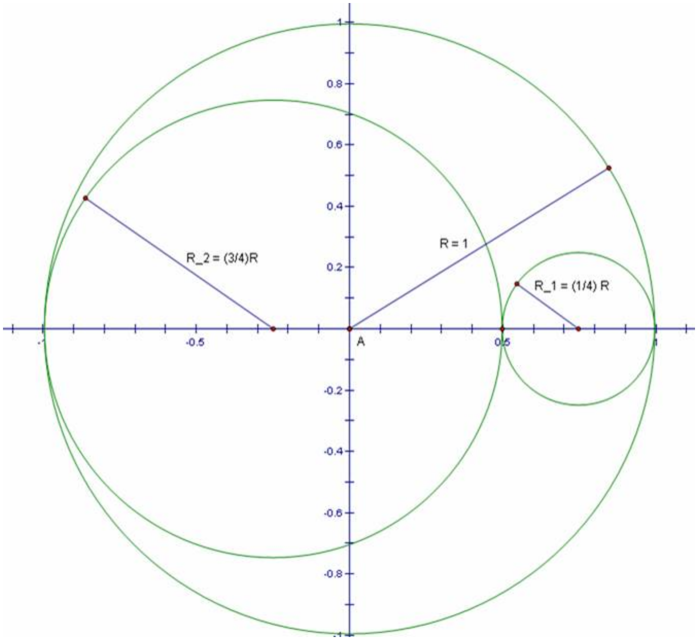

Figure 2: Double Generation of the Asteroid.

Parameterizing the Asteroid Part 1.1

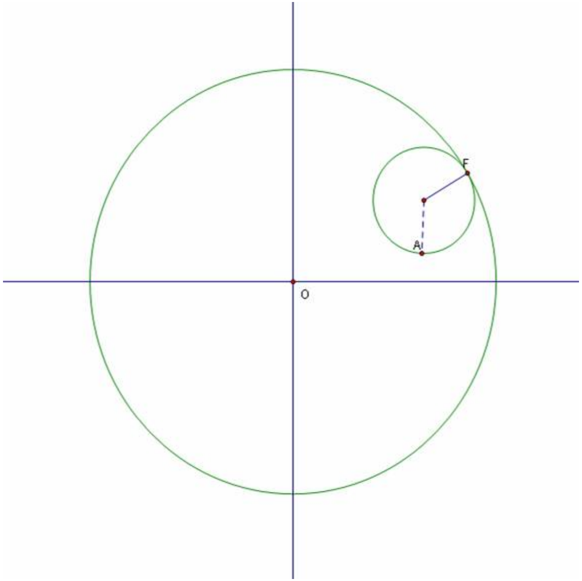

Figure 3: Initial Setup.

Parameterizing the Asteroid 1.2

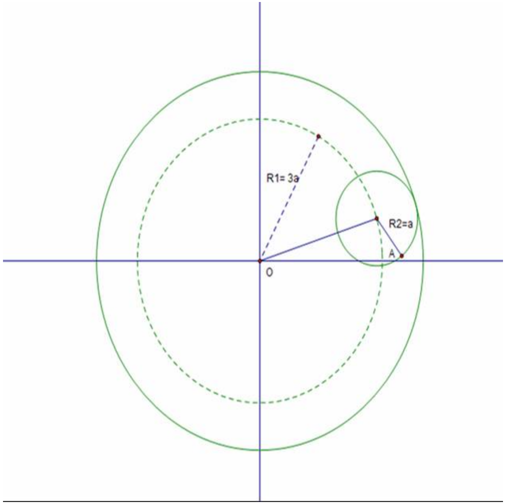

Figure 4: Large Circle Parametrization

$$x_1 = 3a\cos(\theta) \quad (1)$$

$$y_1 = 3a\sin(\theta) \quad (2)$$

Parameterizing the Asteroid Part 2

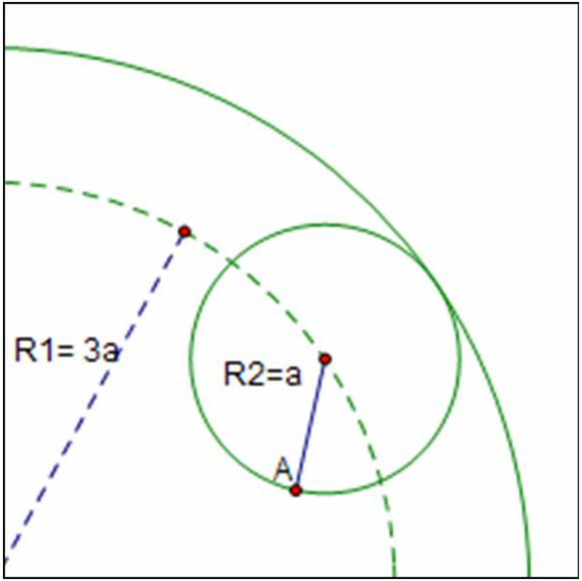

Figure 5: Smaller Circle Parametrization.

$$x_2 = a\cos(\alpha) \tag{3}$$

$$y_2 = a\sin(\alpha) \tag{4}$$

Parameterizing the Asteroid Part 2.2

Parameterized in terms of α, and θ.

$$x = 3a\cos(\theta) + a\cos(\alpha) \tag{5}$$

$$y = 3a\sin(\theta) + a\sin(\alpha) \tag{6}$$

Relating α to θ

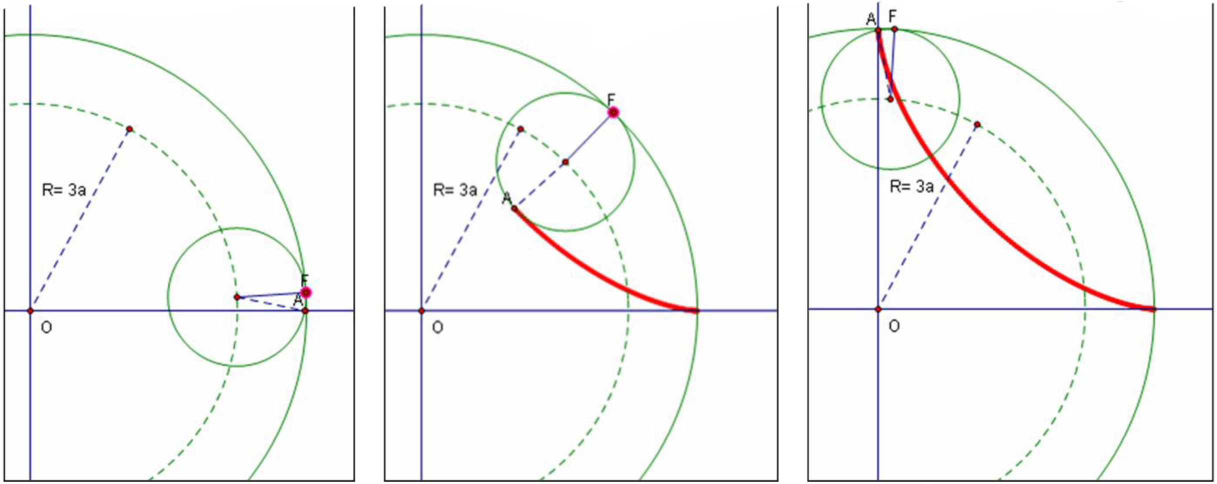

Figure 6: Relating the Angles α to θ.

In terms of theta (θ) only,

$$x = 3a\cos(\theta) + a\cos(-3\theta) \tag{7}$$

$$y = 3a\sin(\theta) + a\sin(-3\theta). \tag{8}$$

Simplifying Parametric Equations, for x

For the X-value equation we have,

$$
\begin{aligned}
x =& 3a\cos(\theta) + a\cos(-3\theta) \\
=& 3a\cos(\theta) + cos(2\theta + \theta) \\
=& 3a\cos(\theta) + a(\cos(2\theta)\cos(\theta) - \sin(2\theta)\sin(\theta)) \\
=& 3a\cos(\theta) + a(\cos^3(\theta) - \sin^2(\theta)\cos(\theta) - 2\sin^2(\theta)\cos(\theta) \\
=& a\cos(\theta)(3 + \cos^2(\theta) - 3\sin^2(\theta)) \\
=& a\cos^3(\theta) + a\cos(\theta)(3 - 3\sin^2(\theta)) \\
=& a\cos^3(\theta) + a\cos(\theta)(3(\cos^2(\theta) + \sin^2(\theta)) - 3\sin^2(\theta)) \\
=& a\cos^3(\theta) + a\cos(\theta)(3\cos^2(\theta)) \\
=& a\cos^3(\theta) + 3a\cos^3(\theta) \\
=& 4a\cos^3(\theta)
\end{aligned}
$$

Simplifying Parametric Equations, for y

And for the Y-value equation

$$
\begin{aligned}
y =& 3a\sin(\theta) + a\sin(-3\theta) \\
=& 3a\sin(\theta) - sin(2\theta + \theta) \\
=& 3a\sin(\theta) - a(\sin(2\theta)\cos(\theta) + \cos(2\theta)\sin(\theta)) \\
=& 3a\sin(\theta) - a(2\sin(\theta)\cos^2(\theta) + \cos^2(\theta)\sin(\theta) - \sin^3(\theta)) \\
=& a\sin(\theta)(3 - 3\cos^2(\theta) + 3\sin^2(\theta)) \\
=& a\sin^3(\theta) + a\sin(\theta)(3(\cos^2(\theta) + \sin^2(\theta)) - 3\cos^2(\theta)) \\
=& a\sin^3(\theta) + a\sin(\theta)(3\sin^2(\theta)) \\
=& a\sin^3(\theta) + 3a\sin^3(\theta) \\
=& 4a\sin^3(\theta)
\end{aligned}
$$

Asteroid Graphed

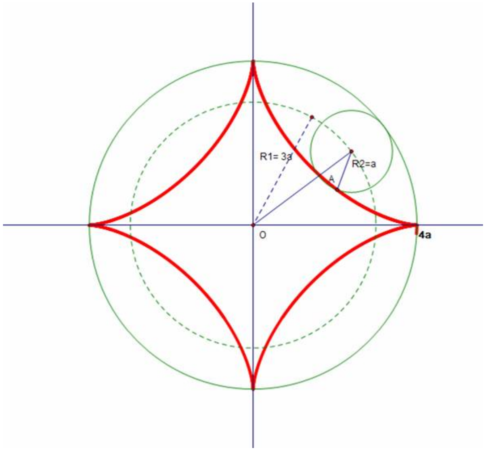

Figure 7: The Asteroid.

$$x = 4a\cos^3(\theta)$$
$$y = 4a\sin^3(\theta)$$

Cartesian Equations

$$x^{2/3} \; + \; y^{2/3} = (4a)^{\frac{2}{3}} \cos^2(\theta) + (4a)^{\frac{2}{3}} \sin^2(\theta)$$
$$= (4a)^{\frac{2}{3}} (\cos^2(\theta) + \sin^2(\theta))$$
$$= (4a)^{\frac{2}{3}}$$

Now replacing 4a with R we get

$$x^{2/3} \; + \; y^{2/3} = R^{\frac{2}{3}} \tag{9}$$

Bibliography

References

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[5] Roemer Picture *From a Unpronoucable Danish Website, http://www.danskekonger.dk/biografi/andre/pict/roemer.jpg*